APPENDIX No. 15.

MEASUREMENTS OF GRAVITY AT INITIAL STATIONS IN AMERICA AND EUROPE.

UNITED STATES COAST AND GEODETIC SURVEY,
ALLEGHENY, PA., *December* 13, 1878.

C. P. PATTERSON,
Superintendent United States Coast and Geodetic Survey, Washington, D. C.

DEAR SIR: I present herewith the first part of my report on the measurement of gravity at initial stations of Europe and America. I here describe the methods employed and communicate the main results of the research. The discussion of the amount and nature of the errors, of the comparison of the present results with those deducible from the experiments of other men, and of the resulting figure of the earth, together with some other matters, are postponed for a subsequent report.

The acceleration of gravity is one of those quantities which it is the business of a geodetic survey to measure. So it has always been considered; and it is usage which fixes the meaning of the word "survey" in its geodetical sense. The geodesist is expected to do more than make a map of the country. He not only determines, for instance, the declination of the magnetic needle, which may be laid down on the chart, but also the other magnetic constants which cannot be so laid down. Were he to omit to determine the total force of magnetism, he would be held by all scientific men to have neglected a part of his duty. Now, in the same relation in which this constant stands to magnetical declination and inclination, just so stands the acceleration of gravity to the latitude and longitude; and by as much more as the latitude and longitude are essential to a survey relatively to the direction of magnetism, by so much more is the measurement of gravity indispensable in comparison with that of the magnetic force. The very first duty of the geodesist—paramount even to the drawing of a map—is the study of the figure of the earth; and an operation of surveying in which this problem was left out of view would neither merit nor receive the name of geodetical work. But it was the variation of gravity with the latitude which first proved the earth's ellipticity; and it may very well yet turn out that this method is the best way of determining it. At all events, the study of local variations of vertical attraction will find an application in the measurement of the level surface of the earth by triangulation. It is, also, quite certain that the solution of some high problems of geology must be facilitated by the integrated soundings which the pendulum virtually makes of the earth's interior.

While the absolute amount of the acceleration of gravity is, no doubt, a geodetical constant necessary to be determined, a very precise knowledge of it can, in the present state of science, find no practical nor theoretical application. What is chiefly of importance is the relative gravity at different places and times. This is also a quantity far easier to measure. To determine the acceleration absolutely, we must accurately measure both an interval of time and a length; to determine it relatively, we have only to carry the same rigid piece of metal from place to place and determine the duration of some phenomenon in which gravity is chiefly concerned. Moreover, we can fix in some measure the probable error of relative determinations. Most of the conditions of the experiments other than the amount of gravity itself are alike at the different stations. If they were precisely so, no constant errors could affect the relative result except in the second order of magnitudes. Now the accidental errors of observations, the only ones which would remain, can readily be determined by the method of least squares. It is not quite true that no conditions other than gravity vary from station to station. The temperature, for example, varies; and in such a manner that an erroneous coefficient of expansion will produce errors in the relative gravity of stations near the poles and near the equator in a constant direction and similar amount; and so will slightly affect the deduced compression of the earth. So an error in the coefficient of atmospheric effect will produce a constantly similar error in the relative gravity of an elevated and a depressed station, and may thus lead to an extremely erroneous value of the absolute modulus of gravitation and of the mean density of the earth. There are, also, various conditions relating to the installation of the instruments which are different at different stations, and which give rise to errors which least squares will fail to detect. Such errors are, however, slight in comparison with those which may affect absolute determinations of gravity, into which the constant errors enter to their full amount. A source of error affecting all modern determinations was lately pointed out by the writer of this paper, which had produced errors in the accepted results amounting to one four or five thousandth part of the quantity measured, and in some cases even to more.

The value of gravity-determinations depends upon their being bound together, each with all the others which have been made anywhere upon the earth. In considering how the necessary connections should be made for our work, it seemed to you, sir, and to Prof. Benjamin Peirce, the consulting geometer, as it did to the writer, that to trust to absolute determinations and to the transportation of meters would be more than hazardous, notwithstanding that such had been the recent practice in continental Europe. Your instructions were accordingly issued for the oscillation of the same pendulum at those fundamental stations of Europe where the chief absolute determinations had been made and whence pendulum-expeditions had set out, and at a station in America which would become the initial one for this continent. Similar action followed on the part of the European surveys; for at the meeting of the International Geodetic Congress in Paris, in 1875, it was resolved, at the suggestion of the writer, that the different states should carry their pendulums to Berlin and swing them in the Eichungsamt there. This has already been done by Switzerland and Austria, and will be done hereafter by every survey which is not willing to sacrifice the solution of a great problem to forms of action based on national exclusiveness. Geodesy is the one science the successful prosecution of which absolutely depends upon international solidarity.

STATIONS.

The stations occupied were as follows:

1. *Geneva.*—The pendulum was swung in the observatory, nearly in the same spot, and on the same wooden stand (see illustration No. 26) used for the purpose by Professor Plantamour, whose advice in regard to the conduct of the experiments was invaluable. His pendulum was set up at the east end and ours at the west end of the main hall. The floor of this hall is (as I remember it) not a meter above the level of the ground which, according to Dufour's map, is 407 meters above the level of the sea. The experiments here were made in August and September, 1875. The station must be pronounced unfavorable for accurate pendulum-work, both from its exposure to changes of temperature and from the slight stability of the asphalt floor and of the tripod.

After the experiments at Geneva the pendulum was injudiciously intrusted to a company in Plainpalais, of whom a vacuum chamber had been ordered. It suffered grave injury in consequence, and was repaired by MM. Brunner, in Paris. In this way the operations at Geneva are completely separated from those at the other stations, and are deprived of much of their value.

2. *Paris.*—M. Wallon, minister of worship and public instruction, authorized pendulum-experiments at the Observatory of Paris. The observations were made in the Hall of the Meridian, in the alcove at the north end. The center of the pendulum-stand was 89 cm. east of the meridian-mark, and opposite the reading 2738 cm. on the meridian-mark. The pendulum-stand stood directly on the floor, and the center of gyration of the pendulum was 29 cm. above the floor. A pendulum-stand, believed to be that of Biot, is by measurement 735 cm. east of the meridian, and opposite 450 cm. on the meridian-mark. Its fulcrum is 171 cm. above the floor. On this subject, M. C. Wolf, astronomer at the observatory, to whose politeness throughout the occupation of the station the writer is much indebted, kindly communicates the following details:

"Borda a opéré dans une salle et contre un mur qui n'existent plus; la hauteur du sol de cette salle au-dessus du niveau de la mer est 67 mètres. L'annuaire du bureau des longitudes donne 65 mètres.

"J'ai pris sur un ancien plan de l'observatoire la position du mur de Borda: son centre était à 14.72 toises (28.69 mètres) de la ligne de la méridienne, à l'est, et au sud à $9^t.89$ ($19^m.28$) du puits de l'observatoire, dont l'axe est sur la méridienne.

"Les coordonées de la station de Biot par rapport à la même ligne et au même puits sont:

Distance à la méridienne	$7^m.34$ est
" au puits suivant la méridienne	$10^m.23$ sud

"Vous étiez placé vous-même presque sur la méridienne, à moins d'un mètre à l'est, et à $12^{m}.70$ du puits vers le nord.

"La hauteur des deux dernières stations [no doubt the floor is meant] au-dessus de celle de Borda est $7^{m}.05$, par conséquent $74^{m}.05$ au-dessus du niveau de la mer."

The level of the ground in the middle of the south face of the observatory above that of the sea is, according to the general staff, 58.8 meters. M. Biot gives as the elevation of his station above the sea 70.25 meters. He erroneously states that Borda's was at the same level. De Freycinet gives 72.28 meters as the altitude of his station. Our experiments at Paris were made during the months of January and February, 1876.

The station at Paris was favorable in regard to the uniformity of temperature, but unfavorable from the excessive instability of the floor.

3. *Berlin.*—The pendulum was swung in Berlin in the large comparison-room of the Imperial Eichungsamt in the garden of the observatory. Plans of this building have been promised by the director, Professor Förster, for this report; but as they have not yet arrived the precise point occupied will be stated in an appendix. The station was very near that of Bessel, but about three meters higher. The experiments were made from April 15 to June 12, 1876. This station was favorable in regard to stillness and stability, but unfavorable owing to changes of temperature. The writer here enjoyed the inestimable advantage of the counsel of the Nestor of geodesy, Genera Baeyer, and also that of great interest in the experiments and attention to everything which could affect the success of them on the part of Professor Förster.

4. *Kew.*—In England, the pendulum was swung at the Kew Observatory in the old deer-park at Richmond, Surrey. The observatory is a meteorological station kept up by a committee of the Royal Society, but is apparently as fundamental a station as there is available in England. The ground is 24 feet above the level of the sea and our pendulum was nearly at the level of the ground. The experiments were made in July, 1876. It proved an excellent place both for steadiness of temperature and for stability Fortunately, the director of the observatory, Mr. Whipple, thoroughly understands the art of oscillating the pendulum, and was most obliging in furthering the investigation in many ways.

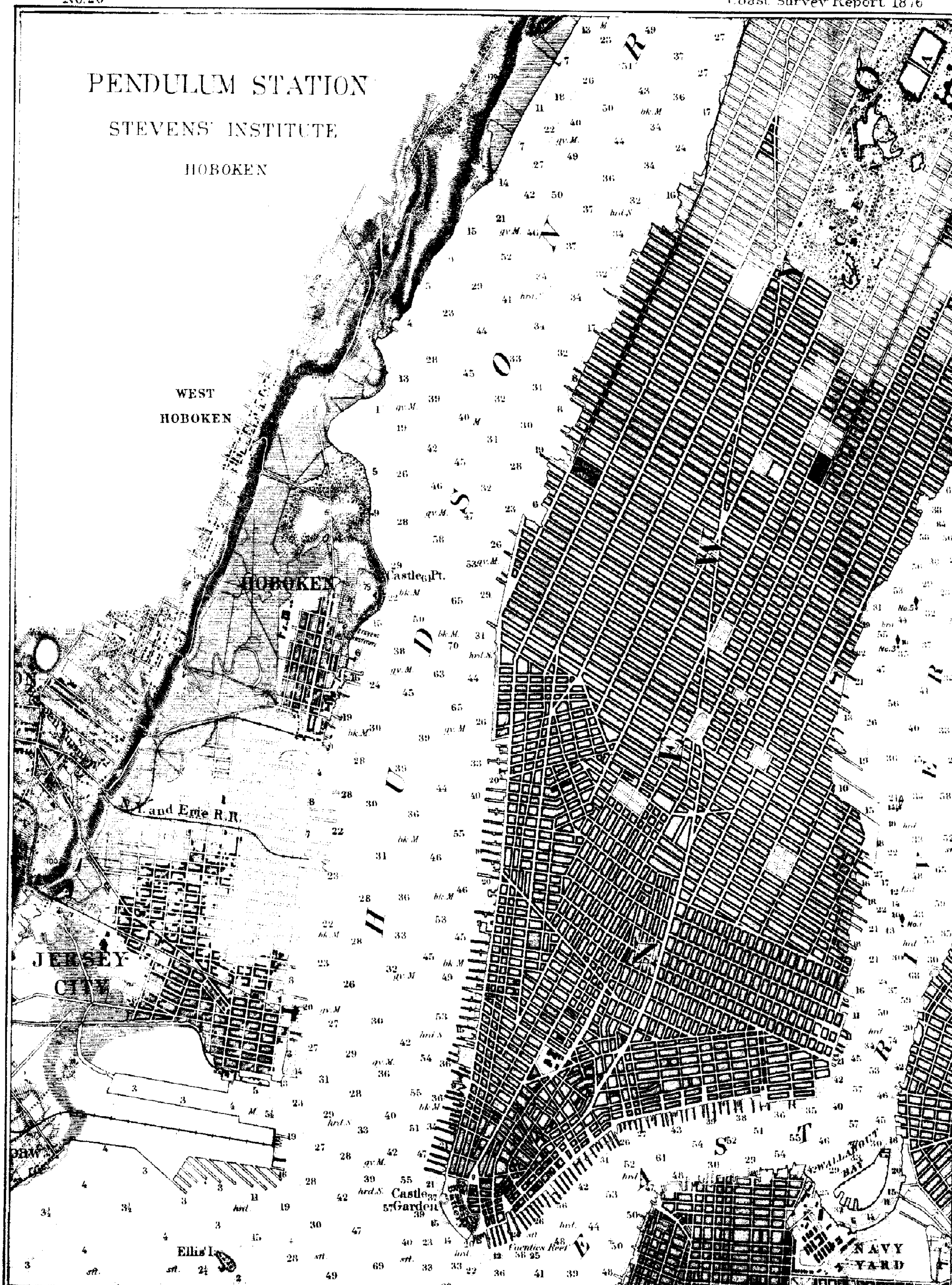
5. *Hoboken.*—The pendulum was swung in a dark chamber in the cellar of the Stevens Institute of Technology. Notwithstanding the kindness of the authorities of the institute in permitting and facilitating the experiments in various ways, and the advantage of the counsels of the eminent physicists resident there, especially those of my friend Professor A. M. Mayer, this station is objectionable from its being situated in a private institution. Otherwise, however, it is a suitable place, except that it is impossible to measure the length of the pendulum there with any accuracy owing to effects of temperature. The latitude of the station is 40° 44′.5, the longitude is 74° 02′ west, and the height above the mean sea-level is about 10 meters. The position in reference to the harbor of New York is shown in the illustration No. 26*a*.

INSTRUMENTS.

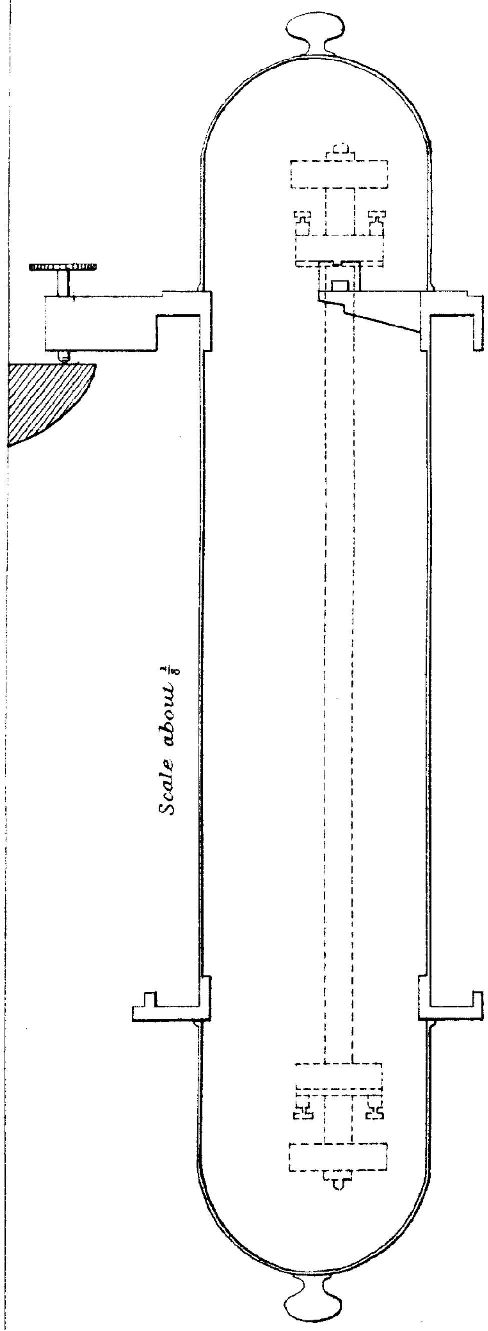
The chief instrument was a Bessel reversible pendulum of one meter length between the knife-edges, admirably constructed by Messrs. Repsold, and nearly an exact copy of the Prussian instrument described by Bruhns in his account of Dr. Albrecht's experiments. One-half this pendulum is shown in illustration No. 27. Its mass is 6308 grams. The dimensions of its principal parts are as follows:

	Centimeter.
Height of cone at end	0.5
Length of little cylinder	1.2
Diameter of little cylinder	1.0
Diameter of collar outside bob	4.9
Height of collar	0.9
Diameter of bob, heavy	11.48
Diameter of bob, light	11.42
Height of bob, heavy	3.25
Height of bob, light	3.18
Diameter of collar below bob	4.78
Height of collar below bob	2.4
Diameter of stem	4.33
Distance (nearest) bob to bob	115.25
Length of knife	9.55

	Centimeter.
Height of knife	1.8
Thickness of knife	1.4
Height from bottom of brass oblong to top of knife	3.4
Thickness of brass piece	1.32
Height of tops of thumb-screws above top of knife	2.05
Breadth of brass for screws	2.35
Length of upper projection on stem below knife	1.4
Diameter of upper projection on stem below knife	5.0
Length of lower projection on stem below knife	1.4
Diameter of lower projection on stem below knife	5.0
Length of hole for tongue	7.5
Breadth	2.6
Thickness of metal	0.18

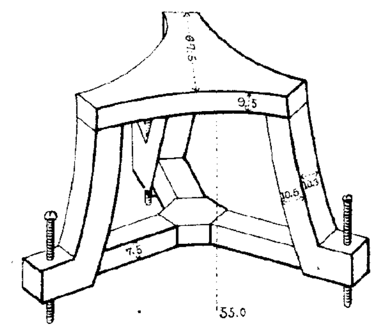

PENDULUM STATION
STEVENS' INSTITUTE
HOBOKEN
WEST
HOBOKEN
HOBOKEN
Castle Pt.
N.Y. and Erie R.R.
JERSEY
CITY
HUDSON
RIVER
Castle Garden
Ellis I.
EAST
RIVER
NAVY
YARD

No. 28



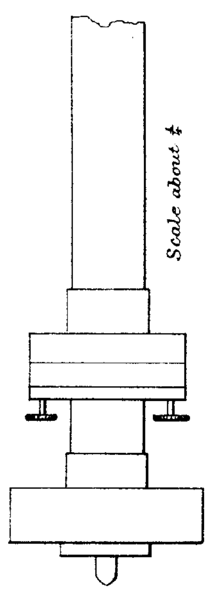
Geneva Pendulum Support.

No. 26



Geneva wooden stand
(measures in centimeters.)

No. 27



Bessel Reversible Pendulum.

In using the vertical comparator which forms a part of the instrument, the intention of the makers seems to have been that the pendulum-meter should be set vertical by means of a spirit-level on a brass straddle provided for the purpose. Instead of this, a plumb-line has been used. This is so made as to be capable of movement in any horizontal direction. The point of support can also be raised or lowered and the whole can be rotated on a vertical axis. In this way, glass scales may be observed, which form part of the plumb; and any error in their verticality is eliminated by reversal. The instrument is first approximately adjusted; the axis of rotation of the comparator is made accurately vertical and the upper microscope is focused on the knife-edge. Then the vertical wire of the lower microscope is made to bisect the plumb-line and the upper microscope is turned about the vertical axis until it also bisects the same vertical line. Afterward, the plumb-line having been brought into the focus of the upper microscope, the lower one is advanced or retracted until it is in focus on the plumb-line below. The microscopes having been so adjusted the meter is adjusted by means of them.

A separate pendulum-support, with a vacuum chamber, constructed under my direction by the Plainpalais company, and called the Geneva support, was used at Hoboken. A vertical section of this support (suppressing various details) is shown on illustration No. 28. The supporting part consists essentially of a solid brass ring with three projections for screw-feet and a tongue to receive the knife-edge cast in one piece. The vacuum chamber is a metallic cylinder covered with bell-glasses at the two ends.* Each screw-foot is furnished with two powerful binding-screws. Only that part of the tongue below the level of the upper surface of the brass ring is a part of the same casting. The upper part is fixed by screws. The instrument is provided with apparatus for raising the pendulum off the knife-edges and letting it down again, another for setting it in motion, supports for thermometers, graduated arc, &c. The graduated arc is divided into thousandths of the radius. Messrs. Stackpole and Brothers, of New York, have made this troublesome graduation with extreme accuracy, upon the arc now in use, and have generously presented it to the survey.

Various other instruments were used which will be described in giving an account of the observations made with them.

OBSERVATIONS OF THE DURATION OF AN OSCILLATION.

The duration of oscillation was ascertained by chronographing 100 transits of the point of the pendulum over the vertical wire of a reading-telescope. Equal numbers of transits were taken from right to left and from left to right, so that any effect from the wire not being at the equilibrium-point was eliminated. At least two seconds elapsed between successive records. The time-keeper was generally a chronometer breaking every two seconds. The chronometer-breaks and signals of pendulum-transits were recorded by the same pen, and interferences were avoided by choosing among the following four methods:

A. 25 transits to the right; 50 to the left; 25 to the right.
B. 50 transits to the left; 50 to the right.
C. 25 transits to the left; 50 to the right; 25 to the left.
D. 50 transits to the right; 50 to the left.

The chronograph is a fillet instrument regulated by a reed and constructed by Breguet. It has three pens. The fillets have been measured to tenths of seconds and the hundredths have been estimated, except in the second readings of the Berlin fillets, where the hundredths were measured with a scale devised and constructed for the purpose.

Observations of the transits were taken when the pendulum reached an arc of oscillation of 2°, $1\frac{1}{2}^\circ$, 1°, and $\frac{1}{2}^\circ$, on each side of the vertical. The object of the intermediate transits at $1\frac{1}{2}^\circ$ and 1° will be seen in the second part of the report, which treats of the errors of the results. By taking the transits at fixed arcs, the condition that the pendulum should be equally affected by the air with heavy end up and heavy end down was secured with certainty. It has been objected that this

* The leakage of the chamber increased the pressure by about that of a tenth of a millimeter of mercury per hour.

plan makes the experiment with heavy end up of too short duration. To remedy this, at Kew the pendulum was swung with heavy end up both before and after every experiment with heavy end down, so that there were twice as many experiments with heavy end up as with heavy end down. The question of the proper arrangement of the experiments in this respect belongs to the theory of the Economy of Research, which is treated in Appendix No. 14.

In order to avoid any possible difference of personal equation in noting the transits when the pendulum was moving rapidly at the beginning and slowly at the end of the experiments, different powers were employed upon the reading-telescope, so that the apparent velocity was about the same at the last as at the first set of transits.

CORRECTIONS.

The observed duration has to receive the following corrections:

1. The correction for the rate of the time-keeper;
2. The correction for amplitude of oscillation;
3. The correction for pressure and temperature of the air;
4. The correction for the expansion of the metal by heat;
5. The correction for the slip of the knives;
6. The correction for the wear of knives;
7. The correction for inequality of knives;
8. The correction for stretching of pendulum by weight of heavy bob when the latter is down;
9. The correction for the flexure of the support;
10. The correction for attractions of sun, moon, and tide;
11. The correction for elevation above sea level.

ON THE CORRECTION FOR THE RATE OF THE TIME-KEEPER.

The stations at Geneva, Paris, and Berlin, being astronomical observatories, the rates of their clocks were determined by the astronomers there.

At Geneva, the transits of the pendulum were registered on a fillet-chronograph which was found there, together with the even seconds of break-circuit chronometer Hutton 202. The second-hand of the sidereal clock was, before and after the experiments of each day, observed through a telescope, and its seconds registered in the same way. The following table, showing the corrections of the clock for 20^h sidereal time of every day on which stars were observed, was kindly communicated by Professor Plantamour:

Date.	Correction, Geneva clock.	Rate.
1875.	*s.*	*s.*
Aug. 27	+ 56.39	
		+ 0.49
31	+ 58.36	
		+ 0.57
Sept. 1	+ 58.93	
		+ 0.60
2	+ 59.53	
		+ 0.65
3	+ 60.18	
		+ 0.63
4	+ 60.81	
		+ 0.70
5	+ 61.51	
		+ 0.62
6	+ 62.13	
		+ 0.68
7	+ 62.81	
		+ 0.78
8	+ 63.59	
		+ 0.67
11	+ 65.60	
		+ 0.71
12	+ 66.31	
		+ 0.45
13	+ 66.76	
		+ 0.60
14	+ 67.36	
		+ 0.70
16	+ 68.77	
		+ 0.56
17	+ 69.33	
		+ 0.76
18	+ 70.09	

These rates have been adopted in the calculations.

The comparisons between the chronometer and clock, and the rate of the chronometer as deduced from them for each day's observations, will appear in their proper place in the full account of the pendulum-work in my next report.

At Paris the pendulum-transits were chronographed together with the beats of the meridian clock. The corrections and rates of this clock were supplied by the observatory in two lists, which are here appended. In transmitting the second, M. Wolf makes the following remarks:

"J'ai examiné avec attention la marche diurne de la pendule sur laquelle vous avez observé et compté la seconde. Sa marche diurne normale, résultant d'un très-grand nombre d'observations faites à différentes époques de l'année, varie de $+0^s.05$ à $+0^s.07$. C'est en effet ce que nous retrouvons pour le mois de février, comme il résulte du tableau ci-dessous. Mais en janvier, il est évident qu'il s'est produit une perturbation dont je ne puis deviner la cause. Quoiqu'il en soit, il me paraît nécessaire d'admettre pour la marche diurne

$+0^s.20$ du 24 janvier au 2 février
$+0\ .07$ du 2 février au 24.

"Voici en effet le tableau des corrections observées en regard desquelles je place les corrections calculées avec les marches précédentes: [Here M. Wolf inserts the second list given below.] Ces corrections se rapportent toutes à 9^h temps moyen à ¼ d'heure près.

"La seule différence trop forte est celle du 4 février; la correction de ce jour a été obtenue par un observateur différent de celui qui a déterminé toutes les autres.

"Quelle que soit d'ailleurs la marche que vous admettiez, il me semble que l'erreur qui peut en résulter sur la durée de la seconde sidérale, c'est toujours de beaucoup inférieure à celle qui résulte de la mesure des oscillations de votre pendule."

Corrections of the Paris meridian-clock.

FIRST LIST.

Date.		Temps sidéral.	Lunette de Gambey. M. Léon.	Cercle méridien. M. Périgord.	Cercle méridien. M. Folain.
1876.		*h. m.*	*s.*		
Jan.	5	3 3	+12.02		
	10	3 40	+12.46		
	15	7 0	+13.37		
	24	4 40	+15.09	+15.35	
	25	7 30	+15.19	+15.60	
	26			+15.85	
	27	4 25	+15.83		
	28	5 40	+15.88	+16.22	
	29	5 0	+15.91	+16.35	
	31	6 3	+16.41	+16.75	
Fév.	1			+17.03	
	4				+17.59
	7			+18.50	
	9			+18.50	
	10				+18.75
	11			+18.75	
	12			+18.82	
	14			+18.46 :	
	15			+19.28	

SECOND LIST.

Date.		Obs.	Calc.
1876.		*s.*	
Jan.	24	+15.35	+15.40
	25	+15.60	+15.60
	26	+15.85	+15.80
	28	+16.22	+16.20
	29	+16.35	+16.40
	31	+16.75	+16.80
Fév.	1	+17.05	+17.00
	2	+17.32	+17.19
	4	+17.62	+17.33
	7	+17.53	+17.54
	9	+17.57	+17.68
	10	+17.72	+17.75
	11	+17.76	+17.82
	12	+17.87	+17.89
	22	+18.50	+18.59
	23	+18.64	+18.66

I cannot presume to review the judgment of the Paris Observatory upon the errors of its own observers, and the rates given in the last column are therefore adopted. Illustration No. 29 exhibits the concordance of the observations with these rates.

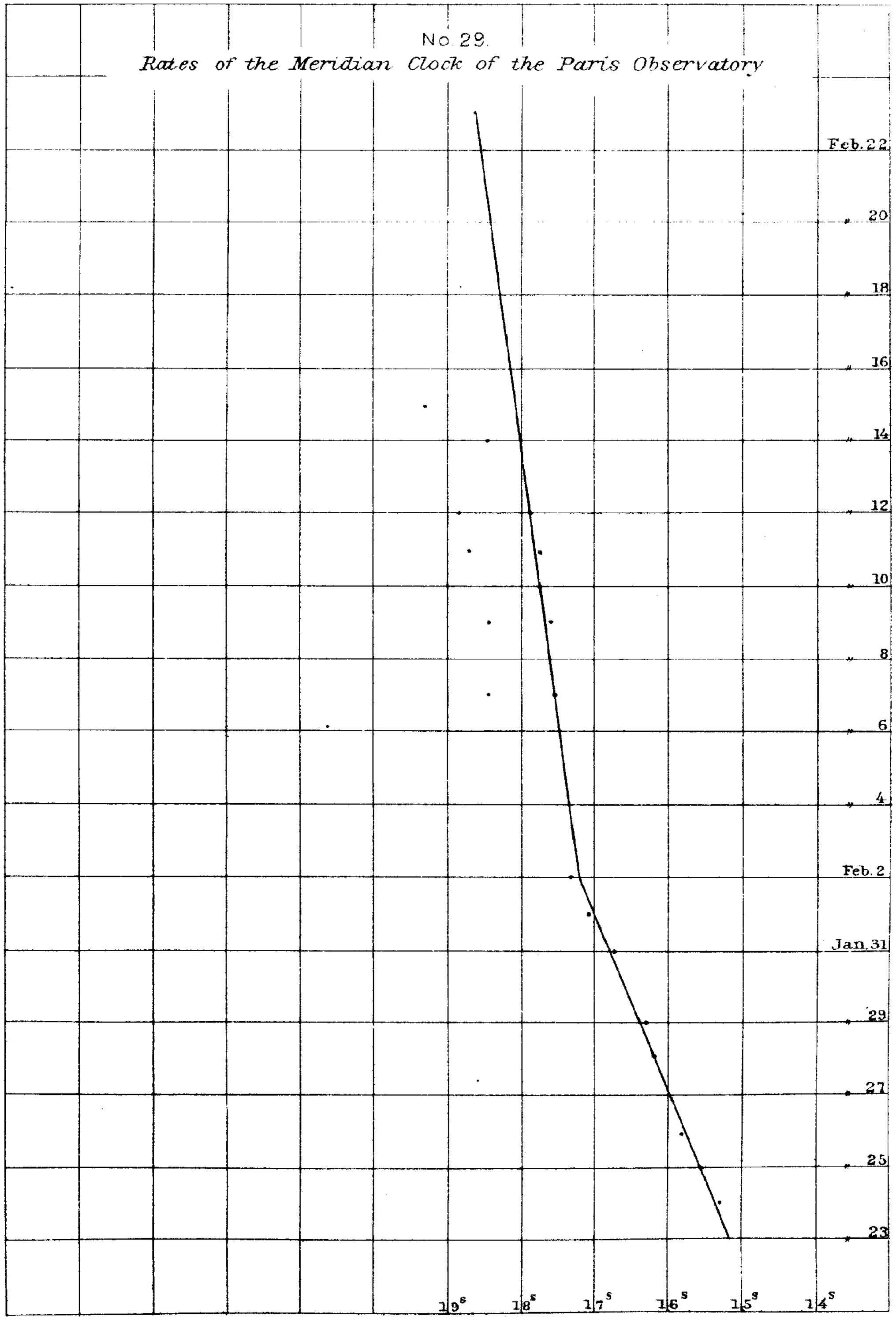
At Berlin the clock, whose seconds were chronographed with the signals of pendulum-transits, was a subsidiary one in the observatory, designated as Serffert. Every day, before and after the experiments, it was put on to one of the observatory chronographs with the clock of the observatory. The corrections were communicated in the following list:

Stand und Gang von Serffert, 1876, *April* 15–*Juni* 16.

1876.		T_3−S	Δ*h*. T_3	Δ*h*. S	Tägl. G. S.
		s.	*s.*		
April	15.63	26.29			
	16.13	26.58			
	.63	26.87			
	17.13	27.14	− 0.17	+26.97	
	.63	27.33			
	18.13	27.48			−0.23
	.63	27.61			
	19.13	27.75	− 1.25	+26.50	
	.63	27.81			
	20.13	27.89			−0.17
	.63	27.95			
	21.13	28.03			
	.63	28.06	− 1.98	+26.08	
	22.13	28.14			
	.63	28.24			
	23.13				
	.63	28.39			−0.03
	24.13				
	.63	28.66			
	25.13	28.90			
	.63	29.09			
	26.13	29.33	− 3.37	+25.96	
	.63	29.52			+0.04
	27.13	29.73	− 3.73	+26.00	
	.63	29.90			
	28.13	30.05			
	.63	30.16			−0.19
	29.13	30.29			
	.63	30.44			
	30.13	30.61			
	.63	30.69	− 5.36	+25.33	
Mai	1.13	30.81			−0.16
	.63				
	2.13	31.03	− 5.94	+25.09	
	.63				
	3.13	31.30			+0.02
	.63	31.53			
	4.13	31.82	− 6.69	+25.13	
	.63	32.15			

No 29.

Rates of the Meridian Clock of the Paris Observatory



Stand und Gang von Serffert, 1876, *April* 15–*Juni* 16—Continued.

1876.		T_3—S	Δh. T_3	Δh. S	Tägl. G. S.
		s.	s.		
Mai	5.13	32.41			
	.63	32.64			+ 0.16
	6.13	32.92			
	.63	33.15			
	7.13	33.31			
	.63	33.46	− 7.77	+ 25.69	(1)
	8.13	33.71			
	.63	33.98			
	9.13	34.28			+ 0.28
	.63	34.57			
	10.13	34.88			
	.63	35.15	− 8.62	+ 26.53	
	11.13	35.49			+ 0.35
	.63	35.78			
	12.13	36.06	− 9.00	+ 27.06	
	.63				(2)
	13.13	36.59			
	.63	36.89			+ 0.11
	14.13	37.19			
	.63	37.41			
	15.13	37.70			
	.63	37.98	−10.54	+ 27.44	
	16.13	38.34			
	.63	38.65			
	17.13	38.96			
	.63	Es wurde eine kleine Verbesserung der Kompensation ausgeführt.			
	18.13				
	.63				
	19.13				
	.63				
	20.13	10.58	−12.26	− 1.68	
	.63	10.45			− 0.39
	21.13	10.38			
	.63	10.24	−12.50	− 2.26	
	22.13	10.08			− 0.53
	.63	9.88			
	23.13	9.69	−12.74	− 3.05	
	.63				− 0.65
	24.13	9.32			
	.63	9.13	−13.15	− 4.02	
	25.13	8.96			
	.63	8.81			− 0.64

Anm. 1). Quecksilbertropfen höher gestellt.

Anm. 2). Die Untersuchung der Gänge 1876, Febr. 25, bis Mai 15, ergiebt

$$\frac{d\,(\text{Täglich. Gang})}{d\ \text{Barom.}} = +\ 0^{s}.018 \text{ für } +1^{mm}.$$

Stand und Gang von Serffert, 1876, *April* 15–*Juni* 16—Continued.

1876.		T_3-S	$\Delta h.\ T_3$	$\Delta h.\ S$	Tägl. G. S.
		s.	*s.*	*s.*	
Mai	26.13	8.69			
	.63	8.54	−13.85	− 5.31	
	27.13	8.40			
	.63	8.27			
	28.13	8.12			− 0.60
	.63	8.03			
	29.13	7.92			
	.63	7.80	−14.91	− 7.11	
	30.13	7.76			
	.63	7.70			
	31.13	7.65			
	.63	7.54			
Juni	1.13				− 0.44
	.63	7.35			
	2.13	7.24			
	.63	7.06			
	3.13	6.94			
	.63	6.72	−16.01	− 9.29	
	4.13	6.49			
	.63	6.31			− 0.50
	5.13	6.16			
	.63	5.96	−16.25	−10.29	
	6.13	5.74			
	.63	5.46			− 0.49
	7.13	5.22			
	.63	4.96	−16.24	−11.28	
	8.13	4.81			
	.63				
	9.13	4.42			
	.63	4.23			− 0.42
	10.13	4.08			
	.63	3.95			
	11.13	3.85			
	.63	3.70	−16.66	−12.96	
	12.13	3.52			
	.63	3.35			
	13.13	3.23			
	.63	2.98			
	14.13	2.78			− 0.50
	.63	2.56			
	15.13	2.36			
	.63	2.17			
	16.13	1.97	−17.19	−15.22	
	.63	1.80			

An independent set of comparisons between Serffert and the normal clock of the observatory is given below, the differences between it and the previous table being usually very slight. In the table below the column headed O gives T_3—S as directly observed, and I as interpolated from the table just given:

Date.	T_3—S, O	T_3—S, I	O—I
1876.	*s.*	*s.*	*s.*
April 19.94	27.88	27.86	+ .02
21.01	28.01	28.01	.00
23.91	28.45	28.47	— .02
24.12	28.50	28.52	— .02
24.89	28.77	28.78	— .01
25.12	28.89	28.90	— .01
25.89	29.22	29.21	+ .01
26.11	29.31	29.32	— .01
27.90	30.00	29.98	+ .02
28.12	30.06	30.05	+ .01
28.88	30.15	30.22:	— .07
29.12	30.30	30.29	+ .01
29.89	30.57	30.53:	+ .04
30.12	30.62	30.61	+ .01
May 1.90	31.03	30.98:	+ .05
2.12	31.03	31.03	.00
2.89	31.25	31.24	+ .01
3.92	31.83	31.70:	+ .13
4.11	31.81	31.81	.00
4.89	32.29	32.29	.00

There are but four cases of discordance, all occurring where the comparison of this set was taken midway between two of the former set.

On comparing the daily rates of the two clocks designated as T_3 and S, during the time of the pendulum-observations, it will be found that the latter went as well as the former, as shown in the following table:

Date.	Daily rate, T_3	Daily rate, S	Diff. from mean T_3	Diff. from mean S
1876.	*s.*	*s.*	*s.*	*s.*
Apr. 19–21	— .49	— .17	— .12	— .07
21–26	— .31	— .03	+ .06	+ .07
26–27	— .36	+ .04	+ .01	+ .14
27–30	— .47	— .19	— .10	— .05
May 0– 2	— .23	— .16	+ .14	— .06
2– 4	— .38	+ .02	— .01	+ .12

For the observations at Kew, the time was kept by the four chronometers—

Hutton 202, sidereal;
Dent 2171, mean solar;
Frodsham 3525, mean solar;
Frodsham 3474, mean solar.

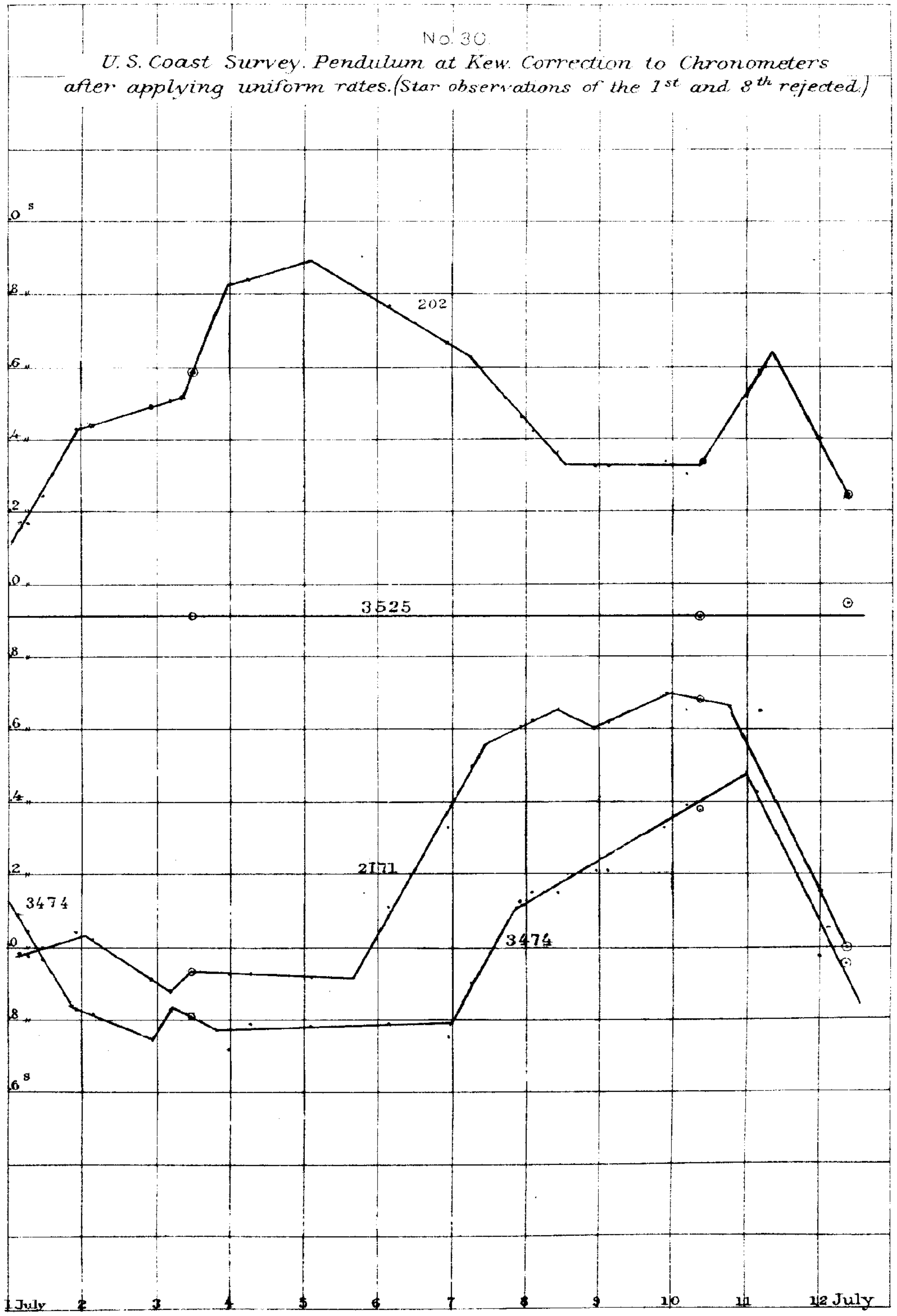
The solar chronometers were all compared with No. 202, by coincidence of beats, at least once a day, from June 30 to July 12, inclusive, but the comparison of June 30 was rejected because the chronometers had not then been in place long enough to acquire uniform rates. The following table gives the results of these comparisons, the excess of each chronometer over No. 3525 being taken.

Kew.—Comparison of chronometers [2171, 3525, *and* 3474 *are reduced to sidereal time*].

	Excess over Frodsham 3525 of—		
	Hutton 202.	Dent 2171.	Frodsham 3474.
1876.	*m.* *s.*	*m.* *s.*	*m.* *s.*
June 30.13	+ 3 8.61	+ 0 1.30	+ 0 20.87
July 1.12	2 57.94	0.14	19.79
1.26	56.43	+ 0.02	19.71
1.47	54.15	— 0.12	19.57
1.92	49.42	0.43	19.31
2.10	47.49	0.57	19.25
2.94	38.42	1.34	18.96
3.16	35.99	1.53	18.97
3.47	32.73	1.70	18.88
3.98	27.40	2.10	18.65
4.25	24.47	2.31	18.65
5.09	15.30	2.96	18.41
6.15	3.60	3.58	18.13
6.92	1 55.17	3.94	17.90
7.25	51.44	4.03	17.94
7.91	44.10	4.42	17.99
8.12	41.77	4.56	17.97
8.45	38.16	4.79	17.88
8.95	32.63	5.22	17.80
9.12	30.79	5.33	17.76
9.91	22.19	5.85	17.67
10.18	19.17	6.11	17.65
10.39	16.87	6.22	17.59
11.11	9.16	6.93	17.43
11.99	0 59.47	7.98	16.73
12.37	+ 0 55.21	— 0 8.43	+ 0 16.63

In the transit-observations, a Dollond transit belonging to the observatory was used. These observations were taken under very great disadvantages. The transit-room, which was not in the observatory building, but located just outside the east wall, was not large enough to allow a place for the chronograph, so that the observer (Mr. H. Farquhar) was obliged frequently to leave this room in order to see to the recording of his signals. The instrument was one of small magnifying power, with a long interval between the wires (about 26^s at the equator), so that a special journey had to be made during the passage of the high northern stars, to wind up the chronograph and see that the pens were working. It was, moreover, in not very good condition, nor very steady. For these reasons, though many stars were observed every fair evening, great difficulty was found in getting a satisfactory correction from them. In illustration No. 30, which shows the correction to each of the chronometers after applying a uniform rate, the comparisons made at the time of the transit-observations are distinguished by circles, and only the observations of the 3rd, 10th, and 12th are taken, it having been found impossible to bring the separate stars, in the observations of the 1st and the 8th, into concordance.

Chronometer 3525 has been considered as having one uniform rate of + $0^s.23$ per day, from the beginning to the end of the pendulum-experiments.


No. 30.
U. S. Coast Survey. Pendulum at Kew. Correction to Chronometers after applying uniform rates. (Star observations of the 1st and 8th rejected.)
202
3525
2171
3474
3474
1 July
12 July

This supposition gives as the rate of No. 202, the chronometer used, during each day when the pendulum was swung—

1876.	Daily rate of 202.
	s.
July 1	+ 10.99
2	11.10
3	11.05
4	11.01
7	11.24
8	11.35
9	11.15
10	11.26

These rates have been adopted in the calculation.

At Hoboken the time was observed with the United States Coast Survey portable transit No. 5, by Simms. It was kept by break-circuit chronometers—

Hutton	202	(sid. time),
Bond	380	(sid. time),
Negus	1589	(sid. time),
Negus	1591	(mean time),

and a part of the time by

Bond	387	(sid. time).

For the observations of April and May, 1877, which were intended to show the difference in the time of oscillation on a stiff and on a flexible stand, and for the regular swings of June, 1877, the time-observations were commenced by Subassistant Edwin Smith on March 24, and were continued until June 29, inclusive. The chronometer used in the observations of time was usually Negus 1589. The following table shows the residuals of the time-observations. The reduction was made by field-methods, by Subassistant Smith:

	P.D.	Z.D.	March. 1	24	April. 6	11	12	17	23	24	25	May. 2	3	7	12
	°	°													
δ Geminorum	68	+19	+.04 W.	..	..	..	..	..	..	..	..	..	..	..	..
P. VII, 67	21	−28	+.01 W.	..	..	..	..	..	..	..	..	..	..	..	..
α Geminorum	58	+ 9	+.20 W.	..	..	..	rej.	..	..	..	..	..	..	..	..
α Canis Minoris	84	+35	−.28 W.	..	..	..	−.03 E.	..	..	..	..	..	..	.	..
β Geminorum	62	+12	+.02 E.	..	..	..	+.14 E.	..	..	..	..	..	..	..	..
3 H Ursæ Majoris	21	−28	+.02 E.	..	..	..	..	..	..	..	..		..	..	..
κ Cephei (L. C.)	−13	−62	..	..	..	..	−.10 W.	..	..	..	..	..	..	..	..
ε Hydræ	83	+34	..	..	..	+.04 W.	+.08 W.	−.10 W.	..	..	..	..	..		..
ι Ursæ Majoris	41	− 8	..	+.09 W.	..	.00 W.	−.09 W.	+.02 W.	..	..	..	..	..	..	..
κ Cancri	79	+30	..	..	−.03 E.	−.04 W.	..	+.04 W.	..	..	..	..	..	..	..
1 H Draconis	8	−41	..	−.09 W.	+.03 E.	..	..	..	−.15 E.	−.24 W.	−.04 E.	..	..	..	..
α Hydræ	98	+49	..	..	..	..	..	+.07 E.	−.16 E.	+.01 W.	−.08 E.	+.07 W.	..	..	..
d Ursæ Majoris	20	−30	..	..	−.05 W.	..	..	.00 E.	+.15 E.	+.25 W.	−.02 E.	..	..	..	..
θ Ursæ Majoris	38	−11	..	+.01 W.	..	..	..	.00 E.	+.10 E.	..	+.02 E.	+.06 W.	..	..	..
ε Leonis	66	+16	..	−.05 E.	−.04 W.	..	..	..	+.01 W.	−.03 E.	+.07 W.	−.13 W.	−.03 E.	..	..
μ Leonis	63	+14	..	+.02 E.	+.07 W.	..	..	..	+.02 W.	+.03 E.	+.06 W.	+.05 E.	+.04 E.	..	..
α Leonis	77	+28	..	..	..	..	..	..	..	..	..	+.04 E.	+.02 W.	..	+.03 E.
32 Ursæ Majoris	24	−25	..	..	..	..	..	..	..	..	..	−.08 E.	.00 W.	..	..
γ_1 Leonis	70	+20	..	..	..	..	..	..	..	..	..	..	−.02 W.	+.03 W.	−.03 E.
9 H Draconis	14	−36	..	..	..	..	..	..	..	..	..	..	..	−.01 W.	..
ρ Leonis	80	+31	..	..	..	..	..	..	..	..	..	..	..	+.02 W.	.00 W.
l Leonis	79	+30	..	..	..	..	..	..	..	..	..	..	..	−.06 E.	+.03 W.
α Ursæ Majoris	28	−22	..	..	..	..	..	..	..	..	..	..	..	+.12 E.	.00 W.
δ Leonis	69	+20	..	..	..	..	..	..	..	..	..	..	..	−.10 E.	..

	P. D.	Z. D.	May. 14	16	26	28	29	June. 1	11	13	14	16	17	18	19
	°	°													
δ Crateris	104	+55	..	−.01 E.	..	..	..	..	..	..	..	..	..		..
τ Leonis	[illegible]	−37	..	+.03 E.	..	..	..	..	..	..	..	..	..	..	..
λ Draconis		−29	..	−.14 E.	..	..	..	..	..	..	..	..	..	..	..
υ Leonis	9	+41	..	+.03 W.	..	..	..	..	..	..	..	..	..	..	..
β Leonis	75	+25	..	−.04 W.	..	..	..	..	..	..	..	..	..	..	..
γ Ursæ Majoris ..	36	−14	..	+.16 W.	..	..	..	..	..	..	..	..	..	..	..
η Virginis	90	+41	−.04 W.	..	..	..	..	..	..	..	..	..	..	..	..
κ Draconis	20	−40	−.02 W.	..	..	−.03 E.	..	..	..	..	..	..	..	..	..
32 H_2 Camelop	6	−43	..	..	..	..	+.02 W.	..	..	..	..	..	..	..	..
12 Canum ven	51	+ 2	..	..	..	+.03 E.	+.07 W.	..	..	+.03 E.	..	..	..	..	..
θ Virginis	95	+46	..	..	..	.00 E.	−.02 W.		..	−.03 E.	..	..	..	..	..
α Virginis	101	+46	+.04 W.	..	..	+.02 W.	−.09 E.	..	..	+.03 W.	..	..	..	..	..
ζ Virginis	90	+41	−.06 E.	..	..	−.02 W.	+.02 E.	..	..	.00 W.	..	+.01 E.	..	..	..
η Ursæ Majoris ..	40	− 9	+.02 E.	..	..	..	..	+.08 E.	−.06 W.	−.02 W.	−.04 W.	−.03 E.	..	+.08 E.	..
η Bootæ	71	+22	+.04 E.	..	..	..	..	−.08 E.	+.01 W.	..	+.03 W.	−.02 E.	..	.00 E.	..
α Draconis	25	−24	..	..	−.03 W.	..	..	.00 E.	..	..	rej.	−.01 E.	..	−.09 E.	..
α Bootæ	70	+21	..	..	+.06 W.	..	..	+.01 W.	+.04 E.	..	+.01 E.	..	+.06 W.	+.03 W.	+.10 W.
θ Bootæ	38	−12	..	..	+.06 W.	..	..	..	..	..	.00 E.	+.01 W.	−.06 W.	..	.00 W.
5 Ursæ Minoris ..	14	−35	..	..	..	..	..	−.03 W.	.00 E.	..	−.01 E.	.00 W.	.00 W.	..	−.02 W.
ε Bootæ	62	+13	..	..	−.08 E.	..	..	..	+.03 E.	..	..	+.02 W.	+.05 E.	..	−.08 E.
α_2 Libræ	106	+56	..	..	..	..	..	..	..	..	..	..	−.02 E.	..	−.03 E.
β Ursæ Minoris ..	15	−34	..	..	..	..	..	..	..	..	..	..	−.02 E.	..	+.03 E.

	P. D.	Z. D.	June. 20	22	23	25	27	29
	°	°						
α Virginis	101	+46	..	..	+.03 E.	..	..	..
ζ Virginis	90	+41	..	..	+.05 E.	..	..	..
η Ursæ Majoris ..	40	− 9	−.04 E.	..	−.08 E.	..	..	−.03 W.
η Bootæ	71	+22	+.03 E.	..	.00 W.	..	..	+.02 W.
α Draconis	25	−24	.00 E.	+.07 W.	.00 W.	..	−.16 E.	.00 E.
α Bootæ	70	+21	+.03 W.	−.04 W.	..	..	.00 E.	−.02 E.
θ Bootæ	38	−12	..	−.02 W.	..	..	+.10 E.	+.01 E.
5 Ursæ Minoris ..	14	−35	.00 W.	−.06 E.	..	..	.00 W.	..
ε Bootæ	62	+13	+.01 W.	+.01 E.	..	..	.00 W.	..
α_2 Libræ	106	+56	..	+.06 E.	..	..	..	..
β Ursæ Minoris ..	15	34	..	..	..	..	..	..
μ_1 Bootæ	52	+ 3	..	..	..	−.06 W.	..	..
γ_2 Ursæ Minoris ..	18	−32	..	..	..	+.02 W.	..	..
α Coronæ Bor	63	+14	..	..	..	+.06 W.	..	..
α Serpentis	83	+34	..	..	..	+.06 E.	..	..
ε Serpentis	85	+36	..	..	..	−.05 E.	..	..
ζ Ursæ Minoris ..	12	−37	..	..	..	−.01 E.	..	..

Table of instrumental constants.

1877.		Level constant.		Azimuth.	Collimation.
March	1	+0.02 W.	+0.06 E.	−5.04	−0.36
	24	+0.01	+0.10	−2.40	0.00
April	6	−0.11	−0.03	−0.42	−0.17
	11	−0.02		+0.13	+0.20
	12	−0.04	+0.03	−0.20	+0.20
	17	−0.12	−0.04	+0.07	+0.22
	23	−0.10	−0.01	+0.04	+0.20
	24	−0.11	0.00	+0.05	+0.31
	25	0.00	+0.04	+0.22	+0.03
May	2	+0.16	+0.24	+0.71	+0.04
	3	+0.07	+0.08	+0.46	0.00
	7	−0.04	0.00	+0.59	+0.02
	12	+0.08	+0.10	+0.92	0.00
	14	+0.09	+0.10	−0.45	0.00
	16	+0.07	+0.13	−0.63	0.00
	26	+0.35	+0.34	+0.72	0.00
	28	+0.31	+0.36	+0.60	+0.04
	29	+0.36	+0.37	+0.47	−0.08

Table of instrumental constants—Continued.

1877.		Level constant.		Azimuth.	Collimation.
June	1	+0.38	+0.44	+0.66	0.00
	11	+0.26	+0.25	+0.56	0.00
	13	+0.22	+0.30	+0.55	—0.03
	14	+0.20	+0.28	+0.54	0.00
	16	+0.32	+0.35	+2.43	—0.03
	17	+0.44	+0.48	+0.33	+0.02
	18	+0.29	+0.38	+0.41	0.00
	19	+0.48	+0.50	+0.43	0.00
	20	+0.36	+0.38	+0.43	—0.05
	22	+0.54	+0.46	+0.30	—0.06
	23	+0.40	+0.40	+0.45	—0.02
	25	+0.31	+0.43	+0.31	+0.05
	27	+0.35	+0.54	+0.52	+0.06
	29	+0.43	+0.50	+0.47	+0.02

The chronometers were compared on the chronograph daily, and also before and after all pendulum-observations. The following table shows the results of these comparisons:

Mean time. Date.		Bond 387— Negus 1589		Bond 380— Negus 1589		Hutton 202— Negus 1589		Negus 1591 (red. to sid.)— Negus 1589	
1877.		*m.*	*s.*	*m.*	*s.*	*m.*	*s.*	*m.*	*s.*
March	1.41	+0	33.61	+0	25.06	+10	58.85	+0	49.34
	24.42	1	19.40	—0	6.55		57.00	1	8.38
	30.98		31.80		14.53	11	2.70		13.30
April	1.20		33.93		15.82		3.20		14.22
	6.38		43.13		21.46		6.36		17.10
	7.00		44.28		22.11		6.87		17.42
	7.20		44.63		22.30		7.10		17.47
	7.96		45.93		23.09		8.69		17.86
	8.05		46.10		23.21		8.80		17.90
	12.27		54.10		27.90		15.80		19.79
	13.06		55.76		29.81		17.09		18.48
	14.08		56.97		30.95		18.45		19.75
	16.18	2	2.41		35.84		24.60		18.13
	17.31		4.73		32.65		27.02		23.72
	19.03		8.77		33.23		31.37		24.22
	20.98		11.10		36.20		34.17		23.87
	23.28		15.99		38.69		38.51		24.64
	24.33		18.03		39.90		40.10		26.46
	25.20		19.80		40.41		41.45		18.53
	25.28		19.90		40.41		41.50		25.12
	26.26		21.74		41.48		42.90		25.20
	27.18		23.59		42.29		44.19		25.54
	28.04		24.97		43.06		45.11		25.65
May	1.21		29.22		46.95		47.40		24.90
	2.30		30.59		48.57		48.51		24.63
	3.29		32.09		50.02		49.69		24.74
	4.23		33.64		51.18		50.70		24.97
	5.24		35.34		52.36		50.04		25.24
	7.11		38.41		54.50		50.00		25.89
	7.32		38.58		54.85		49.79		25.75

Mean time. Date.		Bond 387—Negus 1589	Bond 380—Negus 1589	Hutton 202—Negus 1589	Negus 1591 (red. to sid.)—Negus 1589
1877.		*m.* *s.*	*m.* *s.*	*m.* *s.*	*m.* *s.*
May	7.48	+2 38.73	—0 54.97	+11 49.73	+1 25.76
	8.17	39.80	55.60	49.80	26.04
Chronometers (202) and (387) moved.					
May	8.22	39.40	55.62	49.80	26.04
	9.15	42.16	56.20	50.73	26.53
	10.12	44.92	57.02	51.92	26.84
	11.13	47.87	58.07	53.21	27.16
Chronometers moved.					
	11.23	50.20	56.08	55.33	29.22
	12.27	52.57	56.63	54.17	28.18
	14.35	Excesses over (387)	—3 55.20	+8 53.71	—1 31.45 ?
	14.54		55.80	52.58	32.11 ?
	15.26		57.93	50.00	34.90 ?
	16.31		4 0.86	45.80	37.54 ?
	17.99	+3 14.33	—0 50.75	+11 53.13	+1 30.75
	21.21	27.48	44.17	54.85	36.89
	24.16	33.16	42.64	52.18	31.47
	25.18	36.86	41.38	53.01	31.58
	26.21	40.18	40.32	53.64	31.41
	26.43	40.90	40.10	53.71	31.03
	28.32	47.16	38.17	55.01	31.30
	29.33	50.53	36.97	55.63	31.26
	31.23	57.00	34.30	56.60	31.42
June	1.35	4 1.14	32.46	56.72	31.49
	4.29	10.28	29.72	54.48	30.32
	8.29	21.70	25.60	53.43	27.85
	11.32	29.90	22.00	52.99	25.89
	11.49	30.30	21.90	52.90	25.70
	12.28	32.43	21.16	52.57	24.97
	13.27	35.03	20.37	52.09	24.36
	14.30	38.08	19.36	51.68	23.19
	14.48	38.60	19.20	51.58	23.00
	15.34	41.02	18.36	51.12	22.24
	15.47	41.35	18.22	51.06	22.08
	16.31	43.55	17.45	50.35	21.21
	16.43	43.89	17.31	50.22	21.09
	17.34	46.20	16.49	49.40	21.52
	17.49	46.58	16.28	49.28	
	18.32	48.60	15.67	48.10	19.22
	19.34	51.30	14.61	46.77	18.49
	19.46	51.68	14.42	46.68	18.30
	20.30	54.06	13.63	45.37	17.60
	20.40	54.38		45.30	17.59
	21.29	56.71	12.39	43.91	16.82
	22.31	59.42	11.78	42.22	15.76
	22.42	59.70	11.68	42.07	15.61
	23.28	5 2.10	11.20		14.63
	25.37	7.50	9.49	36.49	12.16

Mean time. Date.	Bond 387—Negus 1589	Bond 380—Negus 1589	Hutton 202—Negus 1589	Negus 1591 (red. to sid.)—Negus 1589
1877.	m. s.	m. s.	m. s.	m. s.
June 26.31	+5 9.82	−0 8.68	+11 34.28	+1 18.76
27.31	12.43	7.77	31.43	10.26
29.14	16.79	6.47	25.54	8.08
29.21	16.96	6.45	25.26	7.98
29.29	17.20	6.37	25.01	7.96
July 4.43	28.69	4.25	5.77	0.29

All the subsequent time-observations were made by Mr. Henry Farquhar, who was instructed to pursue the same system that Mr. Smith had done. For the experiments at various pressures with heavy end down, time-observations were begun on September 18, 1877, and continued till October 5, inclusive. The chronometer used was Negus 1589. The transits of the stars were taken as in Mr. Smith's observations, across the five wires of the middle group only, and read to twentieths of a second.

The observations taken from September 18 to 24 inclusive were not reduced, as the chronometer used in observation was suffered to run down on the 25th, by the janitor of the Stevens Institute, who had been commissioned to attend to winding it. The agreement of the stars observed, instrumental constants, and chronometer-comparisons are given in the following tables:

	P.D.	Z.D.	1877, September. 25	26	27	29	October. 1	5
	°	°						
ε Delphini	79	+30	..	..	+.01 W.	..	..	..
Gr. 3241	18	−31	..	..	+.07 W.	..	..	..
α Cygni	45	− 4	..	+.06 E.	−.10 W.	..	..	..
μ Aquarii	99	+50	..	−.06 E.	−.04 E.	..	..	..
ν Cygni	49	0	..	−.01 E.	..	..	..	..
12 Y. C. 1879	10	−39	..	..	−.05 E.	..	..	−.07 E.
61_1 Cygni	52	+ 3	..	+.04 W.	+.15 E.	+.05 E.	..	+.06 E.
ζ Cygni	60	+11	..	+.04 W.	..	+.05 E.	..	+.03 E.
α Cephei	28	−21	..	−.06 W.	..	−.10 E.	..	−.04 W.
1 Pegasi	71	+21	..	..	..	rej.	..	−.02 W.
β Aquarii	96	+47	−.02 W.	..	..	−.09 W.	..	−.05 W.
β Cephei	20	−29	+.06 W.	..	..	+.08 W.	..	+.07 W.
ξ Aquarii	98	+49	..	..	..	+.11 W.	..	..
ε Pegasi	81	+31	+.02 W.	..	..	−.05 W.	..	..
11 Cephei	19	−30	−.05 W.	..	..	..	..	..
79 Draconis	17	−32	−.01 E.	..	..	..	+.05 W.	..
α Aquarii	91	+42	rej.	..	..	..	−.01 W.	..
θ Aquarii	98	+49	−.01 E.	..	..	..	−.01 W.	..
κ Aquarii	89	+40	..	..	..	..	−.01 E.	..
η Aquarii	91	+41	..	..	..	..	+.04 E.	..
226 B Cephei	14	−35	..	..	..	..	−.05 E.	..

	P.D.	Z.D.	October. 5
	°	°	
α Orionis	83	+33	−.03 W.
22 H Camelop	21	−29	+.03 W.
μ Geminorum	67	+18	−.06 W.
γ Geminorum	73	+24	+.22 E.
α Canis Majoris	107	+57	−.19 E.
51 H Cephei	3	−46	.00 E.

Instrumental constants.

		Level constant.		Azimuth.	Collimation.
September	25	−0.88 W.	−0.80 E.	+0.53	−0.03
	26	+0.30 W.	+0.38 E.	+0.31	−0.03
	27	+0.33 W.	+0.47 E.	+0.42	+0.16
	29	+0.41 W.	+0.49 E.	+0.39	+0.16
October	1	+0.44 W.	+0.50 E.	+0.33	+0.10
	5	+0.57 W.	+0.62 E.	+0.23	+0.14
	5	+0.62 W.	+0.65 E.	+0.48	+0.02

Comparison of chronometers.

1877.	Seconds of excess of Bond & Son 380 ove Hutton 202.	Negus 1589.	Negus 1591, *reduced.*
	s.	*s.*	*s.*
September 18.29:	33.80	30.40	
21.38:	43.23	52.43	
22.36:	48.26	50.45	
23.26:	53.30	48.91	
24.38:	59.88	46.93	
	Chronometer ran down.		
25.41	5.53	15.20	
26.32	10.99	14.28	
26.52	11.96	14.01	
27.32	16.80	13.23	
27.44	17.46	13.13	27.61
29.34	28.62	11.17	25.22
29.77	30.68	10.27	24.45
29.83	30.98	10.17	24.43
October 1.34	37.29	7.34	23.34
1.43	39.82	7.21	23.31
1.73	41.71	6.92	23.35
1.79	42.20	6.89	23.38
3.33	51.54	5.41	
3.57	53.96	5.11	23.71
3.80	54.55	4.83	23.71
5.32	4.41	2.68	23.27
5.76	6.89	1.79	23.12

Time-observations for the experiments at various pressures with heavy end up were made, 1877, November 30 to December 24, inclusive. The chronometer used was Bond and Sons 380. In reducing these observations, the value of the azimuth was taken different in the two positions of the transit-instrument, and the observations of single stars received weights varying with the declination, in accordance with the latest recommendations of the Computing Division.

	P. D.	Z. D.	1877, December. 2
	°	°	
κ Aquarii	89	+40	−.03 E.
η Aquarii	91	+41	+.03 E.
226 B Cephei	14	−35	+.02 E.
ζ Pegasi	80	+31	+.01 W.
ι Cephei	24	−25	.00 W.
λ Aquarii	98	+49	−.01 W.

	P. D.	Z. D.	December. 16	24
	°	°		
α Pegasi	75	+26	..	−.08 E.
ο Cephei	23	−27	..	.00 E.
θ Piscium	84	+35	..	+.25 E.
ι Piscium	85	+36	−.01 W.	.00 W.
γ Cephei	13	−36	+.11 W.	+.03 W.
Gr. 4163	16	−33	−.11 W.	−.09 W.
ω Piscium	84	+35	+.03 W.	+.01 W.
α Andromedæ	62	+12	+.21 E.	..
γ Pegasi	76	+26	−.20 E.	.
α Cassiepeæ	34	−15	.00 E.	..

	P. D.	Z. D.	November. 30	December. 3	10	23
	°	°				
ο Piscium	81	+32	..	..	..	−.06 W.
β Arietis	70	+21	..	−.09 W.	..	+.06 W.
50 Cassiepeæ	18	−31	..	−.06 W.	..	.00 W.
α Arietis	67	+18	..	..	..	−.02 E.
ξ_1 Ceti	82	+32	+.12 E.	+.10 W.	..	+.01 E.
ι Cassiepeæ	23	−26	−.03 E.	+.14 E.	..	−.01 E.
γ Ceti	87	+38	−.10 E.	+.01 E.	..	..
α Ceti	86	+37	−.09 W.	..	−.01 E.	..
48 H Cephei	13	−37	..	−.06 E.	..	..
ζ Arietis	69	+20	..	−.05 E.	.00 E.	..
α Persei	41	− 9	..	..	.00 E.	..
δ Persei	43	− 7	+.02 W.	..	−.01 W.	..
η Tauri	66	+17	+.06 W.	..	+.05 W.	..
ζ Persei	58	+ 9	..	..	−.02 W.	..
γ_1 Eridani	104	+55	..	..	.01 W.	..

	P. D.	Z. D.	December. 7	9	12	14	17	20
	°	°						
γ Tauri	75	+25	−.08 E.	..	..	..	..	..
ε Tauri	71	+22	+.04 E.	..	..	..	..	..
α Tauri	74	+24	+.04 E.	.00 W.	..	+.04 E.	+.01 E.	..
9 Camelop	24	−25	.00 E.	.00 W.	+.01 W.	+.04 E.	+.02 E.	..
ι Aurigæ	57	+ 8	−.01 W.	−.01 W.	−.06 W.	−.09 E.	−.01 E.	..
11 Orionis	75	+26	..	+.08 E.	+.07 W.	.00 W.	+.08 W.	..
α Aurigæ	44	− 5	..	.00 E.	−.04 E.	+.02 W.	−.04 W.	..
β Orionis	98	+49	..	−.08 E.	−.03 E.	+.02 W.	−.05 W.	..
β Tauri	61	+12	..	..	+.07 E.	..	..	+.02 W.
Gr. 966	15	−34	..	..	..	..	..	+.01 W.
δ Orionis	90	+41	..	..	..	..	..	.00 W.
α Orionis	83	+33	..	..	..	..	..	+.12 E.
22 H Camelop	21	−29	..	..	..	..	..	−.01 E.
μ Geminorum	67	+18	..	..	..	..	..	−.14 E.

	P.D.	Z.D.	December. 22
	°	°	
δ Geminorum	68	+19	−.13 E.
P. VII 67	21	−28	−.01 E.
α Geminorum	58	+ 9	+.11 E.
α Canis Min	84	+35	−.23 W.
β Geminorum	62	+12	+.16 W.
φ Geminorum	63	+14	+.05 W.
3 H Ursæ Maj	21	−28	+.02 W.

	P.D.	Z.D.	November. 30
	°	°	
l Leonis	79	+30	.00 W.
α Ursæ Maj	28	−22	+.05 W.
δ Leonis	69	+20	−.04 W.
δ Crateris	104	+55	+.12 E.
λ Draconis	20	−29	−.03 E.
υ Leonis	90	+41	−.10 E.

Instrumental constants.

1877.	Level constant.		Azimuth.		Collimation.
November 30	−0.03 W.	−0.01 E.	+0.15		+0.02
30	−0.08 W.	−0.04 E.	−0.07		+0.01
December 2	−0.05 W.	0.00 E.	−0.24 W.	+0.21 E.	0.00
3	−0.13 W.	−0.08 E.	+0.35		+0.05
7	−0.08 W.	−0.05 E.	+0.04		+0.09
9	−0.07 W.	−0.04 E.	+0.35 W.	+0.03 E.	−0.14
10	−0.17 W.	−0.07 E.	−0.02 W.	−0.20 E.	+0.12
12	−0.12 W.	−0.03 E.	−0.09 W.	+0.08 E.	−0.02
14	−0.06 W.	−0.02 E.	+0.04 W.	−0.36 E.	+0.13
16	−0.14 W.	−0.03 E.	−0.12 W.	−0.76 E.	+0.11
17	−0.19 W.	−0.10 E.	−0.23		+0.14
20	−0.01 W.	+0.01 E.	−0.06 W.	+0.15 E.	+0.15
22	−0.06 W.	+0.04 E.	0.00 W.	+0.19 E.	+0.19
23	+0.04 W.	+0.07 E.	+0.34 W.	−0.01 E.	+0.13
24	+0.02 W.	+0.03 E.	−0.02		+0.06

Comparison of chronometers.

1877.	Seconds of excess of Bond & Sons 380 over Hutton 202.	Negus 1589.	Negus 1591, *reduced.*
	s.	*s.*	*s.*
November 30.50	37.41	19.77	10.26
30.80	38.61	18.86	9.89
December 2.27	44.11	13.28	7.62
3.43	48.19	8.90	6.15
4.45	52.19	5.31	4.68
7.57	5.06	54.87	0.87
8.37	8.64	51.84	59.78
8.86	10.66	50.31	59.23
9.43	12.85	48.35	58.26
10.47	16.59	44.70	57.07
10.83	17.96	43.50	56.73
12.46	24.70	38.37	55.01
12.79	26.11	37.41	54.70
14.44	32.72	32.13	52.92
14.81	34.19	30.88	52.50
16.22	39.30	25.99	51.00
16.51	40.38	25.01	50.26
17.43	43.82	21.90	49.17
17.79	45.10	20.70	48.78
19.52	51.21	14.70	46.60
19.82	52.38	13.71	46.31
20.47	54.90	11.48	45.59
22.53	2.15	3.88	42.14
22.85	3.47	2.84	41.68
23.17	4.61	1.70	41.21
23.51	5.83	0.49	40.68
24.19	8.15	57.76	39.29

The time-observations for the experiments at high temperatures and for the investigations to determine if the chronometers had any diurnal inequality of rate extended from 1878, February 19 to May 23 inclusive. The observation of May 21 is not reduced: on account of the discordance of the separate stars, no values for the time-correction and instrumental constants were resolved upon.

	P. D.	Z. D.	1878, February 24
	°	°	
α Orionis	83	+33	−.03 E.
22 H Camelop	21	−29	−.02 E.
μ Geminorum....	67	+18	+.01 E.
γ Geminorum....	73	+24	+.02 W.
α Canis Majoris..	107	+57	−.01 W.
51 H Cephei	3	−46	+.07 W.

	P. D.	Z. D.	February. 19	February. 27	March. 5	April. 2	April. 14
	°	°					
ι Ursæ Majoris .	41	− 8	..	+.08 W.	..	..	..
σ2 Ursæ Majoris .	22	−27	..	−.10 W.	..	.00 W.	..
κ Cancri	79	+30	..	+.01 W.	..	.00 W.	..
1 H Draconis....	8	−41	..	−.01 E.	−.12 E.	−.25 E.	+.23 W.
α Hydræ	98	+49	+.05 W.	−.02 E.	−.02 E.	−.10 E.	+.02 W.
d Ursæ Majoris .	20	−30	.00 W.	+.05 E.	+.14 E.	+.15 E.	+.19 W.
θ Ursæ Majoris .	38	−11	+.01 W.	−.02 E.	−.02 E.	+.08 E.	−.15 W.
ε Leonis	66	+16	−.06 W.	..	+.02 W.	..	−.09 E.
μ Leonis	63	+14	+.10 E.	..	+.02 W.	..	+.07 E.
α Leonis	77	+28	−.10 E.	..	−.06 W.	..	..
32 Ursæ Majoris .	24	−25	−.01 E.	..	.00 W.	..	..

	P. D.	Z. D.	March. 19	March. 29	April. 6	April. 18	May. 5
	°	°					
γ1 Leonis	69	+20	..	..	−.01 W.	..	..
9 H Draconis....	14	−36	.00 W.	..	−.01 W.	..	..
ρ Leonis	80	+31	−.10 W.		.00 W.	..	..
l Leonis	79	+30	+.11 W.	−.01 E.	.00 E.	..	..
α Ursæ Majoris .	28	−22	.00 E.	.00 E.	.00 E.	+.06 E.	..
δ Leonis	69	+20	.00 E.	+.02 E.	..	+.03 E.	+.03 E.
δ Crateris	104	+55	..	+.09 W.	..	−.09 E.	−.03 E.
τ Leonis	86	+37	..	−.09 W.	..	+.11 W.	+.01 E.
λ Draconis	20	−29	..	+.02 W.	..	−.06 W.	−.01 E.
υ Leonis	90	+41	..	..	..	−.06 W.	−.01 W.
β Leonis	75	+25	..	..	..	..	+.01 W.
γ Ursæ Majoris .	36	−14	..	..	..	..	−.03 W.

	P. D.	Z. D.	1878, April. 3	April. 8	May. 2	May. 6	May. 8	May. 9	May. 10	May. 11	May. 13	May. 13	May. 22	May. 23
	°	°												
β Leonis	75	+25	−.01 E.	..	..	..	..	..	..	..	..	..	..	..
γ Ursæ Majoris	36	−14	.00 E.	..	..	..	..	..	..	..	..	..	..	..
o Virginis	81	+31	..	..	−.04 W.	..	.00 E.	..	..	..	..	..	..	..
4 H Draconis	12	−38	..	..	−.03 W.	..	−.01 E.	..	..	..	..	..	..	..
η Virginis	90	+41	.00 W.	..	..	..	.00 E.	..	..	..	..	..	..	..
β Corvi	113	+63	..	..	+.09 W.	..	+.03 W.	..	..	..	..	..	..	..
κ Draconis	20	−30	+.01 W.	..	..	..	..	..	..	..	..	..	..	..
32 H_2 Camelop	6	−43	..	..	+.06 E.	+.01 W.	..	..	..	..	..	..	..	..
12 Canum Ven	51	+ 2	..	..	−.32 E.	+.01 W.	−.01 W.	..	..	..	..	..	..	..
θ Virginis	95	+46	..	+.01 E.	+.04 E.	−.05 W.	..	..	..	..	..	..	+.06 E.	..
α Ursæ Min. (L.C.)	−1	−51	..	−.05 E.	..	..	..	..	..	..	..	..	..	..
α Virginis	101	+51	..	..	+.04 E.	+.03 E.	..	..	..	..	−.02 E.	..	−.12 E.	−.03 W.
ζ Virginis	90	+41	..	−.03 W.	..	+.05 E.	..	..	..	..	+.02 E.	..	+.05 E.	+.04 W.
η Ursæ Majoris	40	− 9	..	+.01 W.	..	.00 E.	..	..	..	..	.00 E.	..	−.01 E.	−.01 W.
η Bootæ	71	+22	..	..	..	..	..	..	..	..	+.08 W.	..	+.02 W.	+.05 E.
α Draconis	25	−24	..	..	..	..	..	..	..	..	+.01 W.	..	+.01 W.	+.02 E.
α Bootæ	70	+21	..	..	..	..	..	..	..	..	−.06 W.	..	−.01 W.	−.04 E.
θ Bootæ	38	−12	..	..	..	..	..	..	..	..	..	+.11 W.	..	..
5 Ursæ Minoris	14	−35	..	..	..	..	..	..	..	..	..	−.13 W.	..	..
ε Bootæ	62	+13	..	..	..	..	..	..	..	..	..	−.06 W.	..	..
α2 Libræ	106	+56	..	..	..	..	..	..	..	..	..	−.03 E.	..	..
β Ursæ Minoris	15	−34	..	..	..	..	..	−.01 W.	..	..	..	+.03 E.	..	..
β Bootæ	49	0	..	..	..	..	..	..	..	..	..	−.03 E.	..	..
β Libræ	99	+50	..	..	..	..	..	−.02 W.	..	.00 W.	..	..	..	..
μ1 Bootæ	52	+ 3	..	..	..	..	..	+.02 E.	+.01 E.	−.02 W.	..	..	..	..
γ2 Ursæ Minoris	18	−32	..	..	..	..	..	.00 E.	.00 E.	+.01 W.	..	..	..	..
α Coronæ	63	+14	..	..	..	..	..	+.05 E.	.00 E.	+.07 E.	..	..	..	..
α Serpentis	83	+34	..	..	..	..	..	−.07 E.	−.06 W.	+.05 E.	..	..	..	..
ε Serpentis	85	+36	..	..	..	..	..	..	+.07 W.	−.09 E.	..	..	..	..
ζ Ursæ Minoris	12	−37	..	..	..	..	..	..	+.01 W.	.00 E.	..	..	..	..

Instrumental constants.

1878.		Level constant.		Azimuth.		Collimation.
February	19	+0.17 W.	+0.23 E.	+0.25 W.	−0.22 E.	+0.11
	24	+0.74 W.	+0.84 E.	−0.13 W.	−0.38 E.	+0.12
	27	+0.72 W.	+0.80 E.	−0.37		+0.13
March	5	+0.78 W.	+0.78 E.	+0.15 W.	−0.25 E.	+0.07
	19	+0.86 W.	+0.94 E.	−0.31 W.	+0.22 E.	+0.03
	29	+0.85 W.	+0.97 E.	+0.24 W.	−0.38 E.	+0.09
April	2	+0.78 W.	+0.82 E.	−0.41 W.	−0.22 E.	+0.12
	3	+0.73 W.	+0.87 E.	−0.17 W.	−0.47 E.	+0.18
	6	+0.79 W.	+0.88 E.	−0.21 W.	−0.02 E.	+0.06
	8	+0.79 W.	+0.90 E.	−0.22		+0.22
	14	+0.86 W.	+0.96 E.	−0.08		+0.19
	18	+0.87 W.	+0.93 E.	−0.08		+0.17
May	2	−0.24 W.	−0.14 E.	−0.05		+0.20
	5	−0.01 W.	+0.02 E.	−0.27 W.	−0.09 E.	+0.05
	6	−0.12 W.	−0.07 E.	−0.04 W.	−0.33 E.	+0.02
	8	−0.21 W.	−0.13 E.	−0.02		+0.04
	9	−0.08 W.	−0.02 E.	+0.17 W.	+0.04 E.	−0.04
	10	−0.04 W.	−0.01 E.	+0.03 W.	−0.30 E.	+0.15
	11	−0.06 W.	+0.04 E.	+0.21 W.	+0.03 E.	+0.03
	13	−0.12 W.	−0.10 E.	+0.21		−0.01
	13	−0.14 W.	−0.04 E.	+0.06		−0.08
	21	−0.32 W.	−0.20 E.			
	22	−0.21 W.	−0.11 E.	+0.14 W.	−0.21 E.	+0.05
	23	−0.25 W.	−0.17 E.	−0.18		+0.11

Comparisons of chronometers.

1878.		Seconds of excess of Bond & Sons 380 over Hutton 202.	Negus 1589.	Negus 1591, *reduced.*
		s.	*s.*	*s.*
February	19.52		39.39	
	24.30	55.83	35.72	51.67
	27.43	11.50	29.28	
March	5.42	37.56	11.89	34.74
	19.48	39.72	40.06	11.42
	29.46	31.03	21.40	59.94
April	2.38	47.65	24.96	55.35
	3.40	52.44	12.15	54.11
	6.43	10.12	7.39	51.55
	8.48	17.31	0.39	45.74
	14.31	49.18	51.02	41.22
	18.43	10.36	46.14	38.95
	24.42	35.01		34.73
	26.37	42.60	37.98	32.70
	26.54	43.33	37.21	32.56
	30.37	4.39	32.90	29.87
	30.63	5.89	32.69	29.83
May	1.16	8.80	32.16	29.57
	1.38	13.30	31.15	29.13
	2.16	14.26	30.90	
	2.35	15.27	30.76	30.95
	2.94	18.32	30.21	28.71
	4.02	23.99	29.38	28.67

1878.		Excess of Bond & Sons 380 over Hutton 202.	Negus 1589.	Negus 1591, *reduced.*
		s.	s.	s.
May	4.21	24.95	29.17	28.60
	4.39	25.85	29.00	28.41
	4.48	26.38	28.92	28.39
	5.34	30.61	28.18	28.33
	5.47	31.24	28.05	28.16
	5.91	33.39	27.65	28.07
	5.99	33.73	27.53	28.00
	6.18	34.58	27.33	27.91
	6.46	35.88	27.95	28.74
	6.54	36.23	28.13	28.92
	6.97	38.42	28.99	28.78
	7.20	39.85	29.43	28.59
	7.96	44.59	28.59	28.32
	8.20	45.94	28.30	28.23
	8.36	46.85	28.13	28.20
	8.49	47.66	27.99	28.16
	8.98	50.53	27.37	28.06
	9.18	51.66	27.01	27.96
	9.48	53.35	26.64	28.56
	9.99	56.15	25.90	28.37
	10.39	58.23	25.18	28.13
	10.50	58.81	24.99	28.10
	11.00	1.39	24.03	27.69
	11.38	3.24	23.25	27.36
	11.48	3.70	23.04	27.27
	13.39	11.24	19.02	25.56
	13.50	11.64	18.79	25.46
	16.87	23.84	11.25	22.11

During all the pendulum-experiments, except those of September and October in 1877, the chronometers were wound at two different times of day,

Hutton 202 and Bond & Sons 380 at about 8 a. m., and
Negus 1589 and 1591 at 4 to 5 p. m.

A series of special comparisons between these chronometers was made after the completion of the pendulum-work, to ascertain if any diurnal correction, consequent on the time of winding, existed. It will be seen from the following table, and the illustration hereafter to be explained, that there is no such correction to be found.

Comparison of chronometers.

1878.		Seconds of excess of Bond & Sons 380 over Hutton 202.	Negus 1589.	Negus 1591, *reduced.*
		s.	*s.*	*s.*
May	16.87	23.84	11.25	22.11
	21.41	42.63	3.19	19.09
	21.48	42.96	3.10	19.07
	21.89	44.94	2.58	18.97
	22.01	45.70	2.36	18.86
	22.15	46.52	2.14	18.77
	22.23	46.99	2.01	18.72
	22.36	47.80	1.80	18.64
	22.47	48.44	1.64	18.59
	22.89	50.76	0.97	18.38
	23.02	51.41	0.72	18.26
	23.13	51.98	0.54	18.20
	23.23	52.50	0.39	18.12
	23.37	53.39	0.16	18.03
	23.49	54.14	59.98	17.94
	23.90	56.59	59.26	17.71
	24.02	57.22	59.06	17.66
	24.13	57.86	58.89	17.61
	24.22	58.06	58.54	17.33
	24.36	58.14	57.54	16.47
	24.47	58.19	56.75	15.79
	24.88	0.47	56.10	15.61
	25.02	1.21	55.87	15.51
	25.12	1.79	55.73	15.44
	25.22	2.34	55.60	15.40
	25.36	3.15	55.44	15.34
	25.49	3.92	55.30	15.31
	26.90	12.35	53.77	14.88
	27.02	13.02	53.61	14.82
	27.19	13.97	53.43	14.77

Rates of chronometers graphically represented.

Illustrations Nos. 31, 32, 33, 34, and 35 show these comparisons, with the corrections of the chronometers, as graphically represented. From the comparisons made at the time of the transit-observations, the correction of each chronometer is deduced; an approximate mean uniform rate is then applied to each, and the excess of the correction over this mean rate plotted. It is then determined by inspection which chronometer is going most nearly uniformly, between one set of star-observations and the next, and this chronometer is taken as a standard. The rates of the other chronometers, from comparison to comparison, are taken as given by the supposition of its entire uniformity. The comparisons made a few hours apart, at the beginning and end of each day's pendulum-work, are not combined for a definitive rate of the chronometer used, however; their chief service is in guarding against any sudden change of rate. The comparisons made at the time of the star-observations are distinguished by a circle on the illustrations.

1877. *Experiments in June. Illustration No.* 31.—The chronometer whose rate was most nearly uniform during this month was found to be different at different times. The following were found to be best in this respect:

	Best chronometer.
June 1 to 14	202
14 to 16	380
16 to 19	1589
19 to 22	1591
22 to 29	1589

The subjoined table gives the rates adopted for the chronometers used in the experiments, as found on these assumptions:

	Chronometer used.	Rate in seconds.	In decimals of a day.
		s.	
June 11	380	−0.49	−.00000057
11	202	+0.81	+ 094
14	387	−2.49	− 288
15	380	−0.59	− 068
16	380	−0.59	− 068
17	202	+1.23	+ 142
19	202	+2.07	+ 240
20	202	+2.07	+ 240
22	202	+2.90	+ 336
29	202	+3.68	+ 426

1877. *Experiments in September and October. Illustration No.* 32.—A uniform rate of $-0^s.91$ for chronometer 1589 from the beginning to the end of the experiments was taken, and the following rates of the chronometer used (No. 380) during each night of pendulum-work thus deduced.

	Rate in seconds.	Rate in decimals of a day.
	s.	
September 25	+0.11	+.0000013
26	+0.11	013
27	+0.11	013
29	+1.10	128
October 1	+0.08	009
3	+0.08	009
5	+1.11	129

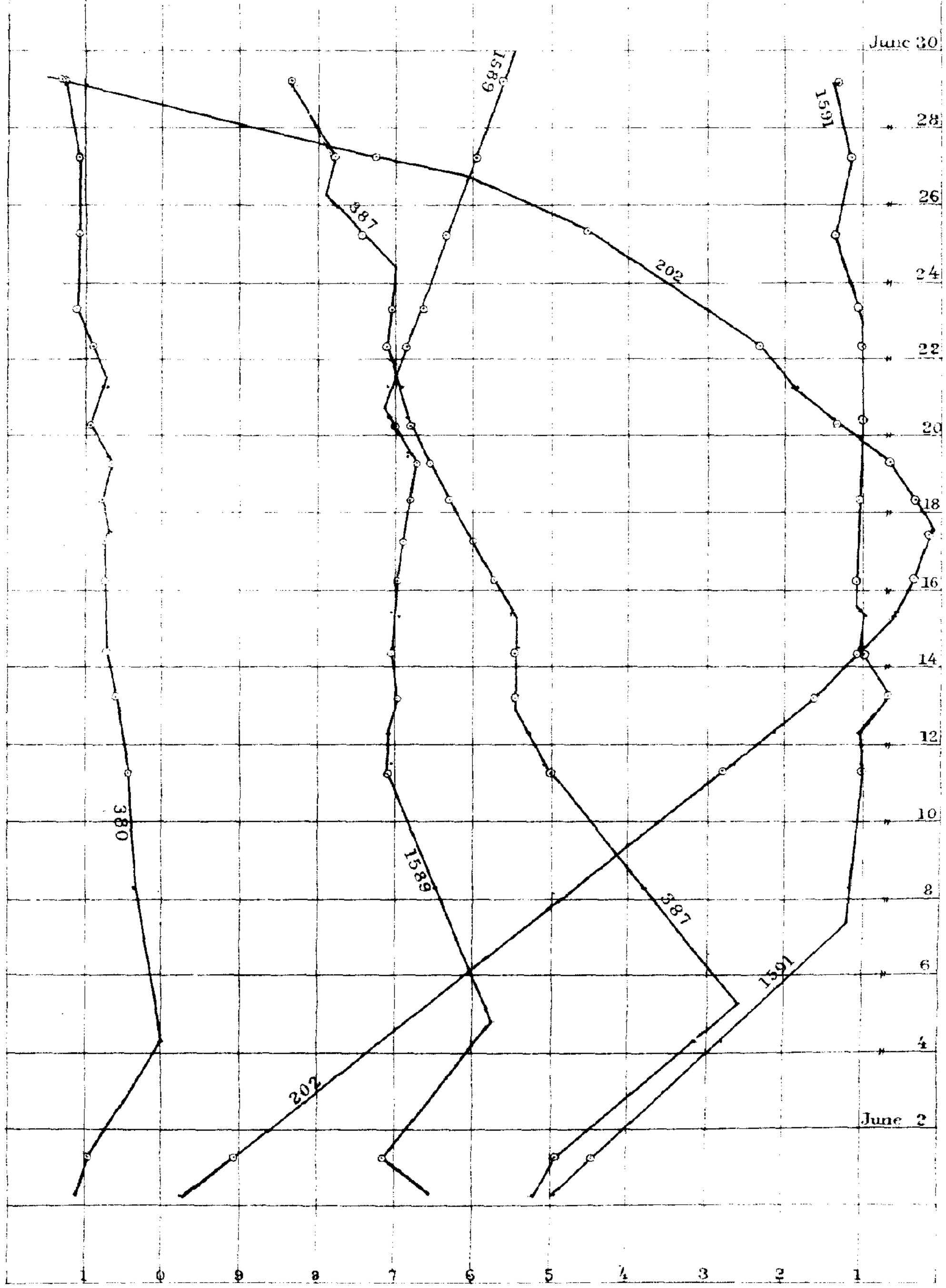
1877. *Experiments in December. Illustration No.* 33.—Chronometer 202 was found to have a uniform rate of $+7^s.225$ from the beginning until December 11.7, then a uniform rate of $+6^s.813$ until the transit-observations of the 20th. After the 20th, No. 380 was itself adopted as the standard.

Rate of chronometer 380.

	s.	Decimal of a day.
November 30	+3.17	+.0000367
December 4	3.19	369
8	3.05	353
10	3.36	389
12	2.45	283
14	2.84	329
16	3.08	356
17	3.25	376
19	2.89	335
22	3.51	406
23	3.51	406

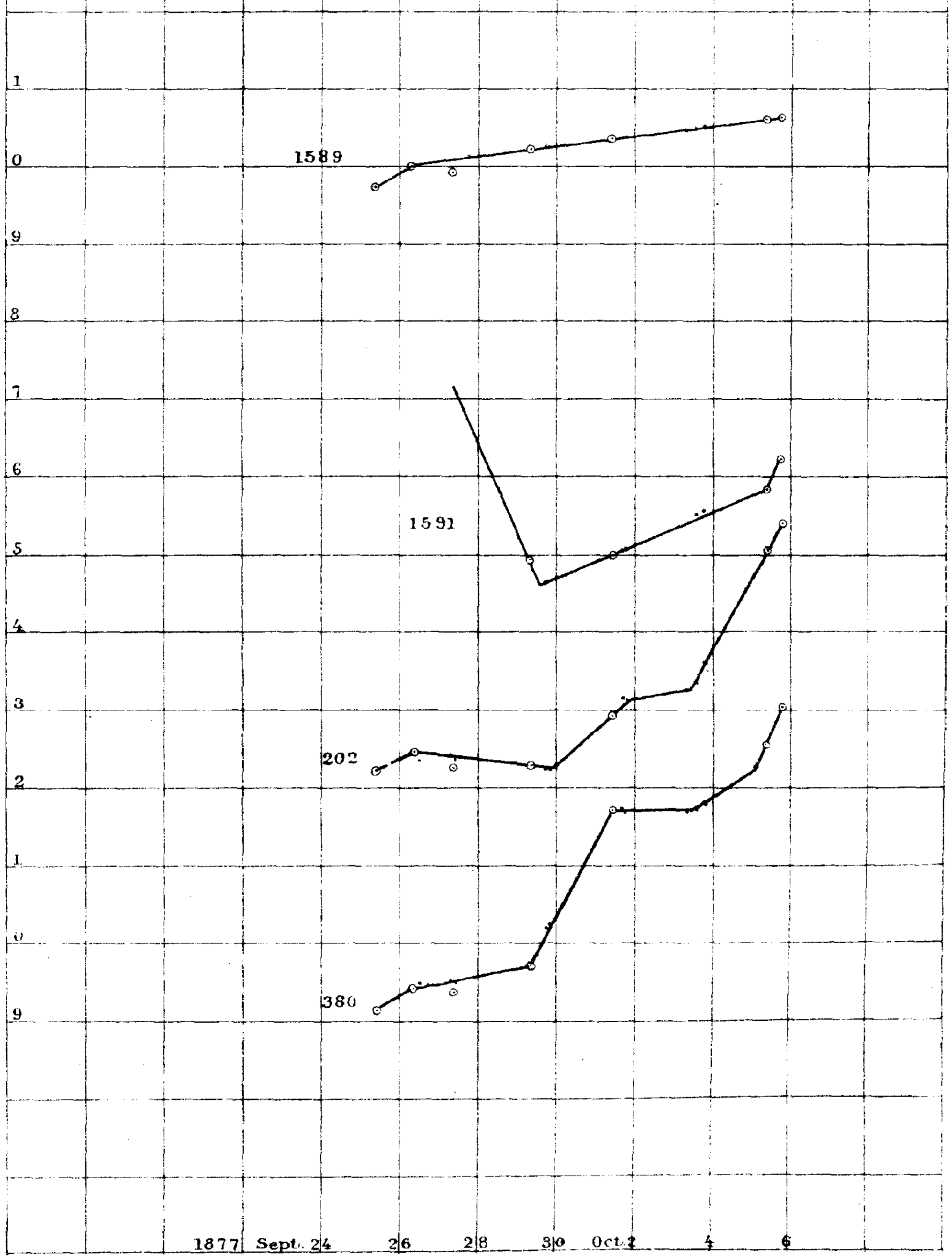
No. 3[illegible]

U. S. Geodetic Survey Pendulum at Hoboken. June 1877. Corrections of Chronometers after applying mean rates.



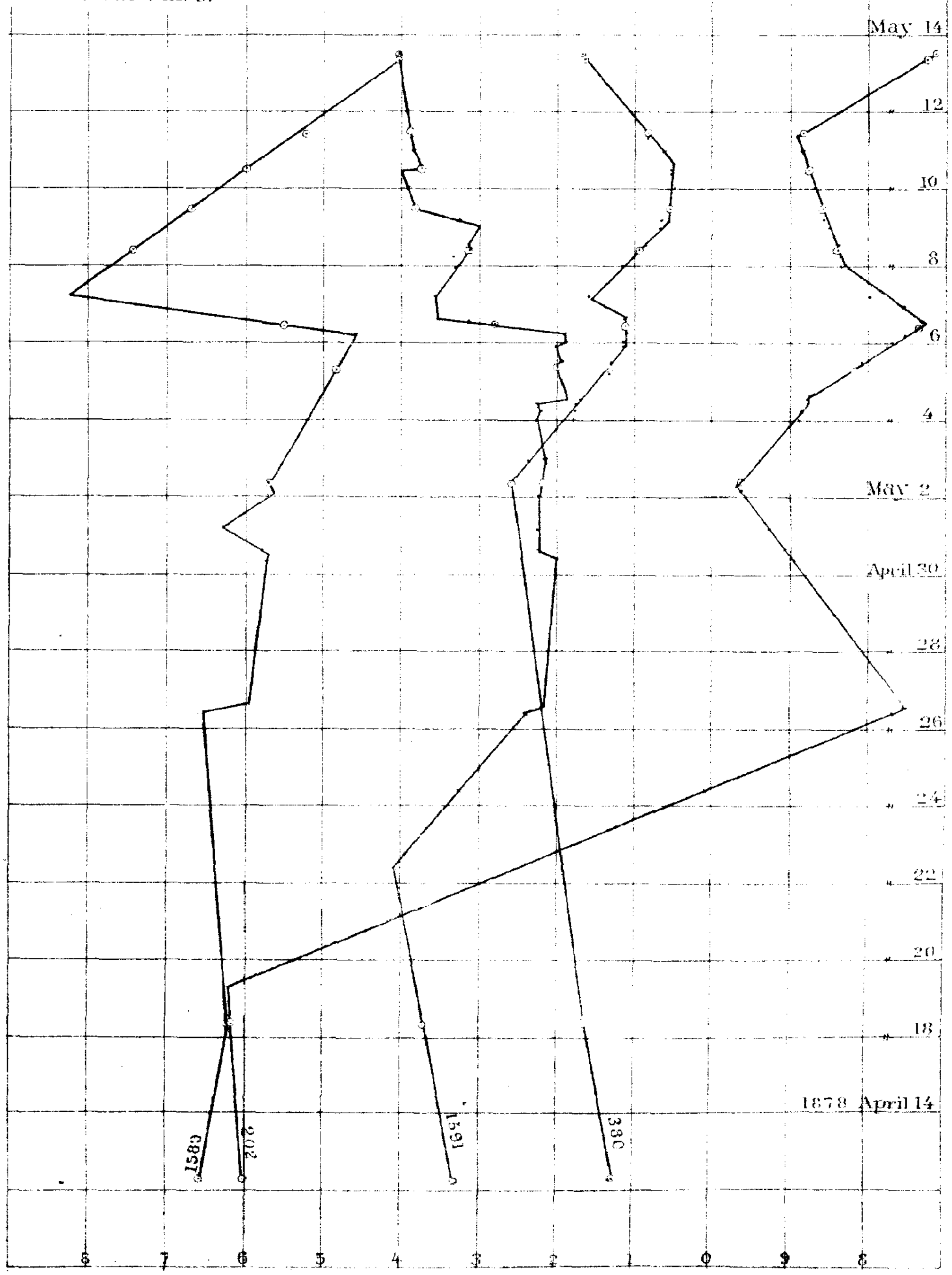
No. 32.

U. S. Geodetic Survey Pendulum at Hoboken. Experiments at Varying Pressures. Corrections to Chronometers after applying uniform rates.

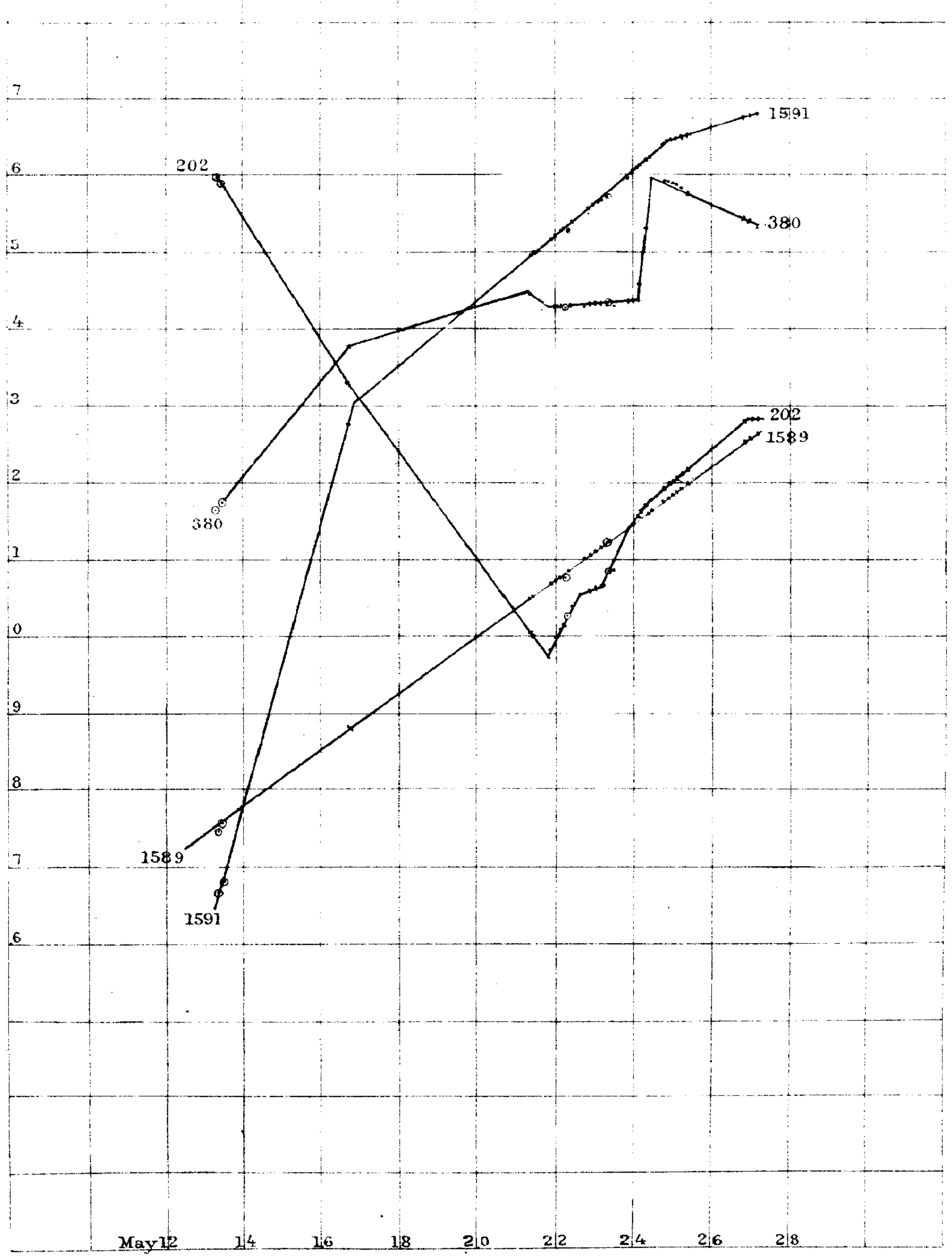


N° 34.

U. S. Geodetic Survey Pendulum at Hoboken. Experiments at High Temperatures. Corrections of Chronometers after application of uniform rates.



1878. Observations for Diurnal Variation of Rate of Chronometers.



1878. *Experiments in April and May. Illustration No.* 34.—Before heating up the room in which the pendulum was swung the chronometers were moved outside, to a place which was not so favorable in uniformity of temperature and in other respects. Their rates were therefore not so good as they had previously been. Chronometer 1589 was taken as the standard, and supposed to run uniformly until the 2d, and from the 2d to the 5th of May. Between the 5th and the 9th there were evidently two changes of rate; the times when these occurred were determined from the comparisons with the other chronometers. The rate was uniform again from the 9th to the 13th.

				Rate of 1589.
				s.
April	18.43	to	32.35	+1.036
May	2.35	to	6.18	+1.29
	6.18	to	7.20	−2.52
	7.20	to	13.50	+1.667

The rates of No. 380 during the nights of pendulum-work were then found to be—

		Rate in seconds.	In decimals of a day.
April	24	+0.07	+.0000008
	26	+0.07	+ 008
	30	−0.08	− 009
May	2	−0.39	− 045
	4	−0.39	− 045
	5	−0.39	− 045
	6	+0.47	+ 054
	8	−0.51	− 059
	10	+0.24	+ 028
	11	+0.24	+ 028

CORRECTION FOR ARC.

The factor for reducing the time of oscillation of the pendulum to an infinitesimal arc is best developed according to powers of the arc itself. Such a development is far more convergent than those found in the books. The factor is

$$1 - \frac{1}{64} A^2 + \frac{1}{49152} A^4 - \frac{5}{1179648} A^6 + \text{etc.},$$

where A represents the whole amplitude of the oscillation expressed in parts of the radius.

The Repsold pendulum tripod is provided with a metallic arc for reading the amplitude of oscillation. This is divided into spaces of 10′ each. In the experiments on the Geneva support an arc divided into thousandths of the radius was made use of.

At Geneva, the amplitude was read by bringing the vertical wire of the telescope so as to bisect the point of the pendulum at the extremity of an oscillation, the wire having been turned in a direction radial from the line of the knife-edge. The time was noted and the position of the wire between two lines of the graduated arc was estimated at leisure. At the other stations a far better method was used. The wire was placed in exact coincidence with a line of the graduated arc and the time was noted at which the pendulum was bisected by it at the extremity of its oscillation. The arc was so placed that its zero was 1′ or 2′ away from the vertical, so as to permit the observation to be made both to the right and the left.

The Geneva observations of arc were plotted on a curve for each experiment. Then values of the arc were read off at six equal intervals between every two sets of pendulum-transits. These values were squared, and the mean square was obtained by Mr. Weddle's rule—

$$\int_0^{6h} u_x \,.\, \mathrm{d}x = \frac{3h}{10} \left\{ u_0 + u_2 + u_4 + u_6 + 5\,(u_1 + u_5) + 6\,u_3 \right\}.$$

To obtain the correction for arc at Paris, Berlin, and Kew, the first step was to tabulate the times of decrement from a fixed value of the arc to each of the others for all cases in which there were good observations both to the right and to the left. The following tables show—first, the minute and second at which the pendulum was observed to reach each amplitude, and, second, the differences of the times from those of reaching the arc of 1° 10′ on each side of the vertical. A colon signifies that the observation to which it is attached was noted as poor at the time. Brackets inclose numbers derived from observations made only to the right or only to the left, in the manner described below. In the second table a star shows that the time of reaching 1° 10′ was not observed, but was deduced from the time of reaching some other arc, the observation of which is therefore omitted and a star put in its place.

Table showing the minutes and seconds of the times of reaching the different half-amplitudes.

PARIS.

HEAVY END UP.

	Jan. 26.	Jan. 28.	Jan. 29.	Jan. 29.	Feb. 2.	Feb. 4.	Feb. 9.	Feb. 14.	Feb. 21.	Feb. 22.
° ′	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.
2 20	......	......	......	......	......	......	......	......	6 17	......
10	......	......	......	......	......	......	......	......	......	......
00	......	......	......	......	......	......	......	......	......	......
1 50	......	27 21:	......	......	......	39 53	......	59 32	......	......
40	......	......	......	......		......	......	......	13 29	......
30	......	......	......	56 20	......	......	......	......	......	......
20	[42 23]	34 36	......	59 13	[51 35]	47 20	25 10	6 56	18 51	52 24
10	46 4	38 7	......	62 42	55 25	50 56	29 2	10 27	22 27	55 59
00	......	......	......	......	......	......	......	......	......	......
0 50	55 39	......	......	72 41	64 38	60 49	38 40	......	32 34	......
40	62 54	......	......	79 32	71 44	67 52	45 38	27 12	39 30	72 20
30	......	......	......	......	......	......	......	37 44:	......	......

HEAVY END DOWN.

	Jan. 26.	Jan. 28.	Feb. 2.	Feb. 2.	Feb. 3.	Feb. 4.	Feb. 9.	Feb. 14.	Feb. 21.	Feb. 22.
° ′	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.
2 20	......	......	......	......	......	......	......	......	......	......
10	......	32 25:	......	......	......	......	......	......	15 31	......
00	......	......	......	......	......	......	......	......	......	......
1 50	30 20	39 56:	32 7	53 48	......	25 53	......	......	23 32	52 36
40	35 1:	44 37	36 48	58 28	......	30 39	51 39	......	28 23	57 33
30	......	......	......	......	25 42:	......	......	......	......	......
20	47 9	56 55:	[48 51]	70 17	32 6	42 6	63 47	44 47	40 45	69 25
10	55 58	......	......	77 52	40 0	49 25	72 3	......	49 3	78 16:
00	......	......	......	......	48 48	......	......	62 57	58 54	......
0 50	77 17	85 52	......	99 00	59 46	69 6	94 41	75 15	71 37	101 2
40	91 28	101 49	......	115 13	74 15	82 29	110 49:	91 47	87 7:	117 20
30	......	......	......	......	94 2	......	132 8	......	109 21	......

Table showing the minutes and seconds of the times of reaching the different half-amplitudes—Continued

BERLIN.

HEAVY END UP.

° ′	Apr. 20.	Apr. 24.	Apr. 25.	Apr. 26.	Apr. 28.	Apr. 29.	Apr. 30.	May 2.	June 1	June 3.	June 3.	June 4.	June 5.	June 5.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
2 20	35 38	....	36 58	....	....	....	53 55	....	....	....	....	....	....	....
10	....	....	....	....	....	....	....	....	....	....	....	....	....	....
00	....	....	....	....	....	....	....	....	....	....	....	....	....	....
1 50	....	....	....	....	....	....	....	....	....	....	....	....	....	....
40	42 47	....	43 52	....	....	....	60 51	....	....	....	....	....	....	....
30	....	....	....	....	....	....	....	....	....	....	....	....	....	....
20	48 8	48 00	49 27	....	34 41	57 57	66 10	17 17	....	....	....	....	....	....
10	51 46	[51 38]	53 00	[36 32]	38 17:	[61 35]	[69 41]	20 55	25 55	40 36	11 23	2 29	58 43	30 44
00	....	....	....	....	....	....	....	....	....	....	....	....	....	....
0 50	61 40	61 19	....	46 11	48 00	71 16	79 15	30 15	....	....	....	....	....	....
40	78 35	68 26	69 41	53 23	....	....	85 50	36 50	....	....	....	....	....	....
30	88 42	....	79 36	....	....	....	....	....	[52 19]	67 2	[37 4]	[29 40]	[85 40]	[57 29]

HEAVY END DOWN.

° ′	Apr. 19.	Apr. 20.	Apr. 24.	Apr. 25.	Apr. 26.	Apr. 28.	Apr. 29.	Apr. 30.	May 2.	May 4.	June 1.	June 3.	June 4.	June 5.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
2 20	49 5	....	51 57	....	18 13	....	58 00	42 3	2 88	....	....	....	....	....
10	52 18	42 51	....	....	....	....	61 12	45 15	5 18	....	....	....	....	....
00	....	....	....	....	....	....	....	....	....	....	....	....	....	....
1 50	....	50 36	62 48	56 42	....	0 29	68 58	53 7	13 28	....	....	....	....	....
40	....	....	67 65	....	33 50	5 16	73 50	57 40	18 11	....	....	....	....	....
30	....	....	....	....	....	....	....	....	....	....	....	....	....	....
20	77 18	68 18	79 32	73 43	46 18	17 34	86 6	70 0	....	....	....	....	....	....
10	[84 42]	75 41	87 39	81 34	53 52	25 31	[93 55]	77 59	38 17	27 35	4 45	15 16	48 24	43 15
1 00	95 11	....	....	....	....	....	104 24	....	....	....	....	....	....	....
50	107 23	97 56	110 3	103 16	75 30	47 24	116 30	99 55	....	....	....	....	....	....
40	....	113 25	124 30	119 40	....	64 11	131 47	115 37	75 40	....	....	....	....	....
30	146 12	135 43	146 34	....	113 10	86 27	156 2	....	97 20	86 26	64 22	75 28	107 22	102 13

KEW.

HEAVY END UP.

° ′	July 1.	July 2.	July 2.	July 3.	July 3.	July 4.	July 4.	July 4.	July 4.	July 7.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
2 40	....	5 00	....	....	....	....	....	....	....	....
30	....	6 10:	....	....	....	....	....	....	....	....
20	0 51	7 27:	22 51:	33 18	35 57	[42 54]	32 21	2 16	59 37	18 54
10	....	8 57:	....	....	....	....	....	....	....	....
00	....	10 28	....	....	....	....	....	....	....	....
1 50	....	....	....	....	....	....	....	....	....	....
40	8 00	....	29 55	40 25	43 8	49 54	39 28	9 30	6 48	26 0
30	....	17 4	....	....	....	....	....	....	....	....
20	13 22	....	35 25	45 57	48 37	55 30	44 50	15 00	12 13	31 38
10	16 58	23 50	39 00	....	52 2	59 5	48 25	18 29	15 51:	....
00	....	28 10:	....	....	....	....	....	....	....	....
0 50	27 2	33 15:	48 47	59 7	61 54:	68 47	58 9	28 9	25 26	44 43
40	....	40 33	55 38:	....	69 1	75 55	65 4	....	....	51 24
30	....	50 29	....	....	....	....	....	....	....	....
20	....	65 2	....	....	....	....	....	....	....	....

Table showing the minutes and seconds of the times of reaching the different half-amplitudes—Continued.

KEW—Continued.

HEAVY END UP—Continued.

	July 7.	July 7.	July 7.	July 8.	July 8.	July 9.	July 9.	July 9.	July 10.	July 10.
° ′	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
2 40	....	....	....	....	....	....	....	....	....	....
30	....	....	....	....	....	....	....	....	....	....
20	37 30	27 7	19 43	18 22	9 43	11 35	8 35	13 6	24 52	32 46
10	....	....	....	....	....	....	9 57	....	....	....
00	....	....	....	....	....	....	11 33	....	....	....
1 50	....	....	....	....	....	....	....	....	....	....
40	44 38	34 21	....	25 38	16 53	18 43	....	20 7	31 56	39 56
30	....	....	....	....	....	....	....	....	....	....
20	50 13	39 55	32 31	....	22 22	....	....	....	37 23	45 28
10	53 56	43 36	35 48	34 48	....	27 49	....	28 58	40 58	48 58′
00	....	....	....	....	....	....	....	....	....	....
0 50	63 41	53 30	46 1	....	35 30	37 57	....	38 35	50 43	58 47:
40	70 27	60 13	52 56	51 38	42 25	....	....	45 30	....	65 45
30	....	....	....	....	....	....	....	....	....	....
20	....	....	....	....	....	....	....	....	....	....

HEAVY END DOWN.

	July 1.	July 2.	July 3.	July 4.	July 4.	July 7.	July 7.	July 8.	July 9.	July 10.
° ′	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
2 40	....	....	....	....	....	....	....	....	....	....
30	....	....	....	....	....	....	....	....	....	....
20	....	....	....	....	....	....	....	....	....	....
10	[18 24]	39 31	49 2	44 1	5 46	21 6	28 1	24 41	13 22	19 11
00	....	....	....	....	....	....	....	....	....	....
1 50	[26 24]	47 33	56 47	....	13 32	....	36 1	32 46	21 25	27 6
40	[31 13]	52 33	....	56 47	18 31	....	....	37 43	26 15	....
30	....	....	....	....	....	....	....	....	....	....
20	44 19:	64 42	74 22	69 8	31 2	46 50	53 37	50 12	38 57	....
10	[51 21]	72 40	82 8	77 18	39 0	55 5	62 5	58 36	....	52 36
00	....	....	....	....	....	....	....	....	....	....
0 50	[72 50]	94 56:	104 27	98 38	60 35	....	84 38	70 57	69 35	74 49
40	88 41	109 55	120 2	114 45	77 40	93 46	101 19	96 44	85 59	91 6
30	....	....	....	....	....	....	....	....	....	....
20	....	....	174 47	....	....	....	....	....	....	....

Table showing the time of decrement of the arc from 1° 10′.

HEAVY END UP.

PARIS.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
Jan. 26	....	....	....	....	[− 3 41]	+ 9 35	+16 50
28	....	....	−10 46:	....	− 3 31	....	....
29	....	....	....	....	− 3 29	+ 9 59	+16 50
Feb. 2	....	....	....	....	[− 3 50]	+ 9 13	+16 19
4	....	....	−11 3	....	− 3 36	+ 9 53	+16 56
9	....	....	....	....	− 3 52	+ 9 38	+16 36
14	....	....	−10 55	....	− 3 31	....	+16 45
21	−16 10	....	....	− 8 58	− 3 36	+10 7	+17 3
22	....	....	....	....	− 3 35	....	+16 21
Means...	−16 10	....	−10 59	− 8 58	− 3 38	+ 9 52	+16 45

Table showing the time of decrement of the arc from 1° 10′—Continued.

HEAVY END UP—Continued.

BERLIN.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
Apr. 20	−16 8	....	....	− 8 59	− 3 38	+ 9 54	+16 55
24	....	....	....	....	− 3 38	+ 9 41	+16 48
25	−16 2	....	....	− 9 8	− 3 33	....	+16 41
26	....	....	....	....	....	+ 9 39	+16 41
28	....	....	....	....	− 3 36	+ 9 43	....
29	....	....	....	....	− 3 38	+ 9 41	....
30	−15 46	....	....	− 8 50	− 3 31	+ 9 34	+16 9
May 2	....	....	....	....	− 3 38	+ 9 20	+15 55
Means...	−15 59	....	....	− 8 59	− 3 36	+ 9 39	+16 32

KEW.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
July 1	−16 7	....	....	− 8 58	− 3 36	+10 4	....
2	−16 23:	−14 53:	....	....	....	+ 9 25:	+16 43
2	−16 9:	....	....	− 9 5	− 3 35	+ 9 47	+16 38:
3	−16 13	....	....	− 9 6	*	+10 24	....
3	−16 5	....	....	− 8 54	− 3 25	+ 9 52:	+16 59
4	[−16 11]	....	....	− 9 11	− 3 35	+ 9 42	+16 50
4	−16 4	....	....	− 8 57	− 3 35	+ 9 44	+16 39
4	−16 13	....	....	− 8 59	− 3 29	+ 9 40	....
4	−16 20	....	....	− 9 3	− 3 38	+ 9 35	....
7	−16 18	....	....	− 9 12	*	+ 9 55	+16 12
7	−16 26	....	....	− 9 18	− 3 43	+ 9 45	+16 46
7	−16 29	....	....	− 9 15	− 3 41	+ 9 54	+16 37
7	−16 5	....	....	....	− 3 17	+10 13	+16 8
8	−16 26	....	....	− 9 10	....	....	+16 50
8	−16 13	....	....	− 9 3	*	+ 9 34	+16 29
9	−16 14	....	....	− 9 6	....	+10 8	....
9	*	−14 50	....	....	....	....	....
9	−15 52	....	....	− 8 51	....	+ 9 37	+16 32
10	−16 6	....	....	− 9 2	− 3 35	+10 45	....
10	−16 12	....	....	− 9 2	− 3 30	+ 9 49	+16 47
Means...	−16 14	−14 51	....	− 9 5	− 3 33	+ 9 54	+16 38

HEAVY END DOWN.

PARIS.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
Jan. 26	....	....	−25 38	−20 57:	− 8 49	+21 19	+35 30
28	....	−32 39:	−25 8:	−20 27:	*	+20 43:	+36 45
Feb. 2	....	....	−24 4	−19 24	− 7 35	+21 8	+37 21
3	....	....	....	....	− 7 54	+19 46	+35 15
4	....	....	−23 35	−18 46	− 7 19	+19 41	+33 4
9	....	....	....	−20 24	− 8 16	+22 38	+38 46:
14	....	....	....	....	*	+22 19	+38 51
21	....	−33 32	−25 31	−20 40	− 8 18	+22 34	+38 4:
22	....	....	−25 40	−20 43	− 8 51:	+22 46	+39 4
Means...	....	−33 1	−24 55	−20 10	− 8 9	+21 26	+36 51

Table showing the time of decrement of the arc from 1° 10′—Continued.

HEAVY END DOWN—Continued.

BERLIN.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
Apr. 19	−35 37	−32 24	....	....	− 7 24	+22 41	....
20	....	−32 50	−25 05	....	− 7 23	+22 15	+37 44
21	−35 42	....	−24 51	−20 04	− 8 7	+22 24	+36 51
25	....	....	−24 52	....	− 7 51	+21 42	+38 06
26	−35 39	....	....	−20 02	− 7 34	+21 38	....
28	....	....	−25 02	−20 15	− 7 57	+21 53	+38 40
29	−35 55	−32 43	−24 57	−20 05	− 7 49	+22 35	+37 52
30	−35 56	−32 44	−24 52	−20 19	− 7 59	+21 56	+37 38
May 2	−35 39	−32 59	−24 49	−20 06	....	....	+37 23
Means...	−35 45	−32 44	−24 58	−20 9	− 7 46	+22 8	+37 45

KEW.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
July 2	....	−33 9	−25 07	−20 07	− 7 58	+22 16	+37 15
3	....	−33 6	−25 21	....	− 8 06	+22 19	+37 54
4	....	−33 17	....	−20 31	− 8 10	+21 20	+37 27
4	....	−33 14	−25 33	−20 29	− 7 58	+21 35	+38 40
7	....	−33 59	....	....	− 8 15	....	+38 41
7	....	−34 4	−26 4	....	− 8 26	+22 33	+30 14
8	....	−33 55	−25 50	−20 53	− 8 24	+22 21	+38 8
9	....	−33 44	−25 41	−20 51	*	+22 29	+38 53
10	....	−33 25	−25 30	....	....	+22 13	+38 30
Means...	....	−33 31	−25 35	−20 36	− 8 09	+22 07	+38 16

The foregoing tables show the amount of discrepancy between the observations of different days. The point of the pendulum is distant 113½ cm. from the knife-edge, so that one minute of arc measures ⅓ of a millimeter. The reading-telescope was placed at a distance of about 3 meters. It may, therefore, be supposed that a single observation of the half-amplitude would be in error by something like ¼ of a minute. The following table shows how much error this would produce in the noted time of attaining the different amplitudes:

Half amplitude.	Time of decrement of ¼. Heavy end up.	Heavy end down.
° ′	*s.*	*s.*
2 10	2	5
1 50	3	7
1 40	3	8
1 20	5	11
1 10	6	13
50	9	21
40	12	·27

It will be seen that the observed discrepancies are several times as large as these values, and cannot therefore well be attributed to errors of observation. The daily discrepancies are, however, less than 7 times the numbers just given, that is less than 1¾ minutes. Such errors would produce an error in the correction for arc proportional to the arc itself and amounting to only 2 millionths for $\Phi = 2°$. It has, therefore, been judged proper to find one function to express the relation between the amplitude and the rate of decrement, and to apply this to all the observations at ordinary pressures and temperatures for the purpose of finding the correction for arc.

The pendulum being symmetrical in form in reference to its two knife-edges, the air resists its motion with the same force whichever end is up. Consequently, the rate of decrement of the arc (produced by this cause) is in the two positions inversely proportional to the moments of inertia. But it is the property of the reversible pendulum that the moments of inertia about its two knife-edges are proportional to the distances of the center of mass from those knife-edges. These distances are in our pendulum very nearly in the ratio of 3 to 7. Hence, the times of decrement of the arc (so far as it is the effect of the air) must be in the ratio of 7 with heavy end down to 3 with heavy end up. The same would be true for any proper effect of friction on the knife-edges. But the decrement of the amplitude is no doubt partly caused by the energy of motion of the pendulum itself. For example, the pendulum sets its support in vibration and this vibration is resisted by internal friction, thus exhausting the energy. Such a decrement of the arc will be more nearly equal with heavy end up and with heavy end down, or it may even be greater with heavy end down. In point of fact it will be seen that the times of decrement are a little more nearly equal than if they were in the ratio of the distances of the knife-edges from the center of mass. This is shown by the following table:

STATION, PARIS.

Decrement.	Time, heavy end down.	Calculated time, heavy end up.	Observed time, heavy end up.	O − C
° ′ ° ′	*m.*	*m.*	*m.*	*m.*
1 40 to 1 20	12.0	5.2	5.4	+0.2
1 20 1 10	8.9	3.9	3.6	−0.3
1 10 50	21.4	9.3	9.9	+0.6
50 40	15.4	6.7	6.9	+0.2
1 40 to 40	57.7	25.2	25.8	+0.6

STATION, BERLIN.

Decrement.	Time, heavy end down.	Calculated time, heavy end up.	Observed time, heavy end up.	O − C
° ′ ° ′	*m.*	*m.*	*m.*	*m.*
1 40 to 1 20	12.4	5.4	5.5	+0.1
1 20 1 10	7.8	3.4	3.6	+0.2
1 10 50	22.1	9.6	9.6	0.0
50 40	15.6	6.8	6.9	+0.1
1 40 to 40	57.9	25.2	25.6	+0.4

STATION, KEW.

Decrement.	Time, heavy end down.	Calculated time, heavy end up.	Observed time, heavy end up.	O − C
° ′ ° ′	*m.*	*m.*	*m.*	*m.*
1 40 to 1 20	12.5	5.5	5.5	0.0
1 20 1 10	8.1	3.5	3.6	+0.1
1 10 50	22.1	9.6	9.8	+0.2
50 40	16.1	7.0	6.9	−0.1
1 40 to 40	58.8	25.6	25.8	+0.2

These numbers, however, show that for the purpose of calculating the correction for arc it will be quite sufficient to assume that the times of decrement are in the ratios of the moments of inertia. In order to obtain the law of decrement, therefore, the times with heavy end up and with heavy end down have been added together; and the means have then been taken for all three stations (Paris, Berlin, and Kew). We thus obtain

Half amplitude.	Sum of times.
′	s.
130	−2880
110	−2187
100	−1779
80	− 706
70	0
50	+1927
40	+3304

The time for 140′ has been neglected as not having been generally observed with heavy end down.

To satisfy these values a form of equation has been assumed which has been copied from Professor Benjamin Peirce's *Analytic Mechanics*, and which is Coulomb's equation with a constant term added. It is—

$$\mathrm{D}_t\,\Phi = -a - b\,\Phi - c\,\Phi^2\,.$$

The integral of this equation is

$$\Phi = \sqrt{\frac{a}{c} - \tfrac{1}{4}\frac{b^2}{c^2}}\cot\left\{c\sqrt{\frac{a}{c} - \tfrac{1}{4}\frac{b^2}{c^2}}(t - t_0)\right\} - \tfrac{1}{2}\frac{b}{c}\,.$$

The values of Φ for the different values of t, as given in the table above, are sufficiently satisfied by putting (for t in seconds of time and Φ in minutes of arc)

$$a = 1547 \times 10$$
$$b = 6418 \times 10^{-8}$$
$$c = 1421 \times 10$$

The errors are shown in the following table:

Sum of times.	Obs. Φ.	Calc. Φ.	(O—C) Φ.
s.	′	′	′
−3191	140	138.82	+1.18
−2880	130	130.03	−0.03
−2187	110	110.33	−0.33
−1779	100	100.00	−0.00
− 706	80	79.97	+0.03
0	70	70.03	−0.03
+1927	50	49.98	+0.02
+3304	40	39.99	+0.01

By least squares, better values of the constants could be obtained; but these are evidently sufficient for our purpose.

The law of decrement of the amplitude having been made out, it was requisite to apply it to the observations. The constant t_0, being different for each experiment, had first to be determined. In doing this, it was desirable to use observations in which the arc had only been noted on the right or on the left. For this purpose it was necessary to calculate the inclination of the zero of the metallic arc to the vertical. This was readily determined from the difference of the time of reaching a given division to the right and to the left. The following tables show the results so obtained. The figures in parentheses at the top of the column show the estimated inclination from observations of the pendulum at rest.

UNITED STATES COAST SURVEY—PARIS.—DIMINUTION OF ARC.

Calculation of inclination of zero-point.

ϕ	Jan. 26. Heavy end up. dt	(+2′) $d\phi$	Jan. 26. Heavy end down. dt	(−2′) $d\phi$	Jan. 28. Heavy end down. dt	(+1′)	Jan. 28. Heavy end up. dt	(1½′) $d\phi$	Jan. 29. Heavy end up. dt	(−2′) $d\phi$
′	*s.*	′	*s.*	′	*s.*	′	*s.*	′	*s.*	′
140										
130					+10:	+0.5				
120										
110			− 57	−2 0	+11:	+0.4	−30:	−2.5		
100			− 63:	−2.0	+11:	+0.3				
90										
80			−104	−2.4	+ 7:	+0.2	−60	−3.1	−67:	−3.5
70	+22	+0.9	−132	−2.5			−79	−3.4	−69:	−3.0
60										
50	+57	+1.6			+33?	+0.4				
40	+74	+1.6	−222	−2.0	+13	+0.1				
30										
Means		+1.5		−2.1		+0.3		−3.3		−3.3

ϕ	Jan. 29. Heavy end up. dt	$d\phi$	Feb. 2. Heavy end up. dt	(+1′) $d\phi$	Feb. 2. Heavy end down. dt	(−1½′) $d\phi$	Feb. 2. Heavy end down. dt	$d\phi$	Feb. 4. Heavy end down. dt	(+½′) $d\phi$
′	*s.*	′	*s.*	′	*s.*	′	*s.*	′	*s.*	′
140										
130										
120										
110					−53	−2.0	− 57	−2.1	+17	+0.6
100					−74	−2.3	− 77	−2.4	+ 4:	+0.1
90	− 57	−3.5								
80	− 69	−3.6					−102	−2.3	+ 2:	0.0
70	− 88	−3.8	+19	+0.8			−115	−2.2	−15	−0.3
60										
50	−103	−3.0	+38	+1.0			−176	−2.1	−14	−0.2
40	−172	−3 6	+53	+1.1			−293	−2.7	−11	−0.1
30										
Means		−3.6		+1.0		−2.2		−2.3		0.0

ϕ	Feb. 4. Heavy end up. dt	(−2′) $d\phi$	Feb. 9. Heavy end down. dt	(−2′) $d\phi$	Feb. 9. Heavy end up. dt	(+½′) $d\phi$	Feb. 14. Heavy end up. dt	(−2′) $d\phi$	Feb. 14. Heavy end down. dt	(+$\frac{4}{10}$′) $d\phi$
′	*s.*	′	*s.*	′	*s.*	′	*s.*	′	*s.*	′
140										
130										
120										
110	− 36:	−3.0					− 22:	−1.9		
100			− 68	−2.2						
90										
80	− 51:	−3.1	−116	−2.6	+17	+0.9	− 67	−3.5	+14:	+0.3
70	− 73	−3.1	−146	−2.7	+15:	+0.6	− 86	−3.7		
60									+15	+0.2
50	−116	−3.2	−182	−2.2	+34	+0.9			+23	+0.3
40	−165:	−3.5	−222:	−2.1	+38	+0.8	−144	−3.0	+19	+0.2
30			−298	−2.0			−224:	−3.5		
Means		−3.2		−2.3		+0.8		−3.4		+0.2

Calculation of inclination of zero-point—Continued.

φ	Feb. 21. Heavy end down. *dt*	*d*φ	Feb. 21. Heavy end up. *dt*	(+2′) *d*φ	Feb. 22. Heavy end up. *dt*	(−2′) *d*φ	Feb. 22. Heavy end down. *dt*	(+1′) *d*φ
′	*s.*	′	*s.*	′	*s.*	′	*s.*	′
140			+ 23	+2.9				
130	− 58	−2.8						
120								
110	− 70	−2.6					+17	+0.6
100	− 81	−2.6	+ 34	+2.5			+22	+0.7
90								
80	−109	−2.5	+ 46	+2.4	+ 81	+4.2	+27	+0.6
70	−146	−2.7	+ 55	+2.4	+100	+4.3	+28	+0.5
60	−157	−2.4						
50	−149	[−1.8]	+110	+3.0			+78	+0.9
40	−234	−2.2	+133	+2.8	+202	+4.3	+67	+0.6
30	−328	−2.2						
Means		−2.5		+2.7		+4.3		+0.6

UNITED STATES COAST SURVEY.—PENDULUM AT BERLIN.—DECREMENT OF ARC.—INCLINATION OF ZERO-POINT.

φ	HEAVY END UP: NAME BACK.								HEAVY END UP: NAME FORWARD.							
	April 24.		April 26.		April 29.		May 2.		April 20.		April 25.		April 28.		April 30.	
	dt	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ
′	*s.*	′	*s.*	′	*s.*	′	*s.*	′	*s.*	′	*s.*	′	*s.*	′	*s.*	′
140									+ 29	+3.5	+ 26	+3.1			+ 23	+2.8
130																
120																
110																
100									70	4.9	37	2.6			52	3.6
90																
80							+12	+0.6	89	4.4	75	3.7	+ 81	+4.0	59	2.9
70							12	0.5	102	4.2	76	3.1				
60																
50	12	+0.3	+15	+0.4	15	+0.4			196	5.1			183	4.8	130	3.4
40							91:	1.8:	204	4.1	153	3.1			202	4.0
30									385	5.7	232	3.4				
Means		0.3		0.4		0.4		0.8		4.5		3.1		4.4		3.3

φ	HEAVY END UP: NAME FORWARD—CONTINUED.													
	May 4.		June 1.		June 3.				June 4.		June 5.			
	dt	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ	*dt*	*d*φ
′	*s.*	′	*s.*	′		′	*s.*	′	*s.*	′	*s.*	′	*s.*	′
140														
130														
120														
110														
100														
90														
80														
70	+57	+2.3	+61	+2.5	+ 83	+3.4	+76	+3.1	+106	+4.3	+92	+3.8	+96	+3.9
60														
50														
40														
30					176	2.6								
Means		2.3		2.5		3.0		3.1		4.3		3.8		3.9

PENDULUM AT BERLIN.—DECREMENT OF ARC.—INCLINATION OF ZERO-POINT—Continued.

ϕ	HEAVY END DOWN: NAME BACK.								HEAVY END DOWN: NAME FORWARD.							
	April 24.		April 26.		April 29.		May 2.		April 19.		April 20.		April 25.		April 28.	
	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$
′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′
140	+ 37	+1.9	+ 57	+3.0	+ 55	+2.9	+ 47	+2.5	+ 95	+5.0						
130							53	2.5	114	5.3	+ 13	+0.6				
120																
110	59	2.1			85	3.0	79	2.8			22	0.8	+ 17:	+0.6:	+ 36	+1.3
100	65	2.0	74	2.2	95	2.9	74	2.2							38	1.2
90																
80	83	1.8	111	2.4	144	3.1			255	5.5			22	0.5	62	1.3
70	108	1.9	132	2.3			117	2.1			19	0.4	15:	0.3:	52	0.9
60					252	3.6			353	5.1						
50	309—		256	2.9					506	5.7	41	0.5	71	0.8	114	1.3
40	248	2.2	284	2.5	361:	3.1	331	2.9			125	1.1			203	1.8
30	343	2.2	420	2.7	714:	4.6:	607	3.9	1038	6.7	246	1.6			254	1.6
Means.	....	2.0	....	2.6	....	3.1	....	2.6	....	5.4	....	0.7	....	0.6	....	1.3

ϕ	HEAVY END DOWN: NAME FORWARD—CONTINUED.											
	April 20.		May 4.		June 1.		June 3.		June 4.		June 5.	
	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$
′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′
140	+ 14	+0.7										
130	26	1.2										
120												
110	41	1.4										
100	34	1.0										
90												
80	65	1.4										
70	58	1.0	+ 49	+0.9	+ 56	+1.0	+ 49	+0.9	+ 42	+0.7	+ 61	+1.1
60												
50	165	1.9										
40	104	0.9										
30			110	1.1	311	2.0	215	1.4	243	1.6		
Means.	......	1.2	......	1.0	......	1.5	......	1.1	......	1.1	....	1.1

SUMMARY OF RESULTS.

INCLINATION OF ZERO AT BERLIN.

1876.	Heavy end up.	Heavy end down.
Name back.	′	′
April 24	+0.3	+2.0
26	0.4	2.6
29	0.4	3.1
May 2	0.8	2.6
Name forward.		
April 19	......	5.4
20	4.5	0.7
25	3.1	0.6
28	4.4	1.3
30	3.3	1.2
May 4	2.3	1.0
June 1	2.5	1.5
3	3.0	......
3	3.1	1.1
4	4.3	1.1
5	3.8	1.1
5	3.9	......
Means.		
Name back	+0.5	+2.6
Name forward, excluding April 19	3.6	1.0

UNITED STATES COAST SURVEY—KEW.—DECREMENT OF ARC.—INCLINATION OF ZERO-POINT.

φ	July 1. dt	Heavy end down. dφ	July 1. dt	Heavy end up. dφ	July 2. dt	Heavy end up. d	July 2. dt	Heavy end down. dφ	July 2. dt	Heavy end up. (+2′) dφ	July 3. dt	Heavy end up. (+2½) dφ
′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′
140	....	....	+16	+2.0	+ 25	+3.1	....	....	+ 23	+2.9	+19	+2.4
130	....	....	....	....	+ 26	+2.9	+26	+1.3	....	....	....	....
120	....	....	....	....	+ 22	+2.1	....	....	....	....	....	....
110	....	....	....	....	....	....	+23	+0.9	....	....	....	....
100	....	....	+27	+2.0	....	....	+21:	+0.7	+ 40	+2.9	+42	+3.1
90	....	....	....	....	+ 42	+2.6	....	....	....	....	....	....
80	....	....	+29	+1.5	....	....	+25	+0.6	+ 50	+2.6	+45	+2.3
70	....	....	+68	+3.0	+ 65	+2.8	+53	+1.0	+ 64	+2.7	....	....
60	....	....	....	....	+ 87	+3.0	....	....	....	....	....	....
50	....	....	+36	+1.0	+ 66	+1.8	....	....	+ 66	+1.8	+73	+2.0
40	+38	+0.3	....	....	+ 93	+2.0	+44	+0.4	+101:	+2.1	....	....
30	....	....	....	....	+146	+2.3	....	....	....	....	....	....
Means.	....	+0.3	....	+1.9	....	+2.5	....	+0.8	....	+2.5	....	+2.5

φ	July 3. dt	Heavy end down. (+1.5) dφ	July 3. dt	Heavy end up. (+2.6) dφ	July 4. dt	Heavy end up. dφ	July 4. dt	Heavy end down. (+1) d	July 4. dt	Heavy end up. dφ	July 4. dt	Heavy end up. dφ
′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′
140	....	....	27	+3.4	....	....	....	....	+ 24	+3.0	+19	+2.4
130	+20	+1.0	....	....	....	....	+18	+0.9	....	....	....	....
120	....	....	....	....	....	....	....	....	....	....	....	....
110	+25	+0.9	....	....	....	....	....	....	....	....	....	....
100	....	....	38	+2.8	+ 42	+3.1	+18	+0.6	+ 37	+2.7	+31	+2.2
90	....	....	....	....	....	....	....	....	....	....	....	....
80	+19	+0.4	44	+2.3	+ 55	+2.9	+40	+0.9	+ 59	+3.1	+46	+2.4
70	+29	+0.5	75	+3.2	+ 65	+2.8	+35	+0.6	+ 75	+3.2	+58	+2.5
60	....	....	....	....	....	....	....	....	....	....	....	....
50	+24	+0.3	74:	+2.3	+ 98	+2.7	+83	+1.0	+ 74	+2.0	+68	+1.9
40	+60	+0.6	109	+2.3	+115	+2.4	+63	+0.6	+134	+2.8	....	....
30	....	....	....	....	....	....	....	....	....	....	....	....
Means.	....	+0.6	....	+2.7	....	+2.8	....	+0.8	....	+2.8	....	+2.3

φ	July 4. dt	Heavy end down. dφ	July 4. dt	Heavy end up. dφ	July 7. dt	Heavy end up. dφ	July 7. dt	Heavy end down. dφ	July 7. dt	Heavy end up. dφ	July 7. dt	Heavy end up. dφ
′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′
140	....	....	+28	+3.5	+22	+2.9	....	....	+15	+1.9	+ 23	+2.9
130	+17	+0.8	....	....	....	....	− 12	−0.6	....	....	....	....
120	....	....	....	....	....	....	....	....	....	....	....	....
110	+40	+1.5	....	....	....	....	− 15:	[−0.6]	....	....	....	....
100	+27	+0.9	+35	+2.5	+29	+2.1	− 15:	[−0.5]	+27	+2.0	+ 34	+2.5
90	....	....	....	....	....	....	....	....	....	....	....	....
80	+36	+0.8	+59	+3.1	+41	+2.1	− 28	−0.6	+38	+2.0	+ 45	+2.3
70	+63	+1.2	+87	[+3.7]	....	....	− 41	−0.8	+60	+2.6	+ 46	+2.0
60	....	....	....	....	....	....	....	....	....	....	....	....
50	+36	+0.4	+93	+2.5	+70	+1.9	....	....	+83	+2.3	+ 59	+1.6
40	+53	+0.5	....	....	+87	+1.8	−100	−0.9	+75	+1.6	+100	+2.1
30	....	....	....	....	....	....	....	....	....	....	....	....
Means.	....	+0.9	....	+2.9	....	+2.2	....	−0.7	....	+2.1	....	+2.2

KEW.—DECREMENT OF ARC.—INCLINATION OF ZERO-POINT—Continued.

φ	July 7. dt	Heavy end down. $d\phi$	July 7. dt	Heavy end up. $d\phi$	July 8. dt	Heavy end up. $d\phi$	July 8. dt	Heavy end down. $d\phi$	July 8. dt	Heavy end up. $d\phi$	July 9. dt	Heavy end up. $d\phi$
′	s.	′	s.	′	s.	′	s.	′	s.	′	s.	′
140	......	......	+ 20	+2.5	+ 17	+2.1	......	......	+24	+3.0	+15	+1.9
130	− 16	−0.8	......	......	......	......	−13	−0.6	......	......	......	......
120	......	......	......	......	......	......	......	......	......	......	......	......
110	− 19	−0.7	......	......	......	......	−37	−1.4	......	......	......	......
100	......	......	......	......	+ 31	+2.3	−20	−0.6	+27	+2.0	+29	+2.1
90	......	......	......	......	......	......	......	......	......	......	......	......
80	− 23	−0.5	+ 41	+2.1	+ 35:	+1.8	−45	−1.0	+45	+2.3	......	......
70	− 33	−0.6	+ 56	+2.4	+ 53	+2.3	−19	−0.4	......	......	+58	+2.5
60	......	......	......	......	......	......	......	......	......	......	......	......
50	− 80	−1.0	+ 74	+2.0	......	......	−49	−0.6	+73	+2.0	+69	+1.9
40	−138	−1.3	+107	+2.3	+111	+2.3	−92	−0.8	+84	+1.8	......	......
30	......	......	......	......	......	......	......	......	......	......	......	......
Mean	......	−0.8	......	+2.3	......	+2.2	......	−0.8	......	+2.0	......	+2.1

φ	July 9. dt	Heavy end down. $d\phi$	July 9. dt	Heavy end up. $d\phi$	July 10. dt	Heavy end up. $d\phi$	July 10. dt	Heavy end down. $d\phi$	July 10. dt	Heavy end up. $d\phi$
′	s.	′	s.	′	s.	′	s.	′	s.	′
140	......	......	+ 16	+2.0	+21	+2.6	......	......	+ 21	+2.6
130	− 14	−0.7	......	......	......	......	−19	−0.9	......	......
120	......	......	......	......	......	......	......	......	......	......
110	− 15	−0.6	......	......	......	......	−27	−1.0	......	......
100	− 17	−0.5	+ 33	+2.4	+29	+2.1	−14:	−0.4	+ 36	+2.6
90	......	......	......	......	......	......	......	......	......	......
80	− 36	−0.8	......	......	+47	+2.4	......	......	+ 46	+2.4
70	......	......	+ 48	+2.1	+50	+2.1	−37	−0.7	+ 55	+2.4
60	......	......	......	......	......	......	......	......	......	......
50	−103	−1.2	+ 75	+2.1	+60	+1.6	−94	−1.1	+ 75:	+2.1
40	− 66	−0.6	+105	+2.1	+84:	+1.8	−93	−0.9	+107	+2.3
30	......	......	......	......	......	......	......	......	......	......
Mean	......	−0.7	......	+2.1	......	+2.1	......	−0.8	......	+2.4

		Heavy end up.	Heavy end down.
		′	′
July	1	+1.9	+0.3
	2	+2.5	+0.8
		+2.5	
	3	+2.5	+0.6
		+2.7	
	4	+2.8	+0.8
		+2.8	
		+2.3	+0.9
		+2.9	
July	7	+2.2	−0.7
		+2.1	
		+2.2	−0.8
		+2.3	
	8	+2.2	−0.8
		+2.0	
	9	+2.1	−0.7
		+2.1	
	10	+2.1	−0.8
		+2.4	
Means.			
First days		+2.6	+0.7
Last days		+2.2	−0.8

The inclination of the zero-point having thus been ascertained, the time of each observation of amplitude to the right and to the left was corrected for inclination so as to give the time of reaching an arc on each side of the vertical of an integral number of tens of minutes. The means of the results for right and left were then taken, in cases where observations were made on both sides. A table calculated from the formula was then entered, giving $t-t_0$ for every ten minutes of Φ, and from this the value of t_0 was obtained. The following tables show the result. The *hours* are omitted.

UNITED STATES COAST SURVEY—PARIS.—DECREMENT OF ARC.—CALCULATION OF t_0.

ϕ	Jan. 26.—Heavy end up.				t	t_0	Jan. 26.—Heavy end down.				t	t_0	Jan. 28.—Heavy end down.				t	t_0	Jan. 28.—Heavy end up.				t	t_0
	From E.		From W.		Mean		From E.		From W.		Mean		From E.		From W.		Mean		From E.		From W.		Mean	
′	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*
150																								
140																								
130													32	23	32	27:	32.4	39.3						
120																								
110			35	17	35.3	8.7	30	17	30	20	30.3	29.2	39	54	39	58:	39.9	38.8	27	22	27	20	27.3	0.7
100	37	35			37.6	8.9	35	0	35	3	35.0	29.1	44	36	44	37:	44.6	38.7						
90																								
80	42	45			42.7	8.6	47	15	47	3	47.1	28.8	56	59	56	52:	56.9	38.6	34	34	34	38	34.6	0.5
70	46	11	45	57	46.1	8.4	55	8	54	48	55.0	28.5	64	26:			64.4	37.9	38	17	38	8	38.2	0.5
60																								
50	55	38	55	41	55.7	8.3	76	13	78	23r	76.2	27.3	85	48	85	57	85.9	37.0						
40	63	23r	62	25	62.4	8.0	91	51	92	1	91.9	27.1	101	59	101	40	101.8	37.0						

ϕ	Jan. 29.—Heavy end up.				t	t_0	Jan. 29.—Heavy end up.				t	t_0	Feb. 2.—Heavy end up.				t	t_0	Feb. 2.—Heavy end down.				t	t_0
	From E.		From W.		Mean		From E.		From W.		Mean		From E.		From W.		Mean		From E.		From W.		Mean	
′	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*
150																								
140																								
130																								
120																								
110															43	54	43.9	17.3	32	4	32	11	32.1	31.0
100													45	49			45.8	17.1	36	49	36	48	36.8	30.9
90							56	20	56	21	56.3	25.1												
80	36	20	36	17	36.3	2.2	59	13	59	14	59.2	25.1	51	27			51.5	17.4						
70	39	48	39	57	39.9	2.2	62	40	62	44	62.7	25.0	54	58	54	53	54.9	17.2						
60																								
50			49	34	49.6	2.2	72	27	[72	56]	72.5	25.1	64	37	64	39	64.6	17.2						
40							79	33	79	31	79.5	25.1	71	42	71	47	71.8	17.4						

ϕ	Feb. 2.—Heavy end down.				t	t_0	Feb. 4.—Heavy end down.				t	t_0	Feb. 4.—Heavy end up.				t	t_0
	From E.		From W.		Mean		From E.		From W.		Mean		From E.		From W.		Mean	
′	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*	*m.*	*s.*	*m.*	*s.*	*m.*	*m.*
150																		
140															35	9	35.1	13.5
130																		
120																		
110	53	46	53	51	53.8	52.7	25	46	26	1	25.9	24.8	39	52	39	55	39.9	13.3
100	58	31	58	26	58.5	52.6	30	35	30	35	30.6	24.7			42	0	42.0	13.3
90																		
80	70	17	70	17	70.3	52.0	42	02	42	10	42.1	23.8	47	13	47	24	47.3	13.2
70	77	48	77	57	77.9	51.4	49	36	49	05	49.3	22.8	50	56	50	57	50.9	13.2
60																		
50	98	52	99	8	99.0	50.1	69	17	68	55	69.1	20.2	60	49	60	49	60.8	13.4
40	115	45	115	3	115.4	50.6	82	40	82	19	82.5	17.7	67	59	67	46	67.9	13.5

UNITED STATES COAST SURVEY—BERLIN.—DECREMENT OF ARC.—CALCULATION OF t_0.

HEAVY END UP: NAME BACK.

ϕ	Correction for inclination.	April 24.				Mean	t_0	April 26.				Mean	t_0	April 29.				Mean	t_0	May 2.				Mean	t_0
		W.		E.				W.		E.				W.		E.				W.		E.			
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	2	35	32			35.5	13.9	20	35			20.6	59.0	45	29			45.5	23.9	5	13			5.2	43.6
130	2																								
120																									
110	3																								
100	4	42	29			42.5	13.8	27	34			27.6	58.9							12	2			12.0	43.3
90																									
80	5	47	55	48	5r	47.9	13.8	32	56			32.9	58.8	57	54	58	0r	57.9	23.8	17	16	17	18	17.3	43.2
70	6	51	37:			51.6	13.9	36	33			36.6	58.9	61	33:			61.5:	23.8	20	55	20	55	20.9	43.2
60	8																								
50	10	61	23	61	15	61.3	13.9	46	14	46	9	46.2	58.8	71	19	71	14:	71.3	23.9	29	47r	30	42	30.7	43.3
40	13	68	27	68	25:	68.4	14.0	53	21	53	25:	53.4	59.0	78	28:			78.5:	24.1	36	18r	37	23	37.4	43.0
30	17																								

HEAVY END UP: NAME FORWARD.

ϕ	Correction for inclination.	April 20.				Mean	t_0	April 25.				Mean	t_0	April 28.				Mean	t_0	April 30.				Mean	t_0
		W.		E.				W.		E.				W.		E.				W.		E.			
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	15	35	39	35	38	35.6	14.0	37	0	36	56	37.0	15.4	22	31			22.5	0.9	53	59	53	52	53.9	32.3
130	17																								
120																									
110	22																								
100	26	42	38	42	54	42.8	14.1	44	0	43	45	43.9	15.2	29	17			29.3	0.6	60	51	60	51	60.8	32.1
90																									
80	36	48	0	48	17	48.1	14.0	49	26	49	29	49.5	15.4	34	37	34	46	34.7	0.6	66	16	66	3	66.2	32.1
70	44	51	39	51	53	51.8	14.1	53	6	52	54	53.0	15.3	38	17:	38	18	38.3	0.6			69	31	69.5	31.8
60	55																								
50	69	61	11	62	9	61.7	14.3			62	47	62.8	15.4	47	36	48	21	48.0	0.6	79	19	79	11	79.2	31.8
40	90	68	23	68	47	68.6	14.2	69	55	69	28	69.7	15.3	54	31:	55	4	54.8	0.4	85	39	86	1	85.8	31.4
30	121	77	31	79	54	78.7	15.1	79	41	79	31	79.6	16.0												

HEAVY END UP: NAME FORWARD—CONTINUED.

ϕ	Correction for inclination.	May 4.				Mean	t_0	June 1.				Mean	t_0	June 3.				Mean	t_0	June 3.				Mean	t_0
		W.		E.				W.		E.				W.		E.				W.		E.			
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
70	44	46	45	46	13	46.5	8.8	26	9	25	42	25.9	48.2	40	39	40	34	40.6	2.9	11	29	11	17	11.4	33.7
30	121									52	19	52.3	48.7	67	35	66	29	67.0	3.4			37	4	37.1	33.5

HEAVY END UP: NAME FORWARD—CONTINUED.

ϕ	Correction for inclination.	June 4.				Mean	t_0	June 5.				Mean	t_0	June 5.				Mean	t_0
		W.		E.				W.		E.				W.		E.			
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
70	44	2	21	2	37	2.5	24.8	58	41	58	45	58.7	21.0	30	40	30	48	30.7	53.0
30	121			29	40	29.7	26.1			85	40	85.7	22.1			57	29	57.5	53.9

BERLIN.—DECREMENT OF ARC.—CALCULATION OF t_0—Continued.

HEAVY END DOWN: NAME BACK.

φ	Correction for inclination.	April 24. W.	April 24. E.	Mean.	t_0	April 26. W.	April 26. E.	Mean.	t_0	April 29. W	April 29. E.	Mean.	t_0
′	*s.*	*m. s.*	*m. s.*	*m.*	*m.*	*m. s.*	*m. s.*	*m.*	*m.*	*m. s.*	*m. s.*	*m.*	*m.*
140	25	52 4	51 51	52.0	2.3	18 9	18 16	18.2	28.5	57 57	58 2	58.0	8.3
130	28	55 22	....	55.4	2.3	....	21 17	21.3	28.2	60 57	61 22:	61.1	8.0
120	......	......	......	......	......	......	......	......	......	......	......	......	......
110	37	62 56	62 41	62.8	1.7	......	......	......	......	68 53	69 4	69.0	7.9
100	43	67 46	67 25	67.6	1.7	33 56	33 44	33.8	27.9	73 45	73 54	73.8	7.9
90	......	......	......	......	......	......	......	......	......	......	......	......	......
80	60	79 51	79 14	79.5	1.2	46 22	46 13	46.3	28.0	85 54	86 18	86.1	7.8
70	73	87 58	87 20	87.6	1.1	53 59	53 45	53.9	27.4	....	94 22	94.4	7.9
60	91	....	....	......	......	....	63 42:	63.7:	27.2	103 49	104 49:	104.3	7.8
50	115	109 24	110 43r	109.4	0.5	75 17	75 43	75.5	26.6	116 5:	116 35	116.3	7.4
40	149	124 55	124 5	124.5	59.7	91 24:	91 0:	91.2:	26.4	131 16	132 19:	131.8	7.0
30	201	147 4	146 5	146.6	0.6	113 1	113 19	113.2	27.2	153 26	158 38r	153.4	7.4

HEAVY END DOWN: NAME BACK—CONTINUED.

φ	Correction for inclination.	May 2. W.	May 2. E.	Mean.	t_0
′	*s.*	*m. s.*	*m. s.*	*m.*	*m.*
140	25	2 40	2 37	2.6	12.9
130	28	5 50	5 47	5.8	12.7
120	......	......	......	......	......
110	37	13 36	13 31	13.6	12.5
100	43	18 17	18 5	18.2	12.3
90	......	......	......	......	......
80	60	31 3r	30 16	30.3	12.0
70	73	38 32	38 3	38.3	11.8
60	91	....	....	......	......
50	115	59 24	61 0r	59.4	10.5
40	149	75 23	75 56:	75.7	10.9
30	201	95 38	99 3:	97.3	11.3

HEAVY END DOWN: NAME FORWARD.

φ	Correction for inclination.	April 19. W.	April 19. E.	Mean	t_0	φ	Correction for inclination.	April 20. W.	April 20. E.	Mean	t_0
′	*s.*	*m. s.*	*m. s.*	*m.*	*m.*	′	*s.*	*m. s.*	*m. s.*	*m.*	*m.*
140	52	49 10	49 1	49.1	59.4	140	10	......	......	......	......
130	58	52 19	52 17	52.3	59.2	130	11	42 56	42 47	42.9	49.8
120	......	......	......	......	......	120	......	......	......	......	......
110	77	......	......	......	......	110	14	50 39	50 33	50.6	49.5
100	89	......	......	......	......	100	16	......	......	......	......
90	......	......	......	......	......	90	......	......	......	......	......
80	125	77 16	77 21	77.3	59.0	80	23	68 0	68 36r	68.0	49.7
70	152	......	......	......	......	70	28	76 0	75 23	75.7	49.2
60	188	95 23	95 0	95.2	58.7	60	35	......	......	......	......
50	238	107 8	107 38	107.4	58.5	50	44	98 20	97 33	97.9	49.0
40	310	......	......	......	......	40	57	113 20	113 31	113.4	48.6
30	417	144 30	147 54	146.2	60.2	30	77	134 56	136 28	135.7	49.7

HEAVY END DOWN: NAME FORWARD—CONTINUED.

φ	Correction for inclination.	April 25. W.	April 25. E.	Mean.	t_0	April 28. W.	April 28. E.	Mean.	t_0	April 30. W.	April 30. E.	Mean.	t_0
′	*s.*	*m. s.*	*m. s.*	*m.*	*m.*	*m. s.*	*m. s.*	*m.*	*m.*	*m. s.*	*m. s.*	*m.*	*m.*
140	10	45 52	....	45.9	56.2	49 30	....	49.5	59.8	42 6	42 0	42.1	52.4
130	11	48 58	....	49.0	55.9	52 48	....	52.8	59.7	45 13	45 17	45.2	52.1
120	......	......	......	......	......	......	......	......	......	......	......	......	......
110	14	56 48	56 37:	56.7	55.6	60 25	60 33	60.5	59.4	52 58	53 15	53.1	52.0
100	16	61 33	....	61.5	55.6	65 13	65 19	65.3	59.4	57 49	57 51	57.8	51.9
90	......	......	......	......	......	......	......	......	......	......	......	......	......
80	23	73 55	73 31	73.8	55.5	77 26	77 42	77.6	59.3	69 51	70 10	70.0	51.7
70	28	81 45:	81 4:	81.4	54.9	85 33	85 29	85.5	59.0	77 58	78 0	78.0	51.5
60	35	......	......	......	......	......	......	......	......	......	......	......	......
50	44	103 25	103 8	103.3	54.4	107 11	107 37	107.4	58.5	99 16	100 33	99.9	51.0
40	57	120 9:	118 52	119.5	54.7	123 27	124 56	124.2	59.4	115 42	115 32	115.6	50.8
30	77	......	......	......	......	145 30	147 16	146.5	60.5	....	138 35:	138.6:	52.6

BERLIN.—DECREMENT OF ARC.—CALCULATION OF t_0—Continued.

HEAVY END DOWN: NAME FORWARD—CONTINUED.

φ	Correction for inclination.	May 4. W.	May 4. E.	Mean.	t_0	June 1. W.	June 1. E.	Mean.	t_0	June 3. W.	June 3. E.	Mean.	t_0
′	s.	m. s.	m. s.	m.	m.	m. s.	m. s.	m.	m.	m. s.	m. s.	m.	m.
70	28	27 38	27 31	27.6	1.1	4 45	4 45	4.7	38.2	15 50	15 43	15.8	47.3
30	77	86 13	86 29	86.3	0.3	63 2	65 41	64.4	38.4	74 58	75 59:	75.5	49.5

HEAVY END DOWN: NAME FORWARD—CONTINUED.

φ	Correction for inclination.	June 4. W.	June 4. E.	Mean.	t_0	June 5. W.	June 5. E.	Mean.	t_0
′	s.	m. s.	m. s.	m.	m.	m. s.	m. s.	m.	m.
70	28	48 31	48 17	48.4	21.9	43 12	43 17	43.2	15.7
30	77	106 38	108 7	107.4	21.4	101 43	102 43	102.2	16.2

UNITED STATES COAST SURVEY.—PENDULUM AT KEW.—DECREMENT OF ARC.—CALCULATION OF t_0.

HEAVY END UP: FIRST DAYS.

φ	Correction for inclination.	July 1. S.	July 1. N.	Mean	t_0	July 2. S.	July 2. N.	Mean	t_0	July 2. S.	July 2. N.	Mean	t_0	July 3. S.	July 3. N.	Mean	t_0
′	s.	m. s.	m. s.	m.	m.	m. s.	m. s.	m.	m.	m. s.	m. s.	m.	m	m. s.	m. s.	m.	m.
140	10	0 53	0 49	0.8	39.2	7 25	7 30:	7.5	45.9	22 50	22 53	22.9	1.3	33 19	33 18	33.3	11.7
130	12					8 56	8 58:	8.9	45.8								
120	13					10 30	10 26	10.5	45.8								
110	15																
100	18	8 5	7 56	8.0	39.3		14 36r			29 53	29 57	29.9	1.2	40 22	40 28	40.4	11.7
90	21					17 4	17 04	17.1	45.9								
80	25	13 33	13 12	13.4	39.3					35 25	35 25	35.4	1.3	46 0	45 55	46.0	11.9
70	30	16 54	17 2	17.0	39.3	23 47	23 52	23.8	46.1	38 58	39 2	39.0	1.3		49 33	49.6	11.9
60	37					28 34r	27 47	27.8	45.8								
50	47	27 31	26 33	27.0	39.6	33 29	33 1	33.3	45.9	49 1	48 33	48.8	1.4	59 18	58 57	59.1	11.7
40	62		33 26	33.4	39.0	40 49	40 18	40.6	46.2	55 50r	55 27	55.4	1.0		66 14	66.2	11.8
30	83					50 39	50 19	50.5	46.9								

HEAVY END UP: FIRST DAYS—CONTINUED.

φ	Correction for inclination.	July 3. S.	July 3. N.	Mean	t_0	July 4. S.	July 4. N.	Mean	t_0	July 4. S.	July 4. N.	Mean	t_0	July 4. S.	July 4. N.	Mean	t_0
′	s.	m. s.	m. s	m.	m.	m. s.	m. s.	m.	m.	m. s.	m. s.	m.	m.	m. s.	m. s.	m.	m.
140	10	35 53	36 0	35.9	14.3	42 49	42 56	42.9	21.3	32 19	32 23	32.4	10.8	2 17	2 16	2.3	40.7
130	12																
120	13																
110	15		40 56	40.9	14.3		47 48	47.8	21.2		37 15	37.2	10.6				
100	18	43 7	43 9	43.1	14.4	49 55	50 1	50.0	21.3	39 28	39 29	39.3	10.6	9 33	9 28	9.5	40.8
90	21																
80	25	48 40	48 34	48.6	14.5	55 27	55 32	55.5	21.4	44 45	44 54	44.8	10.7	15 2	14 58	15.0	40.9
70	30	51 55	51 50	51.9	14.2	59 3	59 8	59.1	21.4	48 17	48 32	48.4	10.7	18 30	18 28	18.5	40.8
60	37																
50	47	61 59:	61 49	61.9	14.5	68 45	68 49	68.8	21.4	58 19	57 59	58.1	10.7	28 22	27 56	28.2	40.8
40	62	69 9	68 54	69.0	14.6	75 59	75 50	75.9	21.5	64 59	65 9	65.1	10.7				
30	83																

PENDULUM AT KEW.—DECREMENT OF ARC.—CALCULATION OF t_0—Continued.

HEAVY END UP: FIRST DAYS—CONTINUED.

φ	Correction for inclination.	July 4.				Mean	t_0
		S.		N.			
′	s.	m.	s.	m.	s.	m.	m.
140	10	59	33	59	41	59.6	38.0
130	12	...	...	...	...	...	...
120	13	...	...	...	...	...	...
110	15	...	...	64	35	64.6	38.0
100	18	66	49	66	48	66.8	38.1
90	21	...	...	...	...	...	...
80	25	72	9	72	18	72.2	38.1
70	30	75	38	76	5	75.6	37.9
60	37	...	...	...	...	...	...
50	47	85	27	85	26	85.4	38.0
40	62	92	20	...	...	92.3	37.9
30	83	...	...	...	...	...	...

HEAVY END UP: LAST DAYS.

φ	Correction for inclination.	July 7.				Mean	t_0	July 7.				Mean	t_0
		S.		N.				S.		N.			
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	9	18	52	18	56	18.9	57.3	37	32	37	29	37.5	15.9
130	10	...	...	...	...	...	...	...	...	...	...	...	...
120	11	...	...	...	...	...	...	...	...	...	...	...	...
110	13	...	...	...	...	...	...	...	...	42	34	42.6	16.0
100	15	26	1	26	0	26.0	57.3	44	40	44	37	44.6	15.9
90	18	...	...	...	...	...	...	...	...	...	...	...	...
80	21	31	39	31	38	31.6	57.5	50	15	50	11	50.2	16.1
70	26	...	...	35	0	35.0	57.3	53	52	54	0	53.9	16.2
60	32	...	...	...	...	...	...	...	...	...	...	...	...
50	40	44	48	44	38	44.7	57.3	63	40	63	43	63.7	16.3
40	52	51	33	51	16	51.4	57.0	71	12	70	43	71.0	16.6
30	71	...	...	...	...	...	...	...	...	...	...	...	...

HEAVY END UP: LAST DAYS—CONTINUED.

φ	Correction for inclination.	July 7.				Mean	t_0	July 7.				Mean	t_0	July 8.				Mean	t_0	July 8.				Mean	t_0
		S.		N.				S.		N.				S.		N.				S.		N.			
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	9	27	5	27	10	27.1	5.5	19	42	19	44	19.7	58.1	18	23	18	22	18.4	56.8	9	40	9	46	9.7	48.1
130	10	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
120	11	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
110	13	...	...	32	6	32.1	5.5	...	...	24	47	24.8	58.2	...	...	23	22	23.4	56.8	...	...	14	42	14.7	48.1
100	15	34	19	34	23	34.3	5.6	...	...	...	...	...	...	25	38	25	39	25.6	56.9	16	55	16	52	16.9	48.2
90	18	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
80	21	39	53	39	56	39.9	5.8	32	32	32	31	32.5	58.4	31	24r	31	17	31.3	57.2	22	21	22	24	22.4	48.3
70	26	43	39	43	33	43.6	5.9	36	16	36	20	36.3	58.6	34	48	34	49	34.8	57.1	...	...	...	...	...	...
60	32	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
50	40	53	41	53	20	53.5	6.1	46	4	45	58	46.0	58.6	...	...	...	...	...	...	35	33	35	26	35.5	48.1
40	52	60	15	60	11	60.2	5.8	52	55	52	58	52.9	58.5	51	34	51	41r	51.6	57.2	42	35	42	15	42.4	48.0
30	71	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

HEAVY END UP: LAST DAYS—CONTINUED.

φ	Correction for inclination.	July 9.				Mean	t_0	July 9.				Mean	t_0	July 10.				Mean	t_0	July 10.				Mean	t_0
		S.		N.				S.		N.				S.		N.				S.		N.			
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	9	11	37	11	34	11.6	50.0	13	7	13	5	13.1	51.5	24	51	24	54	24.9	3.3	32	45	32	48	32.8	11.2
130	10	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
120	11	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
110	13	...	...	16	31	16.5	49.9	...	...	18	0	18.0	51.4	...	...	29	49	29.8	3.2	...	...	37	43	37.7	11.1
100	15	18	44	18	43	18.7	50.0	20	6	20	9	20.1	51.4	31	57	31	56	31.9	3.2	39	53	39	59	39.9	11.2
90	18	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
80	21	...	...	24	15	24.2	50.1	...	...	...	...	...	...	37	21	37	26	37.4	3.3	45	27	45	31	45.5	11.4
70	26	27	46	27	52	27.8	50.1	29	0	28	56	29.0	51.3	40	59	40	57	41.0	3.3	48	57	49	0	49.0	11.3
60	32	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
50	40	37	32	37	21	37.4	50.0	38	38	38	33	38.6	51.2	50	53	50	33	50.7	3.3	58	50	58	45	58.8	11.4
40	52	44	24	...	...	44.4	50.0	45	30	45	31	45.5	51.1	57	55r	57	35	57.6	3.2	65	44	65	47	65.8	11.4
30	71	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

PENDULUM AT KEW.—DECREMENT OF ARC.—CALCULATION OF t_0—Continued.

HEAVY END DOWN: FIRST DAYS.

φ	Correction for inclination.	July 1. S.		July 1. N.		Mean	t_0	July 2. S.		July 2. N.		Mean	t_0	July 3. S.		July 3. N.		Mean	t_0	July 4. S.		July 4. N.		Mean	t_0
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	6																								
130	7	18	24			18.4	25.3	39	25	39	37	39.5	46.4	48	59	49	5	49.0	55.9	43	59	44	3	44.0	50.9
120	8																								
110	10	26	24			26.4	25.3	47	32	47	35	47.6	46.5	56	45	56	50	56.8	55.7						
100	11	31	13			31.2	25.3	52	34r	52	33	52.5	46.6							56	49	56	45	56.8	50.9
90	13																								
80	15	44	19:			44.3:	26.0	64	45	64	40	64.7	46.4	74	28	74	17	74.4	56.1	69	3	69	13	69.1	50.8
70	19			51	21	51.4	24.9	72	33	72	48	72.7	46.2	82	13	82	4	82.1	55.6	77	20	77	17	77.3	50.8
60	23																								
50	29	72	50			72.8	23.9	95	15r	94	37:	94.6	45.7	104	44	104	10	104.5	55.6	98	26	98	51	98.6	49.7
40	38	89	0	88	22	88.7	23.9	110	11	109	39	109.9	45.1	120	10	119	54	120.0	55.2	114	52	114	39	114.8	50.0
30	51																								

HEAVY END DOWN: FIRST DAYS—CONTINUED.

φ	Correction for inclination.	July 4. S.		July 4. N.		Mean	t_0
′	s.	m.	s.	m.	s.	m.	m.
140	6						
130	7	5	45	5	48	5.8	12.7
120	8						
110	10	13	22	13	42	13.5	12.4
100	11	18	29	18	34	18.5	12.6
90	13						
80	15	30	59	31	5	31.0	12.7
70	19	38	47	39	12	39.0	12.5
60	23						
50	29	60	46:	60	24:	60.6	11.7
40	38	77	51	77	28	77.7	12.9
30	51						

HEAVY END DOWN: LAST DAYS.

φ	Correction for inclination.	July 7. N.		July 7. S.		Mean	t_0	July 7. N.		July 7. S.		Mean	t_0
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	7												
130	8	21	10	21	2:	21.1	28.0	27	59	27	59:	28.0	34.9
120	9												
110	11	29	28r	29	21	29.4	28.3	36	3	36	0	36.0	34.9
100	13	34	34r	34	23r								
90	15												
80	18	46	54	46	46	46.8	28.5	53	44	53	31	53.6	35.3
70	21	55	6	55	5	55.1	28.6	62	10	62	1	62.1	35.6
60	27												
50	34			77	51	77.9	29.0	84	32	84	44	84.6	35.7
40	44	93	40	93	52	93.8	29.0	100	54	101	44	101.3	36.5
30	59												

HEAVY END DOWN: LAST DAYS—CONTINUED.

φ	Correction for inclination.	July 8. N.		July 8. S.		Mean	t_0	July 9. N.		July 9. S.		Mean	t_0	July 10. N.		July 10. S.		Mean	t_0
′	s.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.	m.	s.	m.	s.	m.	m.
140	7																		
130	8	24	43	24	40	24.7	31.6	13	23	13	21	13.4	20.3	19	9	19	13	19.2	26.1
120	9																		
110	11	32	39	32	54	32.8	31.7	21	29	21	22	21.4	20.3	27	4	27	9	27.1	26.0
100	13	37	46	37	40	37.7	31.8	26	20	26	11	26.3	20.4	32	20r	32	8r		
90	15																		
80	18	50	8	50	17	50.2	31.9	38	57	38	57	38.9	20.6						
70	21	58	48	58	25	58.6	32.1							52	39	52	34	52.6	26.1
60	27																		
50	34	81	7	80	48	81.0	32.1	69	17	69	52	69.6	20.7	74	36	75	2	74.8	25.9
40	44	96	42	96	48	96.7	31.9	86	10	85	48	86.0	21.2	91	4	91	9	91.1	26.3
30	59																		

The colons affixed to observations in the above table indicate those which were considered, at the time of making them, of inferior value; and the rejections have, in almost all cases, been made upon the exclusive authority of the original notes.

It will be seen that the value of t_0, which ought to remain constant for each experiment, frequently undergoes a progressive change. On this account three successive values of the constant have been adopted for each experiment, one from $\Phi = 2°$ to $\Phi = 1\frac{1}{2}°$, a second from $\Phi = 1\frac{1}{2}°$ to $\Phi = 1°$, and a third from $\Phi = 1°$ to $\Phi = \frac{1}{2}°$. Within each of these limits there is in no case any change which could occasion a sensible error in the correction for arc.

The values of t_0 having been obtained, the next step was to get the value of Φ at the mean instant of each set of transits. This was done by subtracting t_0 from the mean time of each set, entering the value of $t - t_0$ in a table, and taking out that of Φ.

The next step was to find the integral $\frac{1}{16}\int \Phi^2 \, dt$. The formula, obtained by the integration of the expression for Φ given above, is,

$$\int \Phi^2 \, dt = -\frac{1}{c}\,\Phi - \frac{1}{2}\,\frac{b}{c^2}\,\text{log. sin.}\left\{\sqrt{ac - \tfrac{1}{4}b^2}\,(t - t_0)\right\} + \left(\frac{1}{2}\,\frac{b^2}{c^2} - \frac{a}{c}\right)\left(t - t_0\right)$$

In some of the calculations, these three terms were calculated for the observed values of Φ and $(t - t_0)$. In other cases the integral was taken out of a table constructed for the purpose.

The correction of each interval between the successive sets of transits was separately calculated. The approximate values of the corrections were generally as follows:

Interval.	Correction. Heavy end down.	Correction. Heavy end up.
	s.	s.
From $\Phi = 2°$ to $\Phi = 1\frac{1}{2}°$	−0.053	−0.023
From $\Phi = 1\frac{1}{2}°$ to $\Phi = 1°$	−0.043	−0.018
From $\Phi = 1°$ to $\Phi = 1\frac{1}{2}°$	−0.033	−0.014

All the observations of arc at Hoboken, when the Geneva support was employed, were made on a scale divided into decimal parts of the radius. This scale being freely movable, was carefully placed with its zero exactly under the point of the pendulum, when hanging free; so that it was not in general found worth while to observe the arc on more than one side, during the observations taken in June, September, and October—but one allowance for position of zero (0.00043 on September 25) being found necessary in these series. In the observations taken in December and in 1878 less care was taken in placing the scale, and the arc was always observed on both sides, except between 0.024 and 0.010 on the left, when it was hidden from view by one of the supports of the lower platform of the pendulum receiver.

The following tables give the observations of arc in detail, followed by the calculation of inclination, made according to the methods before given.

UNITED STATES COAST SURVEY—HOBOKEN, JUNE, 1877.—DECREMENT OF PENDULUM ARC.—FULL PRESSURE.—SOLID SUPPORT.

TIMES OF REACHING DIFFERENT AMPLITUDES.

	HEAVY END UP.										HEAVY END DOWN.									
June	11	14	15	16	17	19	20	22	29	Mean.	11	14	15	16	17	19	20	22	29	Mean.
′	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
380	......	......	......	......	......	......	......	......	......	......	......	54.5	......	......	......	......	......	......	......	......
370	......	......	......	......	......	......	......	......	......	......	......	57.4	......	26.0	52.1	13.3	2.0	29.7	4.7	......
360	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
350	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
340	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
330	......	......	......	......	......	......	......	......	......	......	......	62.5	......	31.3	57.2	......	7.0	34.8	9.7	34.40
320	......	......	......	......	......	......	......	......	......	......	38.8	64.4	......	......	59.0	20.0	8.4	36.2	11.6	36.07
310	......	......	......	......	......	......	......	......	......	......	......	......	......	34.5	60.6	21.7	10.2	38.1	13.4	37.75
300	13.2	......	......	24.9	49.4	13.9	......	......	7.9	20.44	41.8	67.5	......	36.0	62.7	23.4	......	......	15.0	39.41
290	......	......	......	26.1	......	14.7	......	......	8.6	21.30	......	68.9	......	38.0	......	24.9	......	41.2	16.5	40.99
280	14.6	17.6	17 8	......	50.4	15.4	5.1	......	......	21.98	44.8	70.7	......	39.5	66.0	26.7	15.3	42.8	18.4	42.66
270	......	......	......	......	......	......	......	......	......	(22.98)	......	......	......	......	68.2	......	17.8	......	......	45.06
260	......	......	......	......	......	......	......	25.6	......	23.89	......	......	......	......	......	......	......	47.3	23.1	47.61
250	......	......	......	......	......	......	......	......	......	(24.88)	......	77.0	39.2	47.0	72.8	33.8	22.2	50.0	25.6	49.64
240	......	22.0	......	......	......	......	......	......	......	......	54.4	81.1	41.7	......	......	......	25.0	52.3	28.5	52.50
230	......	23.1	23.1	31.6	55.8	20.6	......	......	14.4	27.07	57.3	......	......	......	78.3	39.0	27.1	54.9	......	54.91
220	......	24.5	24.5	......	56.8	......	11.0	30.0	15.3	28.24	......	86.2	47.4	54.9	80.9	41.5	......	58.1	......	57.81
210	21.6	25.7	26.0	......	58.3	22.9	12.4	31.2	16.8	29.56	62.5	88.5	......	58.1	......	44.7	......	61.2	36.4	60.74
200	22.7	27.0	27.2	35.4	......	24.2	13.8	32.6	17.9	30.84	67.3	......	53.6	60.6	87.0	48.2	......	64.1	39.4	64.04
190	24.6	......	......	36.6	61.1	25.6	15.1	34.1	19.3	31.09	70.1	95.9	56.2	64.8	90.7	50.7	......	66.9	43.1	67.30
180	......	......	......	......	62.5	27.1	......	35.6	......	33.91	73.7	......	......	68.5	94.2	......	......	......	......	70.96
170	......	32.2	......	......	......	......	......	......	......	35.42	......	103.5	......	......	......	......	......	74.9	49.5	74.74
160	......	34.6	33.8	41.7	......	......	......	......	24.0	37.34	......	......	......	......	......	......	......	81.3:	......	(79.80)
150	31.2	36.3	35.3	43.6	67.8	32.2	21.6	40.5	26.0	39.11	......	......	......	81.5	107.6	......	......	83.5	59.5	84.36
140	33.0	......	37.3	......	70.1	......	......	......	27.8	41.10	91.6	117.7	......	86.2	112.0	81.6:	......	87.7	......	88.66
130	34.8	40.6	39.5	47.5	72.1	36.0	......	44.4	29.5	43.05	96.7	......	......	90.7	116.4	......	......	......	68.0	93.40
120	37.1	43.5	42.4	49.8	74.1	......	28.1	46.4	31.2	45.34	101.5	127.2	84.1	95.8	121.2	......	......	97.4	73.2	98.36
110	......	46.3	44.9	52.4	......	40.4	30.1	49.2	33.9	47.91	110.6	134.7	90.1	102.6	128.2	95.4	......	104.4	80.9	105.86
100	42.5	49.6	......	55.5	79.8	43.5	32.7	......	37.2	51.01	116.6	142.8	96.2	110.2	136.8	103.5	......	111.2	88.0	113.16
90	45.7	53.9	51.0	59.0	83.2	46.6	36.2	55.5	......	54.39	125.9	152.2	103.4	117.4	144.2	......	......	119.2	......	121.42
80	50.2	58.9	56.0	......	87.5	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......

UNITED STATES COAST SURVEY.—PENDULUM, HOBOKEN, IN VACUUM APPARATUS.—SEPTEMBER AND OCTOBER, 1877.—HEAVY END DOWN.—H. F., OBSERVER.

TIMES OF REACHING THE DIFFERENT AMPLITUDES.

Arc.	Full pressure.			15 inches.		5 inches.	1½ inch.	½ inch.	¼ inch.	Means.	
	30.3 Sept. 27. 8h	30.2 Oct. 5. 13h	30.2 Oct. 5. 14h	Oct. 5. 8h	Oct. 5. 11h	Oct. 1. 8h	Oct. 1. 12h	Sept. 29. 10h	Oct. 3. 9h	m. 30.23	m. 15
	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
.0370	....	26.5	54.3	54.7	7.5	14½	44½	2	16	26.6	7.5
360	....	27.8	55.6	57.0	9.8	....	53½	16	33½	27.9	9.8
350	46.7	....	....	....	....	23	62½	30½	49	29.3	....
340	48.1	....	....	....	....	27	72	48	71	30.7	....
330	....	....	....	63.2	15.8	32½	81	65	96½	....	15.8
.0320	....	31.1	....	65.3	18.2	37½	90½	86½	121	33.1	18.1
310	52.8	34.3	61.8	67.7	20.6	41½	99	....	144	34.6	20.5
300	53.6	35.6	63.2	70.0	22.9	46½	104	....	168	35.8	22.8
290	54.9	36.8	64.6	72.6	24.8	....	111	127½	191	37.1	25.1
280	....	38.3	65.9	75.3	27.2	55	120	144	218	38.3	27.6
.0270	58.1	40.6	68.2	78.4	30.2	....	133½	162	249	40.6	30.6
260	59.5	....	....	81.3	....	....	144	181	274	42.0	34.0
250	....	43.4	....	83.8	35.2	....	154½	197	302	43.5	35.8
240	63.1	45.3	73.0	....	38.5	....	164	217	330	45.4	38.7
230	67.3	47.8	75.6	91.6	43.9	....	179	245	359	47.9	44.1
.0220	....	51.5	78.5	95.5	48.3	....	....	272	394	51.2	48.3
210	....	....	81.2	99.5	52.4	....	210	298	429	53.6	52.3
200	73.1	54.8	82.5	....	....	....	222	317	461	55.1	....
190	75.3	57.6	85.0	107.8	58.3	117	....	335	489	57.6	59.4
180	77.8	61.0	88.2	113.0	65.1	....	244	362	527	60.6	65.4
.0170	81.3	67.1	....	....	70.6	136	260	387	573	63.6	70.8
160	85.0	68.3	95.0	123.3	76.5	144	278	416	....	67.5	76.3
150	89.0	71.3	98.5	130.3	80.8	156	297	442	....	71.2	81.9
140	93.8	75.5	103.0	135.7	88.2	167	....	....	....	75.7	88.3
130	98.4	80.1	107.7	143.1	95.6	182	....	516	....	80.4	95.7
.0120	104.4	85.4	113.5	153.1	106.2	199	371	563	....	85.7	106.0
110	106.6	89.5	....	160.2	112.6	215	396	....	....	89.3	112.7
100	113.2	96.8	124.0	167.7	120.3	....	....	....	....	96.3	120.3
90	120.7	104.4	....	179.1	133.4	241½	....	....	....	103.8	132.6
80	....	....	....	....	....	266½	....	....	....	....	....

UNITED STATES COAST SURVEY.—PENDULUM OBSERVATIONS AT DIFFERENT PRESSURES.—DECEMBER, 1877.—HEAVY END UP.—OBSERVATIONS OF ARC.

TIMES OF REACHING THE DIFFERENT AMPLITUDES.

December	23	23	14		12		12	12		16		16		16		16		16		17	
Barometer	30.55	30.55	30.25		30.0:		29.9:	29+		29.08		29.08		27.23		27.23		22.4±		15.03	
Right or left.	R.	R.	L.	R.	L.	R.	R.	L.	R.	L.	R.	L.	R.	L.	R.	L.	R.	L.	R.	L.	R.
.039																					
38		12.5									2.8										
37		12.9									3.1									17.8	17.9
36	35.0									2.9	3.6				22.4					18.6	18.7
35	35.6												47.4				8.3				
.034										3.8		47.1		22.9		8.0	8.9				
33													48.4						17.8		
32		15.5																18.5	18.4		
31		16.0										48.8									
30		16.5																			
.029		17.1				41.0									26.8					25.4	25.5:
28	39.7	17.7									8.3			26.6			12.9				
27	40.4	18.5																		27.8	
26	41.2				41.7	42.1	0.6			8.7	9.7		53.2			13.2	14.5			29.0:	29.1
25	42.0	20.1				43.0				9.5		52.7	54.0			14.2	15.5		24.3	30.2	30.3
.024	42.9				43.6	43.9						53.6	54.7			15.0		24.6	25.4	31.7	31.9
23	43.7	21.8				44.9			27.4				55.6				17.5		26.4		33.3
22	44.7	22.8		19.7					28.3				56.7				18.5		27.6		34.8
21		23.8		20.4		47.2					14.6		57.7		34.3		19.7		29.1		
20	47.1	25.0		21.8		48.3	6.7		30.2		15.6		58.9		35.7				30.3		38.1
.019	48.2	26.1				49.5									37.1		22.0		31.7		40.2
18	49.4	27.4		24.1		50.8					18.3				38.4		23.7		33.3		42.3
17	50.7	28.7									19.7				39.9		24.7		35.4		
16	52.2	30.2		27.0		53.5					21.3				41.3						46.8
15	53.8	31.8		28.9		55.2					23.0		66.0		42.9		28.2		39.1		49.6
.014	55.8	33.7				57.3	17.2?				25.0		67.9		45.3		30.2		41.5		52.2
13	57.7	35.7		32.1		59.4	17.2?				27.0		70.0		47.3		32.6		43.7		55.2
12	59.9	37.7				61.7	20.5				29.4		72.3		49.7		34.8		46.4		58.9
11	62.3	39.8		36.0		64.2	23.5				32.0		75.1		52.8		37.3				62.7
10	64.8	42.4		38.4		66.8	27.2		49.7	29.8	35.1	72.9	78.0±		55.8	35.1	41.1	52.0	52.6		67.1
.009	67.5	45.5	41.4	41.0		70.3	30.9			32.2		75.2		53.7	59.1	37.1			56.3		71.4
08				44.9			36.0?			34.5		78.0±		56.1		40.4				76.6	76.6
07			48.2	48.4										59.5							
.004									79.9												
.003								84.0	90.6												
02					116.0	114.7															

PENDULUM OBSERVATIONS AT DIFFERENT PRESSURES.—DECEMBER, 1877.—HEAVY END UP.—OBSERVATIONS OF ARC—Continued.

December	17		4		19		19		23	22	10		10		8		8		23	22	0	
Barometer	7.51		7.5±		0.9±		0.75±		0.82	0.81	0.7±		0.7±		0.4±		0.4±		0.46	0.38	0.39	
Right or left.	L.	R.	L.	R.	L.	R.	L.	R.	R.	R.	L.	R.	L.	R.	L.	R.	L.	R.	R.	R.	L.	R.
.039															38							47.7
38									36			8			42	41				17		52.9
37							35	35	40		10	11									58.8	
36									44		14	16?			54	52					63.9	
35							45	45	48		20	21			59	58					70.2	
.034	56.1	56.5					51	50?	52						66	64	36?	37			75.8	
33	57.7	58.1					55	55	57		31	31			72	71	42	42		48	82.8	
32			13.1		49	49	60	60			36	36			80	78	49	47		55	87.8	
31			15.5?	15.0	55	55	65	65			41	41	34	34	87	85?	54	52		62	95.9	
30					61	61	70	70	72		47	46	40	40	93?	93	60	59			103.2	
.029		64.6			66	66	76	76			52	52	45	45	100	100	67	66		77	108.5	
28	65.8	66.4	21.0		71?	72	82	82			59	58	51	51	107	106	74	73			115.6	
27	68.0		23.1	21.9		77	88	88			65	65	57	58	114	114	81	80			125.3	
26	69.9	70.3		24.4	82	83	95	94			71?	71?	63	64	122	121	90	88			134.5	136.5
25	72.0	72.6	26.1	26.8	87	88	101	101			78	78	70	71	129	129					142.2	143.1
.024	74.1	74.8	29.0								85	85	76	78	138	138	106	106	21		150.2	153.1
23		77.4		30.7		101		114				93		85		148		114	28			160.5
22		80.0				107		121				99		92		156		122	36			171.5
21		82.3		36.3		114				26		107		100		165		132	47			
20		85.1							137	33		115		108		176		141	56			194.7
.019		88.1		41.3		130			144	42		124		117		186		152	67			203.5
18		90.8		45.2		138		154	152	50		133		126		197			77			214.3
17		94.5		48.3		147		164?	161	59		143		136		211				191		228.5
16		98.1				157		175	171	69		154		148		222				206		240.5
15		102.4				167		186	183	79		164				234				220		252.0
.014		106.9				177		198	194	91		176		170		247				237		272.8
13		112.0				189		210		103		189		182		262				253		
12						201		222				203		195		280						
11		122.4				215			229					210		301		263				
10		128.3				229								227		322		282				
.009		135.5												245				304				
08		143.0	94.4	95.0														330		365		
07																				388		

UNITED STATES COAST SURVEY.—PENDULUM EXPERIMENTS AT HOBOKEN AT HIGH TEMPERATURES—APRIL AND MAY, 1878.—DIMINUTION OF ARC—Continued.

ARC.	HEAVY END UP.															
Date	April 24.		April 26.	April 24.	April 26.		April 26.		April 30.		April 30.		May 2.		May 2.	
Bar	1.24		2.25	29.91	29.92		29.92		30.03		30.03		29.93		29.93	
Ther	92		100	91	97		98		103		103		95		96	
Right or left.	L.	R.	R.	R.	L.	R.	L.	R.	L.	R.	L.	R.	L.	R.	L.	R.
.034								29.9								
33							29.9	30.6				50.1				
32							30.6				50.1	50.8				51.2
31								31.5			50.8	51.3	7.5	8.0	50.9	
30		59					31.7	32.4			51.4	52.2	8.2	8.6	51.6	51.8
.029	58	65					32.6			7.7		52.7	8.7	9.4		
28	63	70			52.5	53.0			8.0	8.4	52.9	53.4		10.0	52.8	53.3
27	68	76				53.7		34.5	8.7	9.3	53.7		10.3	10.8	53.7	54.1
26	74	82			54.0	54.4	34.7	35.5	9.6	10.0			11.1	11.8	54.5	
25	80	87		14.5	54.7	55.4		36.3	10.4	11.0			12.0	13.5		
.024			5	15.3	55.7			37.3								56.7
23			10	16.7				38.2				58.0				57.6
22			16					39.4		14.0		59.1				58.7
21			21			59.2		40.4		15.3		60.3		16.4		59.8
20			27			60.6		41.6		16.3		61.5		17.6		61.1
.019			33	21.5		61.8				17.6		62.7		18.8		62.2
18				22.8		63.4		44.1		18.9		64.1				
17				23.9		64.9		45.6		20.6				21.8		64.9
16			52	25.8				47.6				67.3		23.3		66.7
15			60			68.1		49.2		24.0		69.2		25.1		68.5
.014			67	29.7		70.0		51.2		25.8		71.2				70.3
13			76			72.2		53.2		28.1		73.3				72.3
12			85	34.2		74.6		55.5		30.4		75.7		31.0		74.6
11				36.7		77.4		58.1		32.8		78.2		33.7		76.9
10				40.1				61.2		35.6		81.0		36.6		79.8
.009				43.6		84.4		64.6		39.1		84.8		40.0		
08				48.1						43.4				43.8		

PENDULUM EXPERIMENTS AT HOBOKEN AT HIGH TEMPERATURES—APRIL AND MAY, 1878.—DIMINUTION OF ARC—Continued.

ARC.	HEAVY END UP—CONTINUED.									HEAVY END DOWN.								
Date....	May 10.		May 10.		May 11.		May 11.		Mean.	May 4.		May 5.		May 6.		May 8.		Mean.
Bar	29.93		29.93		30.00		30.00		29.96	29.83		29.90		30.06		29.88		29.92
Ther ...	92		93		94		94		96.5	97		94		95		95		95.3
Right or left.	L.	R.	L.	R.	L.	R.	L.	R.	R.	L.	R.	L.	R.	L.	R.	L.	R.	R.
.034																		
33											21.9						59.1	
32											23.4					59.4	60.6	
31				57.5				27.0		21.9	24.9		16.9			61.0	62.1	
30				58.3			27.1	27.8		23.1	26.5		18.6			62.4	63.4	
.029			58.3				27.9	28.6	35.20	24.5	28.3	17.2	20.2		8.6	64.1	65.2	29.78
28		14.1	59.0	59.7		44.2	28.7	29.4	35.92	26.2		19.0	22.1			65.7	67.0	31.52
27		15.1	59.7		44.3	44.9		30.1	36.73	28.1		20.7	24.0	8.1	12.0	67.7		33.36
26	15.1		60.7	61.3	44.9	45.9	30.0	31.1	37.59	29.8	34.2	22.5		9.9	14.0			35.28
25	15.9	16.8	61.7	62.2	45.9	46.8	31.0		38.49		36.2	24.4	27.8	11.8			72.6	37.28
.024						47.7			39.44		38.3		30.0				74.7	39.46
23								33.9	40.38		40.6		32.4		21.3		77.0	41.74
22				65.1				34.9	41.42		43.0		34.8		23.3		79.6	44.02
21		20.8		66.3		50.8		36.2	42.55		45.8		37.4		25.8		82.3	46.56
20		22.1		67.6		52.1		37.3	43.78		48.9		40.3		28.6		85.2	49.40
0.19		23.3		68.9		53.4		38.7	45.02		52.1		43.7		31.7		87.9:	52.40
18		24.7		70.3		54.8		40.1	46.42				46.8					55.52
17		26.2		72.0		56.3		41.5	47.95		59.1				38.0		95.7	59.08
16									49.65		62.8		54.4		41.7		99.5	62.72
15		29.5		75.2		59.7		45.1	51.36		67.4		58.6		46.2		103.5	66.86
.014		31.6		77.3		61.7		47.0	53.30		72.1		63.2		51.2		108.0	71.30
13		33.9		79.5		63.9		49.3	55.46		77.5		68.2				113.2	76.32
12		36.1		81.8		66.4		51.9	57.80		83.5						119.3	81.94
11		38.8		84.5		69.0		54.5	60.39								125.5	87.90
10		41.8		87.7		72.0		57.4	63.37								133.6	95.00
.009		45.4		90.9		75.5		61.3	66.92						88.2			103.22
08		49.4				79.6		65.0 †			116.0		106.8					

UNITED STATES COAST SURVEY.—PENDULUM OBSERVATIONS OF DECEMBER, 1877.—CALCULATIONS OF POSITION OF ZERO FOR EACH SWING.

Dec.	0		4		8		8		10		10		12		12		14	
φ	dt	$d\phi$	dt	$d\phi$	dt	(First.) $d\phi$	dt	(Last.) $d\phi$	dt	(First.) $d\phi$	dt	(Last.) $d\phi$	dt	(First). $d\phi$	dt	(Last.) $d\phi$	dt	$d\phi$
.039																		
38					−1	−.00018												
37									+1	+.00033								
86					−2	33			+2	40								
35					−1	17			+1	20								
.034					−2	29	+1?											
33					−1	14	0	−.00000	0	0								
32					−2	29	−2	40	0	0								
31			−0.5	−.00021	−2	25	−2	29	0	0	0	+.00000						
30					0	0	−1	14	−1	− 17	0	0						
.029					0	0	−1	14	0	0	0	0						
28					−1	13	−1	14	−1	− 14	0	0						
27			−1.2	− 48	0	0	−1	13	0	0	+1	14						
26	+2.0	+.00022			−1	12	−2	25	0	0	+1	17	+0.4	+.00044				
25	+0.9	13	+0.7	+ 29	0	0			0	0	+1	14						
.024	+2.9	29			0	0	0	0	0	0	+2	29	+0.3	+ 33				
.010																		
09																	−0.4	−.00012
08			+0.6	+ 08														
.007																	+0.2	+ 4
.003															+6.6	+.00062		
02													−1.3	− 10				
Mean		+.00022		−.00016		−.00014		−.00016		+.00004		+.00010		+.00022		+.00062		−.00004

PENDULUM OBSERVATIONS OF DECEMBER, 1877.—CALCULATIONS OF POSITION OF ZERO FOR EACH SWING—Continued.

Dec.	16		16		16		16		16		17		17		19		19	
	dt	(Pr. 22½). $d\phi$	dt	(Pr. 29, first.) $d\phi$	dt	(Pr. 29, last.) $d\phi$	dt	(Pr. 27, first.) $d\phi$	dt	(Pr. 27, last.) $d\phi$	dt	(Pr. 15.) $d\phi$	dt	(Pr. 7½.) $d\phi$	dt	(First.) $d\phi$	dt	(Last.) $d\phi$
.039																		
38																		
37											+0.1	.00013			0	.00000		
36			+0.7	.00167							+0.1	12						
35															0	0		
.034				167		.00160		.00100	+0.9	.00150			+0.4	.00025	−1	20		
33													+0.4	25	0	0		
32	−0.1	−.00012													0	0	0	.00000
31						120									0	0	0	0
30															0	0	0	0
.029											+0.1	8			0	0	0	0
28								133					+0.6	33	0	0	+1	17
27												8		18	0	0		
26			+1.0	143					+1.3	162	+0.1	8	+0.4	21	−1	14	+1	17
25				129	+1.3	162			+1.3	138	+0.1	8	+0.6	26	0	0	+1	20
.024	+0.8	+ 73			+1.1	150				150	+0.2	13	+0.7	32				
......																		
.010	+0.6	+ 19	+5.3	185	+5.1	179			+6.0	188								
09				194		196	+5.4	170		208								
08				219		200		191		218	0.0	0						
.007								188										
......																		
.003																		
02																		
Mean	...	+.00026	......	+.00172	.....	+.00166	.....	+.00156	...	+.00174	.....	+.00008	.....	+.00026	.....	−.00002	.....	+.00008

ADOPTED MEANS.

Dec. 0	+.00022
4, 8	−.00016
10	+.00006
12	+.00038
14	−.00004
16 (Pr. 22½)	+.00026
16 (Pr. 27, 29)	+.00166
17 (Pr. 15)	+.00008
17 (Pr. 7½)	+.00026
19, 22, 23	.00000

UNITED STATES COAST SURVEY.—HOBOKEN, 1878, APRIL AND MAY.—DIMINUTION OF ARC.—INCLINATION OF ZERO-POINT, RIGHT *minus* LEFT.

[For each reading of arc apparent L the difference is given.]

	HEAVY END UP.																	
	April 24.		April 26.		April 26.		April 30.		April 30.		May 2.		May 2.		May 10.		May 10.	
	dt	Pr. 1.24 $d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$
	m.		*m.*		*m.*		*m.*		*m.*		*m.*		*m.*		*m.*		*m.*	
.0330					0.7	.00100												
20						100			0.7	.00100								
10									0.5	100	0.5	.00071		.00050				
300					0.7	78			0.8	89	0.4	67	0.2	33				
290	7	.00118				71					0.7	87						.00100
.0280	7	133	0.5	.00063			0.4	.00057	0.5	71			0.5	63			0.7	100
70	8	14					0.6	67		63	0.5	63	0.4	50				100
60	8	133	0.4	57	0.8	80	0.4	57			0.7	70		50		.00100	0.6	75
50	7	133	0.7	70			0 6	60				78			0.9	100	0.5	56
40				62														
Mean		.00132		.00064		.00086		.00069		.00084		.00072		.00050		.00100		.00084

	HEAVY END UP—CONTINUED.				HEAVY END DOWN.							
	May 11.		May 11.		May 4.		May 5.		May 6.		May 8.	
	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$	dt	$d\phi$
	m.		*m.*		*m.*		*m.*		*m.*		*m.*	
.0330												
20											1.2	.00080
10					3.0	.00200					1.1	73
300			0.7	.00088	3.4	220					1.0	77
290			0.7	88	3.8	227	3.0	.00182			1.1	61
.0280			0.7	88		219	3.1	175			1.3	72
70	0.6	.00086				211	3.3	174	3.9	.00231		61
60	1.0	100	1.1	110	4.4	217		179	4.1	223		
50	0.9	100		111			3.4	179		212		
40												
Mean		.00096		.00098		.00216		.00178		.00222		.00070

SUMMARY.

		Heavy end up.	Heavy end down.
April	26	.00064	
	26	86	
	30	60	
	30	84	
May	2	72	
	2	50	
	10	100	
	10	84	
	11	96	
	11	98	
	4		.00216
	5		178
	6		222
	8		
Means		.00078	.00204

The December observations were corrected for inclination by the proper additions to the time of arcs observed on the right side only. For the observations at high temperatures, the observed times on the right side were taken, and the arcs corrected,

In heavy-end-up observations, all days except the first, by	−0.0004
In heavy-end-down observations, May 4, 5, and 6, by	−0.001
In heavy-end-down observations, May 8, by	−0.00035

The differential formula connecting the arc and the time was next found. Only three constants were employed, as has been already stated, the observations not being sufficiently numerous or sufficiently exact to admit of four. The equation, then, is

$$D_t\phi = -b\,\varphi - c\,\varphi^2.$$

$$\text{Hence, } \varphi = \frac{b}{c}\left(\mathfrak{S}^{bt} - 1\right)^{-1},$$

$$\frac{1}{16}\int \varphi^2\,dt = \frac{b}{16\,c^2}\left\{ b\,t - \text{Nat. log.}\left(\mathfrak{S}^{bt}-1\right) - \left(\mathfrak{S}^{bt}-1\right)^{-1} \right\} + \text{C}.$$

$$= \frac{b}{16\,c^2}\left\{ \text{Nat. log.}\left(1 + \frac{c}{b}\,\varphi\right) - \frac{c}{b}\,\varphi \right\} + \text{C},$$

$$\text{and } t = \frac{1}{b}\ \text{Nat. log.}\left(1 + \frac{b}{c\,\varphi}\right).$$

The constants b and c, for heavy end up, calculated from the observed times of decrement, but not corrected by least squares, are given in the subjoined table.

From the observations of decrement were deduced, first, an equation of the form

$$b + c\,\varphi_m = n,$$

φ_m denoting the mean value of φ; and, second, a value of c. As c has been supposed proportional to the density of the air, we find

$$c = 0.03125\,\frac{\sqrt{p}}{\tau}$$

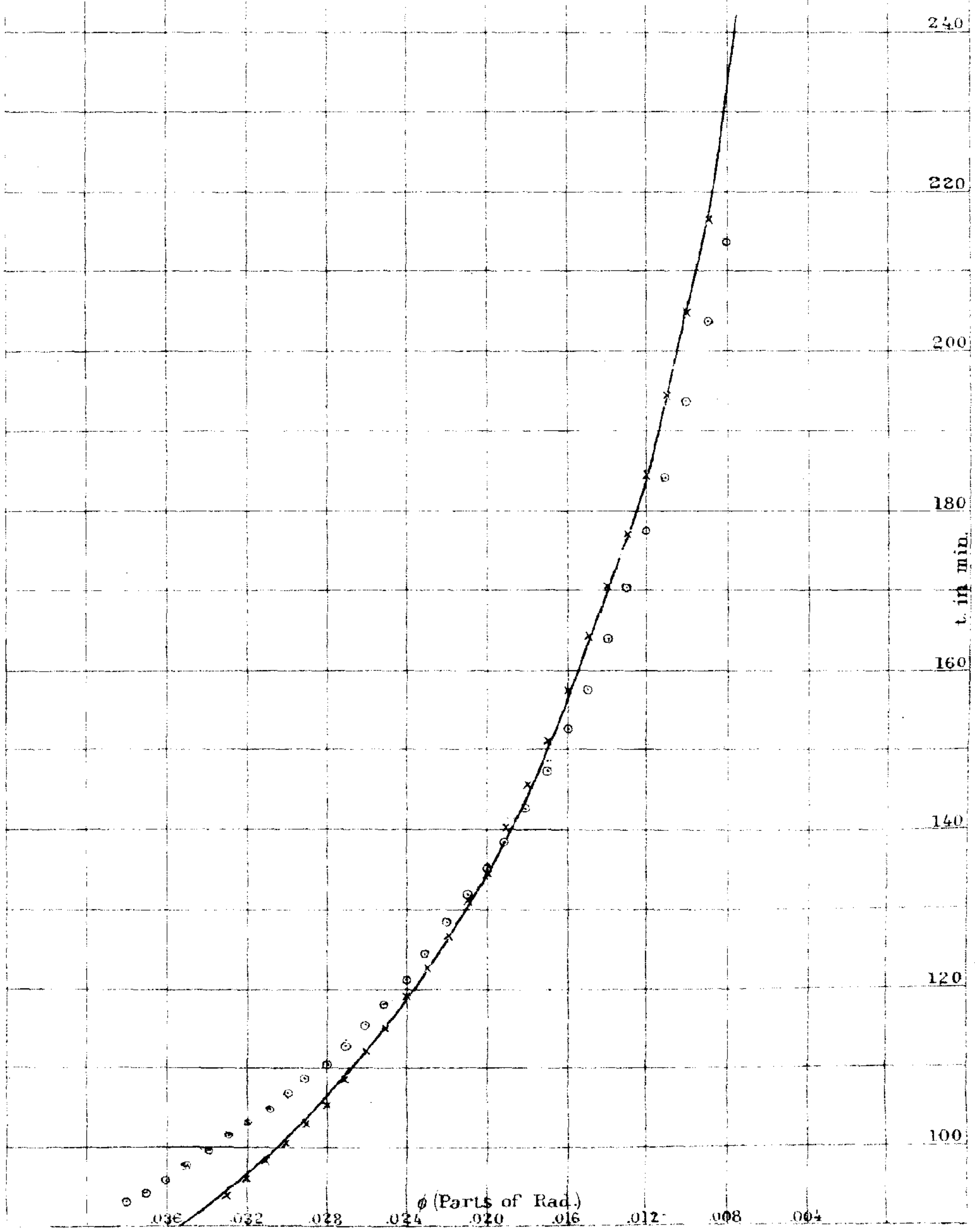
By substituting these values in the equations $b + c\,\varphi_m = n$, we obtain new values of b, called "second reduction" in the table; the "first reduction" having been obtained with the original values of c. These second values answer to the formula

$$b = 0.0013\,\tau^{1/4} + 0.00435\,p^{1/2}\,\tau^{-1/2}.$$

For the observations in which the bell-glasses were on the receiver, these alone being comparable with one another, the values of b, thus calculated, are given in the table, along with the values from the reduction of observations of decrement. Also in illustration No. 36, before alluded to.

No. 36

U. S. Geodetic Survey. Pendulum at Hoboken. Decrement of Arc
Observations of June 1877. Geneva support with bell glasses off (×)
and Obrs. of Sept. – Dec. at pressures of about 30 in. with glasses
on (⊙) compared with curve given by European Obs. Repsold Support.
Sum of h. e. up & h. e. down.



UNITED STATES COAST SURVEY.—PENDULUM AT HOBOKEN.—DECREMENT OF ARC.

[The constants b and c, as deduced from the observations, compared with values dependent on pressure and temperature of the air.]

	Pressure.	Temp. C.	τ	$\frac{p}{\tau}$	$\frac{\sqrt{p}}{\sqrt[8]{\tau}}$	ϕ m	$b+c\phi$ m	c	$0.03125\frac{p}{\tau}$	Excess of c.	b, 1st red'n.	b, 2d red'n.	$\left\{0.0013\,\tau^{\frac{3}{4}} + 0.00435\frac{\sqrt{p}}{\sqrt[8]{\tau}}\right\}$	Excess of b.
1877.	*in.*	°												
June	30. 12	19. 7	1. 016	29. 65	5. 48	. 021	0. 0364	+0. 58	....	......	0. 0242	......	....	......
Sept. and Oct.	30. 22	18. 0	1. 010	29. 92	5. 49	. 023	0. 0469	+0. 91	0. 93	—0. 02	0. 0259	0. 0254	0. 0252	+. 0002
	15. 08	18. 8	1. 013	14. 89	3. 88	. 0205	. 0270	. 39	. 46	— . 07	. 0189	. 0175	. 0182	— 7
	4. 99	20. 5	1. 019	4. 90	2. 23	. 023	. 0146	. 13	. 15	— . 02	. 0117	. 0110	. 0110	0
	1. 50	20. 3	1. 018	1. 48	1. 22	. 024	. 0084	. 09	. 05	+ . 04	. 0063	. 0073	. 0066	+ 7
	0. 50	20. 5	1. 019	0. 49	0. 71	. 0345	. 0048	+ . 02	. 02	. 00	. 0042	. 0044	. 0044	0
	0. 25	20. 6	1. 019	0. 25	0. 50	. 027	. 0032	— . 03	. 01	— . 04	. 0040	. 0029	. 0035	— 6
December ...	29. 81	10. 5	0. 984	30. 30	5. 47	. 0225	0. 0470	+1. 05	0. 95	+ . 10	0. 0250	. 0257	0. 0251	+. 0006
	27. 20	10. 4	0. 984	27. 64	5. 23	. 018	. 0421	0. 84	. 87	— . 03	. 0270	. 0265	. 0240	+ 25
	22. 45	10. 8	0. 985	22. 80	4. 75	. 017	. 0327	. 57	. 71	— . 14	. 0230	. 0206	. 0219	— 13
	15. 02	10. 5	0. 984	15. 26	3. 88	. 0185	. 0264	. 50	. 48	+ . 02	. 0171	. 0175	. 0182	— 7
	7. 46	9. 8	0. 982	7. 60	2. 74	. 021	. 0176	. 19	. 24	— . 05	. 0136	. 0126	. 0132	— 6
	0. 75	9. 9	0. 982	0. 76	0. 87	. 022	. 0062	. 01	. 02	— . 01	. 0059	. 0056	. 0051	+ 5
	0. 40	10. 0	0. 983	0. 41	0. 63	. 023	. 0046	. 01	. 01	. 00	. 0045	. 0043	. 0040	+ 3
1878.														
Apr. and May	29. 92	35. 7	1. 072	27. 91	5. 42	. 0186	0. 0400	+0. 84	0. 87	— . 03	0. 0243	0. 0238	0. 0249	—. 0011

From the calculated b and c we obtain values of t_0, which are given in the following tables.

For the observations of June, 1877, without the bell-glasses, the values of b and c given by the observed decrement of arc, were used unaltered.

UNITED STATES COAST SURVEY.—PENDULUM AT HOBOKEN, JUNE, 1877.—CALCULATION OF t_0.

[Quantities in brackets correspond to observations not on the even thousandth, but on arcs differing a little from those given in the first column.

φ	HEAVY END UP.									HEAVY END DOWN.								
	June 11.	June 14.	June 15.	June 16.	June 17.	June 19.	June 20.	June 22.	June 29.	June 11.	June 14.	June 15.	June 16.	June 17.	June 19.	June 20.	June 22.	June 29.
′	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
.038	......	......	......	......	......	......	......	......	......	[25.4]	50.7r	......	......	......	......	......	......	......
37	......	......	......	......	......	......	......	......	......	......	53.4	[14.7]	21.0	47.1	8.3	57.0	24.7	59.7
36	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
35	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
34	......	......	......	......	......	......	......	......	......	[26.2]	......	......	......	......	......	......	......	......
33	......	......	......	......	......	......	......	......	......	[26.6]	51.9	......	20.7	46.6	[8.0]	56.4	24.2	59.1
32	......	......	......	......	......	......	......	......	......	26.6	52.2	14.5?	......	46.8	7.8	56.2	24.0	59.4
31	......	......	......	......	......	......	......	......	......	26.3	......	......	20.7	46.8	7.9	56.4	24.3	59.6
30	40.3	......	......	52.0	16.5	41.0	......	......	35.0	26.2	51.9	13.7?	20.4	47.1	7.8	......	[24.3]	59.4
29	[39.8]	......	......	52.3	[15.9]	40.9	......	......	34.8	......	51.5	14.3?	20.6	......	7.5	......	23.8	59.1
28	40.0	43.0	43.2	......	15.8	40.8	30.5	......	......	25.5	51.4	......	20.2	46.7	7.6	56.0	23.5	59.1
27	......	......	......	......	......	......	......	......	......	......	......	......	......	46.9	......	56.5	......	......
26	......	......	......	......	......	......	......	49.2	......	......	......	......	......	......	[8.1]	......	23.8	59.6
25	......	......	......	......	......	......	......	......	......	[25.3]	51.3	13.5	21.3	47.1	8.1	56.5	24.3	59.9
24	......	43.6	......	[52.2]	......	......	......	......	[34.9]	26.3	53.0	13.6	[21.8]	......	[8.6]	56.9	24.2	60.4
23	[40.1]	43.6	43.6	52.1	16.3	41.1	......	......	34.9	26.6	[52.9]	......	......	47.6	8.3	56.4	24.2	[60.2]
22	......	43.8	43.8	[52.0]	16.1	......	30.3	49.3	34.6	......	52.9	14.1	21.6	47.6	8.2	......	24.8	[60.0]
21	39.7	43.8	44.1	[52.1]	16.4	41.0	30.5	49.3	34.9	26.3	52.3	......	21.9	......	8.5	......	25.0	60.2
20	39.4	43.7	43.9	52.1	......	40.9	30.5	49.3	34.6	28.0	[53.5]	14.3	21.3	47.7	8.9	......	24.8	60.1
19	39.9	......	43.6	51.9	16.4	40.9	30.4	49.4	34.6	27.5	53.3	13.6	22.2	48.1	8.1	......	24.3	60.5
18	......	......	......	......	16.3	40.9	......	49.4	......	27.6	......	......	22.4	48.1	......	......	[2500]	......
17	......	44.3	......	......	......	......	......	......	......	[27.5]	53.6	......	[22.6]	[48.5]	[22.4]r	......	25.0	59.6
16	......	44.9	44.1	52.0	......	......	......	......	34.3	[28.2]	......	......	......	......	......	......	27.3	......
15	39.6	44.7	43.7	52.0	16.2	40.6	30.0	48.9	34.4	[28.8]	......	......	23.0	49.1	......	......	25.0	61.0
14	39.3	......	43.6	[51.8]	16.4	......	......	......	34.1	28.2	54.3	......	22.8	48.6	18.2r	......	24.3	......
13	38.7	44.5	43.4	51.4	16.0	39.9	......	48.3	33.4	28.0	52.9	[11.8]	22.0	47.7	......	......	......	59.3
12	38.4	44.8	43.7	51.1	15.3	......	29.4	47.7	32.5	26.9	52.6	9.5	21.6	46.6	[15.2]	......	22.8	58.6
11	......	44.8	43.4	50.9	[15.5]	38.9	28.6	47.7	32.4	29.4	53.5	8.9	21.4	47.0	14.2	......	23.2	59.7
10	37.8	44.9	......	50.8	15.1	38.8	28.0	[47.5]	32.5	28.0	54.2	7.6	21.6	48.2	14.9	......	22.6	59.4
09	37.4	45.6	42.7	50.7	14.9	38.3	27.9	47.2	......	29.0	55.3	6.5	20.5	47.3	......	......	22.3	......
.008	37.7	46.4	43.5	[50.5]	15.0	......	......	[50.8]	......	[31.0]	......	......	......	......	......	......	......	......

UNITED STATES COAST SURVEY.—PENDULUM OBSERVATIONS AT HOBOKEN, SEPTEMBER-OCTOBER, 1877.—CALCULATED t_0 FOR EACH SWING.

φ	Full pressure.			Pr. 15.		Pr. 5.	Pr. 1.5.	Pr. 0.5.	Pr. 0.25.	φ	Pr. 30.	φ	Pr. 15.
	Sept. 27.	Oct. 5.	Oct. 5.	Oct. 5.	Oct. 5.	Oct. 1.	Oct. 1.	Sept. 29.	Oct. 3.		Sept. 25.		Sept. 26.
	m.	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*		*m.*		*m.*
										.03473	13.1		
										3363	13.4		
.037		24.0	−3.8	36.9	84.1	208.5	491.9	1120.0	1715.0	3343	12.7		
36		23.8	4.0	36.4	83.6		490.3	1118.7	1715.1	3173	13.3	.0321	61.5
35	6.0					208.1	488.8	1117.1	1717.4	2763	11.1	320	61.2
34	5.9					207.6	487.1	1113.2	1713.4	2633	11.4	319	60.9
33				36.2	83.6	206.3	486.2	1109.9	1706.9	2533	11.8	301	61.3
32		23.4		36.3	83.4	205.7	485.1	1102.6	1701.6	2483	11.7	299	61.0
31	5.1	23.6	3.9	36.1	83.2	206.2	485.2		1698.9	2433	11.2	276	59.5
30	5.7	23.7	3.9	36.2	83.3	205.9	489.3		1695.4	2263	11.0	273	59.3
29	6.0	24.1	3.7:	36.1	83.9		491.7	1107.3	1694.0	2223	10.9	270	59.1
28		24.2	3.4	36.0	84.1	207.5	492.5	1107.4	1689.2	2163	10.3	254	59.3
27	6.1	23.6	4.0	35.6	83.8		489.1	1106.5	1681.6	1983	11.0	248	59.3
26	6.5			35.6			489.3	1105.5	1680.8	1913	11.9	244	60.1
25		24.5		36.2	84.8		489.8	1108.0	1677.7	1843	12.6	236	58.8
24	6.8	24.6	3.1		84.8		488.0	1107.4	1675.9	1733	11.6	230	59.8
23	4.8r	24.3	3.5	35.1	82.8		489.2	1099.7	1674.3	1683	12.3	226	60.7
22		22.9	4.1	34.7	81.9			1093.8	1668.1	1653	12.4	210	59.3
21			4.3:	34.7	81.8		484.5	1090.4	1663.2	1543	12.5	208	58.9
20	6.4	24.7	3.0				486.8	1094.9	1662.1	1513	12.7	207	57.0
19	7.0	24.7	2.7	34.9	84.4	210.7		1101.5	1667.8	1493	12.5	200	60.1
18	7.6	24.4	2.8	34.5	82.4		495.8	1100.9	1664.8	1363	13.2	190	60.5
17	7.4	21.6r			82.0	205.2	496.7	1103.4	1655.9	1343	13.8	180	60.8
16	7.3	24.0r	2.7	34.8	81.6	207.3	496.8	1104.2		1333	12.7	170	60.6
15	7.2	24.9	2.3	33.8	83.3	206.2	497.1	1109.7		1243	11.3	160	58.9
14	6.8	25.1	2.4	34.9	82.4	207.0				1193	12.0	149	58.2
13	6.9	25.2	2.4	34.7	82.2	204.8		1106.2		1173	12.3	147	58.8
12	6.2r	25.2	2.9	32.5	79.4	202.4	491.2	1098.5		1033	13.0	139	59.1
11	10.2	27.3		34.2	81.8	201.3	493.0			0997	14.3	138	59.3
10	9.9	26.3	−0.9	36.5	83.9					963	15.9	130	59.8
09	9.9	26.2		36.1	81.8	205.0				957	15.8	120	59.7
.008						207.6				943	16.4	110	59.9
										937	15.8	100	60.2
										943	15.4	.0090	57.7
										917	16.8		
										953	13.6		
										937	13.8		
										933	14.1		
										917	14.8		
										853	15.7		
										847	15.3		
										843	15.6		
										.00847	14.8		

UNITED STATES COAST SURVEY.—PENDULUM OBSERVATIONS OF DECEMBER, 1877.—CALCULATIONS OF t_0 FOR EACH SWING.

D = Density of the air compared with one absolute atmosphere at 0° C.

D...	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.893	0.893	0.729	0.479	0.245
Dec	23	23	14	12	12	12	16	16	16	16	16	17	17
.039													
38		+9.1					+18.8						
37		9.1					18.9					22.1 22.0	
36	−12.5						19.6 18.9		+1.7			22.1 22.0	
35	12.6							−24.4		+16.3			
.034							19.7	23.6	2.2	17.1 16.2			16.6 16.2
33								24.3			+12.9		16.4 16.0
32		9.1									12.9 13.0		
31		9.2						23.6					
30		9.4											
.029		9.4		−14.5					1.5			22.0 21.9:	15.7
28	12.5	9.5					18.9		2.5	16.2			16.3 15.7
27	12.4	9.5										21.9	15.9
26	12.4			12.9 13.3	+28.2		19.9 19.1	24.4		17.5 16.2		21.9: 21.8	15.9 15.5
25	12.4	9.5		13.4			20.1	23.1 24.4		17.4 16.1	13.2	22.1 22.0	15.8 15.2
.024	12.4			13.1 13.4				23.1 24.2		17.5	14.0 13.2	22.0 21.8	15.9 15.2
23	12.3	9.6		13.5		+4.0		24.2		16.0	13.3	21.9	14.8
22	12.3	9.6	+12.7			4.1		24.5		16.0	13.3	22.0	14.6
21		9.7	13.1	13.7			18.9	24.2	1.4	16.0	13.1		14.8
20	12.5	9.6	12.8	13.7	27.9	4.4	19.0	24.3	1.2		13.3	22.2	14.7
.019	12.3	9.8		13.6					1.1	16.2	13.4	22.0	14.5
18	12.2	9.8	13.1	13.6			18.9		1.2	15.9	13.4	22.0	14.9
17	12.0	10.0					19.0		1.2	16.4	13.0		14.4
16	12.0	10.0	13.2	13.3			18.9		1.5			22.1	14.3
15	11.9	10.1	13.0	13.3			18.9	24.1	1.7	16.4	13.3	21.9	13.8
.014	12.0	10.1		13.5	{ 26.6?		18.8	24.1	1.2	16.3	13.1	22.2	13.4
13	11.8	10.2	13.8	13.5	{ 28.7?		18.9	24.1	1.4	16.1	13.4	22.3	12.8
12	11.7	10.5		13.5	27.7		18.8	24.1	1.4	16.3	13.4	22.0	
11	11.4	11.1	14.9	13.3	27.4		18.9	24.2	1.0	16.5		22.0	12.7
10	11.2	11.2	15.2	13.2	26.4	3.9	23.8 18.5	19.3 24.4±	1.0	21.7 15.7	14.2 13.4	21.9	12.8
.009	−10.6	11.4	15.5 15.9	13.4	26.0		24.7	18.3	6.5 1.1	23.1	13.8	22.4	13.3
08			15.8		24.7?		26.2	17.3±	8.1	23.8		22.8	12.5
07			16.9 16.7						9.3				
.004						5.1							
.003						11.9 5.3							
02				4.1 2.8									

PENDULUM OBSERVATIONS OF DECEMBER, 1877.—CALCULATION OF t_0 FOR EACH SWING—Continued.

D... Dec.	0.245 4		0.025 19		0.025 19		0.025 23	0.025 22	0.025 10		0.025 10		0.013 8		0.013 8		0.013 23	0.013 22	0.013 0	
.039													511.4							501.7
38							324.2			352.2			513.1	514.1				538.1		502.2
37			329.5	329.5			324.5		354.5	353.5									502.2	
36							325.0		355.0	353.0?			513.0	515.0					503.1	
35			328.5	328.5			325.5		353.5	352.5			514.3	515.3					503.1	
.034			327.3	328.3?			326.3						513.7	515.7	543.7	542.7			503.9	
33			328.1	328.1			326.1		352.1	352.1			514.3	515.3	544.3	544.3		538.3	503.5	
32	62.5		328.2	328.2	339.2	339.2			352.2	352.2			513.2	515.2	544.2	546.2		538.2	505.4	
31	61.6?	62.1	328.4	328.4	338.4	338.4			352.4	352.4	359.4	359.4	513.2	515.2?	546.2	548.2		538.2	504.3	
30			328.7	328.7	337.7	337.7	326.7		351.7	352.7	358.7	358.7	514.6?	514.6	547.6	548.6			504.4	
.029			328.4	328.4	338.4	338.4			352.4	352.4	359.4	359.4	515.2	515.2	548.2	549.2		538.2	506.7	
28	61.1		328.1	328.1	339.1?	338.1			351.1	352.1	359.1	359.1	516.2	517.2	549.2	550.2			507.6	
27	60.8	62.0	328.3	328.3		339.3			351.3	351.3	359.3	358.3	517.2	517.2	550.2	551.2			505.9	
26		61.4	327.6	328.6	340.6	339.6			351.6?	351.6?	359.6	358.6	517.9	518.9	549.9	551.9			505.4	503.4
25	61.7	61.0	328.1	328.1	342.1	341.1			351.1	351.1	359.1	358.1	519.8	519.8					506.6	505.7
.024	61.0								351.1	351.1	360.1	358.1	520.0	520.0	552.0	552.0	637.0		507.8	504.9
23		61.5		329.3		342.3				350.3		358.3		519.7		553.7	639.7			507.2
22				329.8		343.8				351.8		358.8		522.0		356.0	642.0			506.5
21		60.8				344.8		434.8		351.8		358.8		523.7		556.7	641.7			
20							330.2	433.2		352.2		359.2		523.8		558.8	643.8			505.1
.019		61.3				346.0	332.0	434.0		352.0		359.0		525.7		559.7	644.7			508.2
18		60.5		331.4		347.4	333.4	435.4		352.4		359.4		527.1			647.1			509.8
17		60.6		331.3?		348.3	334.3	436.3		352.3		359.3		526.4				546.4		508.9
16				330.9		348.9	334.9	436.9		351.9		357.9		529.6				545.6		511.1
15				331.3		350.3	334.3	438.3		353.3				532.7				546.7		514.7
.014				331.4		352.4	335.4	438.4		353.4		359.4		535.9				545.9		510.1
13				332.5		353.5		439.5		353.5		360.5		538.2				547.2		
12				334.7		355.7				353.7		361.7		539.0						
11						357.3	343.3					362.3		538.7		576.7				
10						360.2						362.2		540.2		580.2				
.009												363.3				583.4				
08	61.1	60.5														585.4				
07																				
.004																				
.003																				
02																				

UNITED STATES COAST SURVEY.—HIGH-TEMPERATURE EXPERIMENTS AT HOBOKEN, APRIL AND MAY, 1878.—CALCULATED t_0.

	HEAVY END UP.													HEAVY END DOWN.					
φ	April 24. (Pr. 1.24.)	April 26. (Pr. 2.25.)	April 24.	April 26.	April 26.	April 30.	April 30.	May 2.	May 2.	May 10.	May 10.	May 11.	May 11.	φ	May 4.	May 5.	May 6.	φ	May 8.
′	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	′	m.	m.	m.	′	m.
.0336					−4.6														
326					4.7		−24.2							.032	+38.5			.03265	+0.4
316							24.2		r−24.6					31	38.5			3165	0.3
306					4.3		24.1	+19.2			−30.3		+0.2	30	38.5	+46.5		3065	0.3
296	246.8				4.5		24.3	19.3	−23.9		30.4		0.1	29	38.5	46.4		2965	0.6
286	245.5					+20.9	24.1	19.2					0.0	28	38.4	46.3	+58.1	2865	0.4
276	245.3			−23.6		21.0	24.0	19.4	23.9	+15.3	30.3	−14.4	0.0	27		46.4		2765	0.3
266	244.5			23.5	4.3	20.9		19.4	23.9	15.1		14.7	0.1	26		46.4	58.4	2665	
256	243.8			23.3	4.4	21.1		19.3			30.2	14.8	0.0	25	38.2		58.4	2565	
246	244.3		+17.5	23.4	4.3	21.0		r18.5		15.2	30.2	14.8		24	38.4	46.8		2465	0.6
236		223.6	17.6		4.4				23.8			14.8		23	38.6	46.9		2365	0.7
226		223.2	17.2		4.3		24.1		23.7				0.0	22	38.7	46.9	58.0	2265	0.5
216		222.0			4.4	21.0	24.1		23.7		30.1		0.1	21	38.9	47.1	58.6	2165	0.6
206		222.1		23.0	4.2	20.9	24.1	19.8	23.6	15.4	30.1	14.6	0.0	20	38.9	47.3	58.9	2065	0.6
196		221.5		23.2	4.2	21.1	24.1	19.8	23.7	15.3	30.2	14.7	0.1	19	38.7	47.3	59.0	1965	0.5
186		221.2	17.3	23.0		21.2	23.9	20.0	23.4	15.5	30.1	14.6	0.1	18	38.8	47.2	59.2	1865	+0.9
176			17.4	23.2	3.9	21.3	23.9			15.5	30.1	14.6	0.1	17		47.5		1765	
166			17.9	23.1	3.8	21.2		20.0	23.1	15.6	30.2	14.5	0.3	16	39.0		60.1	1665	−0.1
156		221.6	17.7		4.1		23.8	20.2	23.2					15	39.5	47.9	60.6	1565	+0.1
146		221.0		22.7	3.8	21.4	23.8	20.3	23.1	15.9	29.8	14.3	0.3	14	39.4	48.2	60.6	1465	0.4
136		222.0	17.7	22.6	3.8	21.6	23.8		22.9	15.8	29.9	14.3	0.4	13	39.6	48.5	60.5	1365	0.5
126		221.7		22.5	3.5	21.6	23.6		22.6	15.8	29.8	14.2	0.4	12	39.8	49.1		1265	0.5
116		222.2	18.0	22.4	3.3	21.8	23.5	21.2	22.4	16.1	29.6	14.2	0.3	11	39.9			1165	0.1
106			18.3	22.4	3.1	22.2	23.2	21.3	21.9	16.2	29.5	14.0	0.5	10				1065	+0.3
096			18.1		3.0	22.6	22.8	21.6	−21.6	16.4	29.5	13.8	0.8	09				.00965	−0.4
086			18.2	−22.6	−2.8	22.7	−23.0	21.8		16.4	−29.1	13.7	0.5	08			+59.1		
.0076			+17.9			+22.6		+22 2		+16.6		−13.6	+1.0?	.007	+41.9	+51.1			

REDUCTION TO A VACUUM.

It is not usual to reduce observations with the reversible pendulum to a vacuum, because the formula for combining the results with heavy end up and heavy end down, namely,

$$T = \frac{T_d h_d - T_u h_u}{h_d - h_u},$$

is supposed to eliminate the atmospheric effect; and it really does so if the hollow bob is staunch. Nevertheless, there is a great advantage in reducing to a vacuum, for in that way we may use the reversible pendulum as two invariable pendulums, combining them by the formula

$$T = \frac{T_d h_d + T_u h_u}{h_d + h_u}.$$

In our pendulum $h_d : h_u = 7 : 3$; hence, the usual formula gives a T which is subject to $\frac{7}{4}$ of the error of T_d and $\frac{3}{4}$ of the error of T_u, while the proposed formula gives a T which is subject to only $\frac{7}{10}$ of the error of T_d and $\frac{3}{10}$ of the error of T_u, so that its error is only $\frac{4}{10}$ of that of the other. It is true that it is somewhat difficult to ascertain the amount of the atmospheric correction which must be applied before the second formula can be used, but it will be shown that it is not impossible. Nor will it be subject to any uncertainty which will be sensible unless under changes of pressure amounting to a considerable fraction of an atmosphere.

But if this is so, it may be asked, why use a reversible pendulum at all? The reply is, that we have thus a continual check upon our work; and also a means of studying knife-edge effects, etc. Moreover, the fatal effect of any accident to the pendulum is thus insured against. Another advantage of the reversible pendulum is that the center of oscillation can be ascertained with exactitude, its position being a necessary datum for calculating the effect of the atmosphere.

The presence of the air lengthens the period of oscillation of the pendulum in no less than four distinct ways: 1st, by its buoyancy; 2d, by being carried along within inclosed parts of the pendulum; 3d, by the hydrodynamic effect of its pressure; and 4th, by its viscosity.

In reckoning the buoyancy of the air, it will make a good deal of difference whether the hollow bob is open or tightly closed. It should be quite staunch; otherwise, the atmospheric effect is not eliminated. If it be so, the air it contains is to be considered as a part of the pendulum; otherwise, not. It is believed that our hollow bob is tight. To ascertain the volume of this air, we have to consider that if the hollow were filled with brass the center of mass would be at the center of figure. The existing brass whose center is at a distance $\frac{1}{2}(h_d - h_u)$ from the center of figure and the brass which would be required to fill the hollow bob and which would be at the distance, say o (which can be measured), from the center of figure, would then be in equilibrium about the center of figure. Hence, their volumes are in the ratio of $\frac{1}{2}(h_d - h_u)$ to o. It is necessary to assume a density for the air in the bob. Then the mass of the pendulum obtained by weighing and corrected for the buoyancy of brass in air must be further increased by the mass of air in the hollow bob, by multiplying it by

$$\left(1 + \frac{\frac{1}{2}(h_d - h_u)}{o}\frac{\rho_3}{\rho_1}\right),$$

where ρ_3 is the supposed density of this air and ρ_1 is that of brass; and this corrected mass of the pendulum must be used in all the corrections affecting the inertia. The value of $h_d - h_u$ must also be diminished by the ratio of the mass of air in the bob to the corrected mass of the pendulum multiplied by $(2o + h_d - h_u)$. These corrections being applied we shall have (putting ρ_2 for the density of the circumambient air)

$$T_d = \odot\sqrt{\frac{l}{g}}\cdot\left(1 + \frac{1}{4}\frac{h_d + h_u}{h_d}\left[1 + \left(1 - \frac{\rho_3}{\rho_1}\right)\frac{h_d - h_u}{2o}\right]\frac{\rho_2}{\rho_1}\right),$$

and a similar formula will hold for T_u replacing h_d by h_u in the denominator.

Besides the air in the hollow bob, a large volume is inclosed within the open tube which forms the stem of the pendulum. The volume has to be calculated from the measured dimensions of the tube. Let this volume be σ_2 and let its radius of gyration about the center of mass be γ_2; then the formula for the period will be, with heavy end down,

$$T_d = \odot\sqrt{\frac{l}{g}}\left(1 + \frac{\sigma_2\rho_2\left[\gamma_2^2 + \frac{1}{4}(h_d + h_u)^2\right]}{M\,l\,h_d}\right),$$

and with heavy end up the same formula will hold after substituting h_u for h_d in the denominator. The $l\,h_d$ in the denominator is best replaced by $(\Gamma^2 + h_d^2)$ where Γ is the radius of gyration of the pendulum about its center of mass. This quantity can be ascertained by an approximate reduction of the times of oscillation to a vacuum.

Thirdly, the ordinary pressure of the atmosphere, due to its weight, makes the pendulum carry air with it, and increases its inertia. This effect was first discovered by du Buat, but particular attention was brought to it by Bessel. It has been subjected to mathematical analysis by Green, who has given the formula for the period of oscillation of any ellipsoid making oscillations, very small as compared with its own dimensions, in an infinite incompressible fluid. The air is sufficiently incompressible under the gentle movement of the pendulum, and the limitation to very small oscillations, though particularly insisted on by Green, is probably immaterial. His formula is as follows:

Let the equation of the ellipsoid be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

and let x be in the direction of the oscillations. Then calculate the quantity

$$F = \tfrac{1}{2}\, a\, b\, c \int_0^\infty \frac{d f}{\sqrt{(a^2+f)^3\,(b^2+f)\,(c^2+f)}}$$

and the effect in question will be as if the inertia of the ellipsoid were increased by $\frac{F}{1-F} \cdot \frac{\rho_2}{\rho_1}$. When the ellipsoid is one of revolution, the integration can be effected in finite terms.

If $a = c$ (that is, if the oscillation is in the equatorial plane) put $\frac{b}{a} = \cos \omega$, and we have

$$F = \tfrac{1}{2} \operatorname{cotan}^2 \omega \left(\frac{2\,\omega}{\sin 2\,\omega} - 1 \right).$$

This formula is suitable for an oblate spheroid. If it is prolate, imaginaries are avoided by putting $\frac{a}{b} = \cos \omega$, when

$$F = \tfrac{1}{2} \operatorname{cosec}^2 \omega \left(1 - \cos \omega \,.\, \operatorname{cotan} \omega \,.\, \log \tan \left(\frac{\pi}{4} + \frac{\omega}{2} \right) \right)$$

If $b = c$ (that is, if the oscillations are on the polar axis), put $\frac{b}{a} = \cos \omega$, and we have

$$F = \cot^2 \omega \left(\operatorname{cosec} \omega \,.\, \log \tan \left(\frac{\pi}{4} + \frac{\omega}{2} \right) - 1 \right).$$

This formula is suitable for a prolate spheroid; for an oblate one, put $\frac{a}{b} = \cos \omega$ and we have,

$$F = \operatorname{cosec}^2 \omega \left(1 - \frac{\omega}{\tan \omega} \right).$$

When the ellipsoid becomes an infinite cylinder swinging transversely, we readily see by the second equation that $F = \frac{1}{2}$, and the coefficient of the effect is unity. In the case of a sphere, all the equations have indeterminate forms. We may consider the last as being the simplest. When ω is made infinitesimal this becomes

$$F = \frac{1}{\omega^2 (1 - \frac{1}{6}\omega^2)^2} \left(1 - \frac{\omega}{\omega (1 + \frac{1}{3}\omega^2)} \right) = \tfrac{1}{3}.$$

Hence, the coefficient of the effect is $\frac{1}{2}$. In the case of a plane oscillating tangentially, the first equation shows that F is a zero of the second order, so that not merely the coefficient, but also the whole effect, vanishes. The same is true of a cylinder oscillating longitudinally. In the case of a circular disk oscillating normally, we have to put in the last equation $\omega = \frac{\pi}{2}$. It will be convenient to substitute $\psi = \frac{\pi}{2} - \omega$ and then put ψ infinitesimal. This gives us

$$F = \sec^2 \psi \left(1 - \left(\frac{\pi}{2} - \psi \right) \tan \psi \right)$$

$$= 1 - \frac{\pi}{2} \psi$$

Then, the coefficient of the effect is

$$\frac{F}{1-F} = \frac{2}{\odot}\frac{1}{\psi}$$

If R be the radius of the disk, the volume of the infinitesimally thin spheroid is $\frac{4}{3}R^3\psi$. Hence, the effect is $\frac{2}{\odot}\frac{4}{3}R^3$. The effect for the sphere erected on the circle is $\frac{1}{2}\frac{4}{3}R^3$. So that we find that the effect of the circular disk is $\frac{4}{\odot}$ that of the sphere erected on it.

The following little table will show the run of the function:

Ratio of polar to equatorial diameter.	EQUATORIAL OSCILLATIONS.		POLAR OSCILLATIONS.	
	Ratio of hydrodynamic effect to buoyancy.	Ratios of successive coefficients.	Ratio of hydrodynamic effect to buoyancy.	Ratios of successive coefficients.
0	0.000		∞	
$\frac{1}{4}$	0.174		2.374	
$\frac{1}{2}$	0.310	1.78	1.115	$\frac{1}{2.13}$
1	0.500	1.61	0.500	$\frac{1}{2.23}$
2	0.704	1.14	0.210	$\frac{1}{2.38}$
4	0.860	1.22	0.082	$\frac{1}{2.56}$
∞	1.000		0.000	

On examining this table, we see that, in reference to equatorial oscillations, 1st, the flatter the spheroid the less the resistance not only absolutely but relatively to the displacement (or cross-section, which, in this case, is in the same proportion); 2d, that this change of the coëfficient with a change of shape of the spheroid is greater and greater the flatter the spheroid and less and less the longer it is, until it must soon become insensible. This shows that a moderately long cylinder may be treated as infinitely long; nay, more, that a moderately long ellipsoid may be treated as an infinite cylinder, the small amount of air which flows over its ends not relieving the flow of the rest, perceptibly. But the flatter the ellipsoid the sharper becomes its edge, which quickly sheds the air. A short ellipsoid thus bears but a very slight resemblance to a short cylinder which has no such edge. Yet a large proportion of air must flow over the ends of a short cylinder. Accordingly, in the absence of any mathematical analysis, it is difficult to treat an object of this form. The effect of the shedding of the air is shown in the table relating to polar oscillations. We see here that the more prolate spheroids not only resist less proportionally to their bulk (the cross-section remaining the same), but also less absolutely. The ratios of successive numbers here for bodies nearly spherical are about the squares of those in the other table. Here, as before, the sharper the points the greater and greater is the effect of further sharpening, and *vice versa*. When the spheroid is moderately flat the absolute hydrodynamic effect is nearly the same as for a circular disk; when it is moderately pointed the effect, relative to its volume, is very small, but, in comparison to its own magnitude, is very variable.

A rough estimate of the effect on a short cylinder oscillating transversely may be obtained as follows: First, compare the effects on a sphere and a cylinder having the same volume and resisting section. The ratio of the diameter to the altitude of such a cylinder will be $\frac{9}{128}\odot^3$, or nearly 1.09. This cylinder will undoubtedly have a greater resistance than the sphere and less than a circular disk equal to the diametral section of the sphere. But the effect on the disk is only $\frac{4}{\odot}$, or about 1.27; so that if we take the effect on the cylinder as 1.18 (for it must be somewhat nearer that on the disk), we cannot be very far out. This would be supposing it to carry 0.59 of its displaced air. This is about the same that is carried by a prolate ellipsoid whose axes are in the ratio of $\sqrt{2}:1$. A shorter cylinder must carry less air and a longer one more, relatively to the volume. But the difference cannot be so great as the difference of ellipsoids from the sphere. We may consistently suppose that every cylinder carries as much air as a spheroid of the same volume, the ratio of whose polar axis to its equatorial diameter is $\sqrt{2} \times 1.09$, or 1.54 that of the ratio of the altitude to the diameter of the cylinder.

Stokes has shown that the hydrodynamic effect is largely increased by the walls of the vessel containing the pendulum. In the case of a sphere of radius a in a spherical vessel of radius b, the ratio of increment is $\frac{b^3 + 2a^3}{b^3 - a^3}$. In the case of an infinite cylinder in a concentric vessel, the ratio is $\frac{b^2 + a^2}{b^2 - a^2}$. For large values of b these expressions coincide. The case of an ellipsoid in a cylindrical vessel has not been solved; but an estimate of the effect may be made, as follows: A cylindrical vessel would probably act on an oscillating sphere to one-half the amount of a spherical vessel of the same diameter; but if the oscillating body is an oblate spheroid, whose polar axis coincides with that of the vessel, the vessel has relatively less effect, because most of the air escapes in that direction. In the case of a sphere, half the air escapes toward the sides and half to the top and bottom. If the ellipsoid, instead of having the coefficient of the hydrodynamical effect $\frac{1}{2}$, like the sphere, has only $\frac{1}{3}$, we may say that only $\frac{1}{3}$ escapes in the equatorial direction and is affected by the cylindrical envelope. In general, therefore, we may estimate this small quantity sufficiently by multiplying the correction for the sphere in a spherical envelope by the coefficient of the hydrodynamic effect.

In the fourth place, even if the air had no weight and consequently no statical pressure, it would still affect the motion of the pendulum in virtue of its *viscosity*. This effect forms the subject of a fine investigation by Stokes. He shows that the viscosity of the air causes a decrement of the amplitude in a constant ratio. This is the cause of the phenomenon represented by the second term of the equation

$$D_t \Phi = - a - b\,\Phi - c\,\Phi^2.$$

In the case of an oscillating sphere this part of the decrement consists entirely of two terms, one proportional to the square root of the viscosity and the other to the viscosity itself. In the case of an infinite cylinder, two similar terms constitute the bulk of the effect. In the case of a plane oscillating tangentially, only the term proportional to the square root of the viscosity appears. In all three cases the formulæ of Professor Stokes exhibit a remarkable relation between the effect on the decrement of the arc and the effect on the period of oscillation; namely, that that term of the former which is proportional to the square root of the viscosity is identical with the only considerable term of the latter. In fact, the viscosity introduces into the differential equation of the motion a term in $\frac{ds}{dt}$ and a term in $\frac{d^2 s}{dt^2}$. The former of these has a part which varies as the square root of the viscosity, and the coefficient of this part is equal to the coefficient of the term in $\frac{d^2 s}{dt^2}$.

By the viscosity is here meant what Stokes terms the index of internal friction and Maxwell the kinematical viscosity. It is the quotient of the retardation of the velocity at any point of the fluid caused by the excess of the velocity at this point over the mean velocity in the neighborhood divided by this excess. Analytically defined, it is

$$\mu^1 = \frac{\dot{v}}{\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right) v}.$$

The dependence of the viscosity of air upon its pressure and temperature has been one of the chief objects of physical inquiry in our day. That it is inversely proportional to the pressure is fully established, this being equally the prediction of the kinetical theory of gases and the result of all the experiments, whether those by Graham and Meyer and Springmühl on transpiration, or those of Maxwell and others on the oscillation of plane surfaces. In respect to the dependence upon temperature, opinion does not seem to be quite unanimous. Nevertheless, the experiments of von Obermayer, of Kundt and Warburg, of Wiedemann, of Holman, and of Puluj, all concur in showing that, in the case of air, the viscosity as here defined varies nearly as the $\frac{7}{4}$ power of the absolute temperature.

Inasmuch as the function of the viscosity always appears in the formulæ multiplied by the density of the air, it follows that the logarithmic part of the decrement will appear as two terms. Namely, for heavy end down

$$b_d = \frac{k_1}{l\,h_d}\,\tau^{\frac{3}{4}} + \frac{k_2}{l\,h_d}\,\frac{\sqrt{p}}{\sqrt[8]{\tau}},$$

and for heavy end up,

$$b_u = \frac{k_1}{l\,h_u}\,\tau^{\frac{3}{4}} + \frac{k_2}{l\,h_u}\,\frac{\sqrt{p}}{\sqrt[8]{\tau}}.$$

And the effect of the resistance on the period of oscillation will be for heavy end down

$$\Delta\,\mathrm{T} = \frac{\mathrm{T}}{\circledcirc}\,\frac{k_2}{l\,h_d}\,\frac{\sqrt{p}}{\sqrt[8]{\tau}},$$

and for heavy end up,

$$\Delta\,\mathrm{T} = \frac{\mathrm{T}}{\circledcirc}\,\frac{k_2}{l\,h_u}\,\frac{\sqrt{p}}{\sqrt[8]{\tau}}.$$

It must be remembered that this supposes b to be expressed in parts of the radius, it being an angular quantity.

The theory of Stokes supposes that the friction between the air and the pendulum is not infinitely less than that between two layers of air, so that no slipping of the air over the pendulum can take place. He adduces some facts in support of this hypothesis, but Messrs. Kundt and Warburg showed in 1875 that this is not true. It is, however, easy to show from their observations that the amount of slipping is insignificant. They find, in fact, that the coefficient of slipping, which is the depth to which the solid must be considered to be composed of air in order to account, on the hypothesis of no slipping, for the degree of motion on its surface, is inversely proportional to the density and is at ordinary pressure and at the freezing-point of water equal to $\frac{1}{10}$ of a micron. Consequently, even at a pressure of only $\frac{1}{3}$ of an inch it would only amount to $\frac{1}{100}$ mm. Now, as the radius of the stem of the pendulum is 2 cm, the friction could not be reduced by this slipping more than $\frac{1}{1000}$ part, which is quite imperceptible.

It has been rendered probable by Stokes that a large envelope about the pendulum will not much affect this correction, but this will not probably be the case when the pendulum approaches near to the envelope in swinging, and the rate of decrement is our only guide in the matter.

It is, now, necessary to apply the general formulæ to the calculation of the values of the coefficients of the atmospheric corrections.

The effect of the buoyancy of the brass is half the ratio of the moment of gravity on the displaced air to its moment on the pendulum. All these calculations will be made in C. G. S. absolute units. We are, however, unacquainted with the absolute unit of temperature, and it becomes necessary to adopt one. The unit chosen will be $273^\circ + 15^\circ = 288^\circ$ of the Centigrade scale; so that the absolute zero being taken as zero, 15° C. will be a temperature of unity. This choice is made because 15° C. is the mean temperature at which it is desirable to make pendulum-experiments. Centigrade degrees will always be spoken of, but τ in the formulæ and calculation will denote the absolute temperature divided by 288 Centigrade degrees above the absolute zero. The coefficients of pressure will relate to ONE ABSOLUTE ATMOSPHERE, by which is meant one million grammes per centimeter (second)^2. This may be converted into the pressure of a height of mercury by a calculation like the following:

	Names of quantities.	Numbers.	Logarithms.
	Density mercury at 0° C. compared with water at 4°	13.5959	1.133405
	Absolute density of water at 4°	$0.99999 \frac{gr}{(cm)^3}$	—4
∴ Multiplying,	Absolute density of mercury at 0°	$13.5957 \frac{gr}{(cm)^3}$	1.133401
	Gravity at Paris (60^m elevation)	$980.88 \frac{cm}{s^2}$	2.991616
∴ Multiplying,	Absolute specific gravity of mercury at Paris	$13335.7 \frac{gr}{cm(s)^2}$	4.125017
	One absolute atmosphere	$1000000 \frac{gr}{cm(s)^2}$	6.000000
∴ Dividing,	One absolute atmosphere expressed in centimeters, pressure of mercury at 0° at Paris at elevation of 60^m	74.986	1.874983

This is equal to 29.63 inches pressure at Hoboken at 15° C. At London it is less by 0.03 inch, a quantity which produces hardly a perceptible effect in the time of oscillation of the pendulum.

To find the density of the air under this pressure, we have, according to the experiments of Regnault (Wüllner's Experimentalphysik, 3ter Band, 3te Auflage, S. 133), the absolute density of dry air free from CO_2 at 0° C. and pressure 76 cm at Paris (6^m elevation), 0.0012932. This supposes a slightly different absolute density of water from that above assumed; but that is a matter of no consequence. Then we have

		Log.
Density dry pure air at Paris, standard atmosphere	.0012932	7.11167
Centimeters, pressure in Paris, standard	76	1.88081
Centimeters in absolute atmosphere	74.986	1.87498
∴ Density pure dry air at 0° C. under one absolute atmosphere	.0012760	7.10584
Absolute temperature of 0° C. $= \frac{273}{288}$		9.97677
∴ Density pure dry air at 15° C. under one absolute atmosphere	.0012095	7.08261
Correction for usual amount CO_2 $(1 + .529 \times 4 \times 10^{-4})$		0.00009
∴ Density common dry air at 15° C. under one absolute atmosphere	.0012097	7.08270

The moisture was not observed during the pendulum-experiments but is believed to have been as much as would be contained in air at a little less than ½ saturation at 15°; or say that the density was diminished by 3 thousandths. The density of the air taken may then be taken at .001206 at 15° C. under a pressure of one absolute atmosphere.

Our pendulum never having been weighed in water, it is necessary to estimate its density. The brass of which it is composed may be supposed to be of the same density as that of the Prussian pendulum-meter, which density is given by Bruhns as 8.5. But the pendulum contains a certain amount of steel. The knives have as their dimensions $9.55 \times 1.8 \times 1.4$, the product being $24\ (cm)^3$. A part is beveled off, but other steel about the pendulum will make up nearly the amount. The entire volume of steel would be therefore $48\ (cm)^3$. We may take it at $45\ (cm)^3$. Assigning to steel the density 7.82 the mass of steel would be 352 gr. The entire mass of the pendulum has been ascertained by weighing to be 6308 grammes. Subtracting the mass of the steel, we have for that of the brass 5956 grammes. Dividing this by the assumed density of the brass we have $701\ (cm)^3$ as the volume of brass, and $746\ (cm)^3$ as the total volume of the metal in the pendulum. Then for the density of the metal we have $\frac{6308}{746}$ or $8.46 \frac{gr}{(cm)^3}$.

But a part of the pendulum is the air within the hollow bob. To find the volume of this, we have the following data, obtained by measurement:

$$h_d - h_u = 39.292$$
$$o \quad = 59.25$$

We then calculate as follows:

$\frac{1}{2}(h_d - h_u)$	19.696	1.29438
o	59.25	1.77269
Ratio volume hollow to that of metal	0.3324	9.52169
Volume metal	746	2.87276
Volume hollow	248	2.39443

The accuracy of this calculation can be checked by another. The hollow of the bob is a cylindrical ring. The thickness of the metal is said to be 0.1 cm. The exterior diameter of the bob is by accurate measurement 11.4 cm. Its interior diameter would, therefore, be 11.2 cm. The exterior diameter of the stem on which this ring fits is 4.35. Then the inner diameter of the hollow ring would be 4.55 cm. The exterior height of the bob is 3.175 cm; then, its inner height would be 2.975. Calculation from these data gives as the volume of the hollow 243 $(cm)^3$; thus confirming, in some measure, the density of brass assumed.

The total volume of the pendulum, so calculated, is 994 $(cm)^3$. Then the absolute density of the whole is $6.35 \frac{gr}{(cm)^3}$. And the ratio of the density of air at 15° and under pressure of one absolute atmosphere to that of the pendulum is $\frac{.001206}{6.35}$ .0001900. To get the effect of buoyancy with heavy end down and up, this ratio is to be multiplied by one-fourth the distance between the knife-edges (which is 100.01 cm), divided by the distance of the center of mass from the point of support. This gives, for heavy end down and up, .0000682 T_d, and .0001565 T_u. Even should the density of the brass be in error by 2 per cent., the resulting error in the relative gravity of two stations, where the barometric difference was 5 inches, would be inappreciable with heavy end up (much more with heavy end down) in the sixth place of decimals.

All the remaining parts of the atmospheric effect are inversely proportional to the moment of inertia of the pendulum. Experiments to be described below show that in vacuo at Hoboken at 15° C.

$$T_d^2 = 1.012045$$
$$T_u^2 = 1.011465$$
$$T_d^2 - T_u^2 = .000380$$
$$\tfrac{1}{2}(T_d^2 + T_u)^2 \quad 1.011755$$

Then it follows that the square of the radius of gyration about the center of mass is

$$h_d h_u \left(1 + \frac{2}{\frac{T_u^2 + T_d^2}{T_u^2 - T_d^2} - \frac{h_d - h_u}{h_d + h_u}} \right)$$

From this we find the two radii of gyration to be $\sqrt{6963.0}$ and $\sqrt{3033.3}$ centimeters; and the two moments of inertia, the mass being 6308 gr, are 4392×10^4 gr $(cm)^2$ and 1913×10^4 gr $(cm)^2$.

To find the effect of the air in the tube forming the stem of the pendulum, we have the following data:

	Centimeters.
Diameter of tube	3.99
Length	123.6
Distance center to knife	50.00
Square of radius of gyration of the pendulum about its center of mass parallel to knife edges	2111

This gives for the solid contents of the tube 1515 $(cm)^3$, for the square of the radius of gyration is—

$$(50.00)^2 + \tfrac{1}{4}\left(\frac{(3.99)^2}{4} + \frac{(123.6)^2}{3}\right) = 3774\ (cm)^2.$$

The two moments of inertia of the pendulum are, with heavy end down, 4387×10^4 gr. $\times$ $(cm)^2$, and with heavy end up 1911×10^4 gr $\times$ $(cm)^2$. Whence the effect of this air is to add to the times of oscillation .0000786 T_d and .0001807 T_u, respectively.

We come now to the hydrodynamic effect. The greater part of this is due to the stem of the pendulum. This is a cylinder whose dimensions are—

	Centimeters.
Length	123.8
Diameter	4.33

From these data the solid contents are found to be 1823 $(cm)^3$. Its radius of gyration being slightly greater than that of the tube, may be taken at $\sqrt{3779}$ cm. Then, the air having the standard density, the moment of inertia is 8310 gr $(cm)^2$, and the effects on the periods of oscillation will be

$$.0000946\ T_d \text{ and } .0002169\ T_u.$$

When the pendulum is on the Geneva stand, the effect of the walls of the cylinder have to be taken into account. If the bells are not on, only the 90 middle centimeters of the stem are affected. The square of the radius of gyration is thus reduced to 3176 $(cm)^2$, only 84 per cent. of that of the whole cylinder. The solid contents are only .727 of the whole, and hence the affected moment of inertia is only $.727 \times .84 = .61$ of the whole. The diameter of the cylinder is 25 cm. Hence, the coefficient of correction is $2\,\frac{(4.33)^2}{(25)^2 - (4.33)^2} \times .61 = .0377$; which gives for heavy end down and up

$$.0000036\ T_d \text{ and } .0000081\ T_u.$$

But when the bells are on the whole is affected, and the corrections are

$$.0000058\ T_d \text{ and } .0000134\ T_u.$$

The two bobs are not precisely of the same size. Their dimensions are

	SOLID BOB.	HOLLOW BOB.
	Centimeters.	Centimeters.
Diameter	11.48	11.42
Height	3.25	3.18

Their solid contents are

$$336.3\ (cm)^3 \qquad 325.0\ (cm)^3.$$

But a part of this volume has already been reckoned as a part of the stem of the pendulum. Owing to the influence of the re-entrant angle at the junction of the bob and stem, which must cause an increase of the effect, it will be better to leave this core as a part of the stem than to include it in the bob. The volume of this core is

$$\odot\ (4.33)^2\ (3.18) = 46.8\ (cm)^3 \text{ for the light bob}$$
$$\text{and } \odot\ (4.33)^2\ (2.25) = 47.9\ (cm)^3 \text{ for the heavy one.}$$

Subtract these from the volumes already obtained, we get as the true volumes

$$278.2\ (cm)^3 \text{ for the light bob}$$
$$\text{and } 288.4\ (cm)^3 \text{ for the heavy one.}$$

The squares of the radii of gyration of these bobs about their centers of mass, parallel to the knife-edges, are

$$\tfrac{1}{4}\left(\frac{(4.33)^2 + (11.42)^2}{4} + \frac{(3.18)^2}{3}\right) = 10.16 \ (\text{cm})^2$$

$$\tfrac{1}{4}\left(\frac{(4.33)^2 + (11.48)^2}{4} + \frac{(3.25)^2}{3}\right) = 10.29 \ (\text{cm})^2.$$

The center of each bob is distant 9.283 cm. from the nearest knife-edge. Hence, the squares of the radii of gyration about the near knife-edges are

$$(9.283)^2 + 10.16 = 96.33 \ (\text{cm})^2 \text{ for the light bob.}$$
$$(9.283)^2 + 10.29 = 96.46 \ (\text{cm}) \text{ for the heavy bob.}$$

And about the far edges the value for both is

$$(109.28)^2 + 10 = 11954 \ (\text{cm})^2.$$

Then, using the standard density of air, we find for the moment of inertia of the air displaced by both bobs, with heavy end down, 4190 gr $(\text{cm})^2$, and with heavy end up, 4045 gr $(\text{cm})^2$.

To calculate what proportion of the displaced air is to be considered as carried along, we first find the ratios of the axes of both cylinders. These are—

For the light bob	.278
For the solid bob	.283

These are to be multiplied by 1.54, to find the cosine ω. We thus find ω for light bob 64°.6 and for the heavy bob 64°.1. Hence, we find, by Green's formula, for the coefficient of the effect, .278 for the solid bob and .272 for the hollow one. This gives for the effective moments of inertia,

With heavy end down	1244 gr. $(\text{cm})^2$
With heavy end up	1185 gr. $(\text{cm})^2$.

Whence, the effects on the time of oscillation are

$$.0000141 \ T_d \text{ and } .0000310 \ T_u.$$

Putting 27 cm as the diameter of the bell-glasses of the Geneva support, it appears that their effects upon the correction for the bob are

$$.0000021 \ T_d \text{ and } .0000046 \ T_u.$$

We have now taken account of the following amounts of displaced air:

Displaced by the stem	1823 $(\text{cm})^3$
Displaced by the light bob	278
Displaced by the heavy bob	288
Total	2389 $(\text{cm})^3$

But the whole amount displaced by the pendulum may be reckoned thus—

Displaced by metal	746 $(\text{cm})^3$
Displaced by hollow of bob	248
Contents of tube of stem	1533
Inclosed in frames	16
Total	2543 $(\text{cm})^3$

There remain, therefore, 154 $(\text{cm})^3$ displaced by the knives, and apparatus for holding them, by the collars which secure the bobs, and by the little cylinders at the ends of the pendulum. The position of the center of mass of this air may be estimated as 3 cm outside the knife-edges and its radius of gyration about its center as 2 cm. The hydrodynamic effect should be taken as equivalent to carrying the displaced air, as a part of this air enlarges the cylindrical stem and the rest is so shaped as to offer very great resistance. Hence the effects are

$$.0000105\ T_d \text{ and } .0000241\ T_u.$$

If we, now, add together the various parts of the effects of buoyancy, of inclosed air, and of air carried outside, we have the total calculable effect proportional to the atmospheric density, as follows:

	Heavy end down.	Heavy end up.
Buoyancy	682×10^{-7}	1565×10^{-7}
Air within stem	795	1826
Air within frames	24	70
Air without stem	946	2169
Air without bobs	141	310
Air without knives, etc	105	241
Sums	2693	6181

When the pendulum swings upon the Geneva support without the bell-glasses we have to increase these effects by

Effect of cylinders	36	71
Sums	2729	6252

When the bell-glasses are in place we have in addition

Effect of bells on stem	22	53
Effect of bells on bobs	21	46
Sums	2772	6351

The result of this calculation is probably a little too small, owing to neglected terms, and can hardly be too large.

The calculation of the effect of viscosity on the time of oscillation depends on the variation of the decrement of the arc with the pressure. Experiments were made upon the Geneva support at Hoboken at various pressures. The observations of arc made during these observations, as has been explained above, were reduced according to the formula

$$\dot{\Phi} = -b\ \Phi - c\ \Phi^2,$$

a being supposed zero in order to diminish the number of unknown quantities. The coefficient c was supposed proportional to the density and one factor was taken for all the experiments, while b was left to be determined independently for each. The result is that all the abnormal variations of the decrement which are considerable are thrown upon b, so that the latter presents an appearance of greater irregularity than properly belongs to it. The results of these experiments are shown in illustration No. 37 *c*. Those with heavy end down have been brought to heavy end up by multiplying them by $\frac{h_d}{h_u}$. The time is expressed in minutes; the pressure in inches pressure at 15° C. at Hoboken. It will be seen that the observations satisfy sufficiently well the formula

$$b = .0013\ \tau^{\frac{1}{2}} + .00435\ p^{\frac{1}{2}}\ \tau^{-\frac{1}{2}}.$$

When the bell-glasses were removed, the time of decrement was noticeably increased; but this is partly due to the change in the value of c. Upon the Repsold support there is scarcely any sensible difference between the time of decrement from that in experiments of the Geneva support with the bells removed. This is shown on illustration No. 36. To compare the observations of arc at

Paris, Berlin, and Kew, with those at Hoboken, they were recalculated with only three constants. These as corrected by least squares are

$$b = .0001082 \quad \text{(units: one second of time and one minute of arc.)}$$
$$c = .000001125$$

t_0, reckoning from $1^\circ\ 10' = 8001^s$. The agreement of these values with observation is shown below:

φ (Obs.)	φ (Calc.)	C − O
130′	129′.96	− 0′.04
110	109.85	− .15
100	100.17	+ .17
80	80.05	+ .05
70	69.89	− .11
50	49.91	− .09
40	40.12	+ .12

Reduced to decimal parts of radius, minutes of time and heavy end up, these values become

$$b = 0.0214, \qquad c = 0.76.$$

Observations of June, 1877, at Hoboken, give

$$b = 0.0242, \qquad c = 0.58.$$

Allowing the European observations a weight of 3 and combining the values of c, we have

$$c = 0.72.$$

Substituting this value, we find for b at Paris, Berlin, and Kew........................ 0.0224
And at Hoboken.. 0.0212
b (weighted mean).. = 0.0221

The curve drawn in illustration No. 36 is calculated from the European coefficients, and the agreement of the observations taken at Hoboken before the bell-glasses were put on is shown by the near coincidence with it of the points distinguished by crosses. The points distinguished by circles are obtained from a combination of all the observations taken in 1877 with the bells on, when the pressure was about 30 inches. The t_0 taken in each case was the one that made the mean excess over the curve equal to zero. The influence of the bell-glasses in arresting the motion of the pendulum is thus very strikingly shown. The value of b, with the bells on, under a pressure of 30 inches, appears by the above formula to be 0.0251 for a minute of time. The mean of the experiments with bells off, as just shown, gives $b = .0221$; so that we may assume that the viscosity effect is one-seventh larger with the bells on than off.

To calculate the effect on the period of oscillation, we take the coefficient .00435, we multiply it by $\sqrt{29.63}$ to bring it to one absolute atmosphere, we divide it by 60 to bring it to seconds, and finally we divide by ⦾, and we get as the effect, with heavy end up, .0001256 T_u. To find the effect with heavy end down, we simply multiply by $\frac{h_u}{h_d}$; which gives .0000548 T_d. When the bells are off, $\frac{7}{8}$ of these values are to be taken.

At excessively low pressures the whole theory of atmospheric viscosity fails, because the fundamental hypotheses are then violated; and, therefore, the real effect of viscosity at $\frac{1}{4}$ inch pressure will probably be somewhat smaller than calculation would make it.

Experiments have been made at Hoboken on the Geneva support, in order to determine the effect of atmospheric pressure *à posteriori*. A series of experiments were made in September, 1877, with heavy end down, and another in December, 1877, with heavy end up. The duration of each experiment was generally long, and the agreement of the results is all that could be expected. These observations were made by Mr. Farquhar. The temperature, during the September experiments, was about 20° C.; that during the December experiments was about 10° C. They have been corrected so as to bring them exactly to these temperatures. The results of these experiments are exhibited on illustrations Nos. 37*a* and 37*b*. It will be seen that the sidereal time of oscillation, with heavy end down, satisfies the formula

$$T_d \overset{s.}{=} 1.006072 + .00000985\,p + .0000081\,\sqrt{p};$$

and those with heavy end up, the formula

$$T_u = 1.005740 + .00002264\,p + .0000234\,\sqrt{p},$$

where p is the pressure in inches at 15° C.

Taking one absolute atmosphere, or 29.63 inches, as the unit of pressure, and reducing the coefficients to 15° C., we have the general formulæ,

$$T_d \overset{s.}{=} 1.006027 + .0002969\,\frac{p}{\tau} + .0000442\,\frac{\sqrt{p}}{\sqrt[8]{\tau}},$$

$$T_u = 1.005785 + .0006598\,\frac{p}{\tau} + .0001271\,\frac{\sqrt{p}}{\sqrt[8]{\tau}}.$$

The values which we have obtained *à priori* are

$$T_d = x \qquad + .0002789\,\frac{p}{\tau} + .0000551\,\frac{\sqrt{p}}{\sqrt[8]{\tau}},$$

$$T_u = y \qquad + .0006388\,\frac{p}{\tau} + .0001263\,\frac{\sqrt{p}}{\sqrt[8]{\tau}}.$$

The difference between observation and *à priori* calculation is perhaps not greater than ought to be expected. The values which have been used in the reductions are

$$T_d = x \qquad + .0002917\,\frac{p}{\tau} + .0000512\,\frac{\sqrt{p}}{\sqrt[8]{\tau}},$$

$$T_u = y \qquad + .0006694\,\frac{p}{\tau} + .0001175\,\frac{\sqrt{p}}{\sqrt[8]{\tau}}.$$

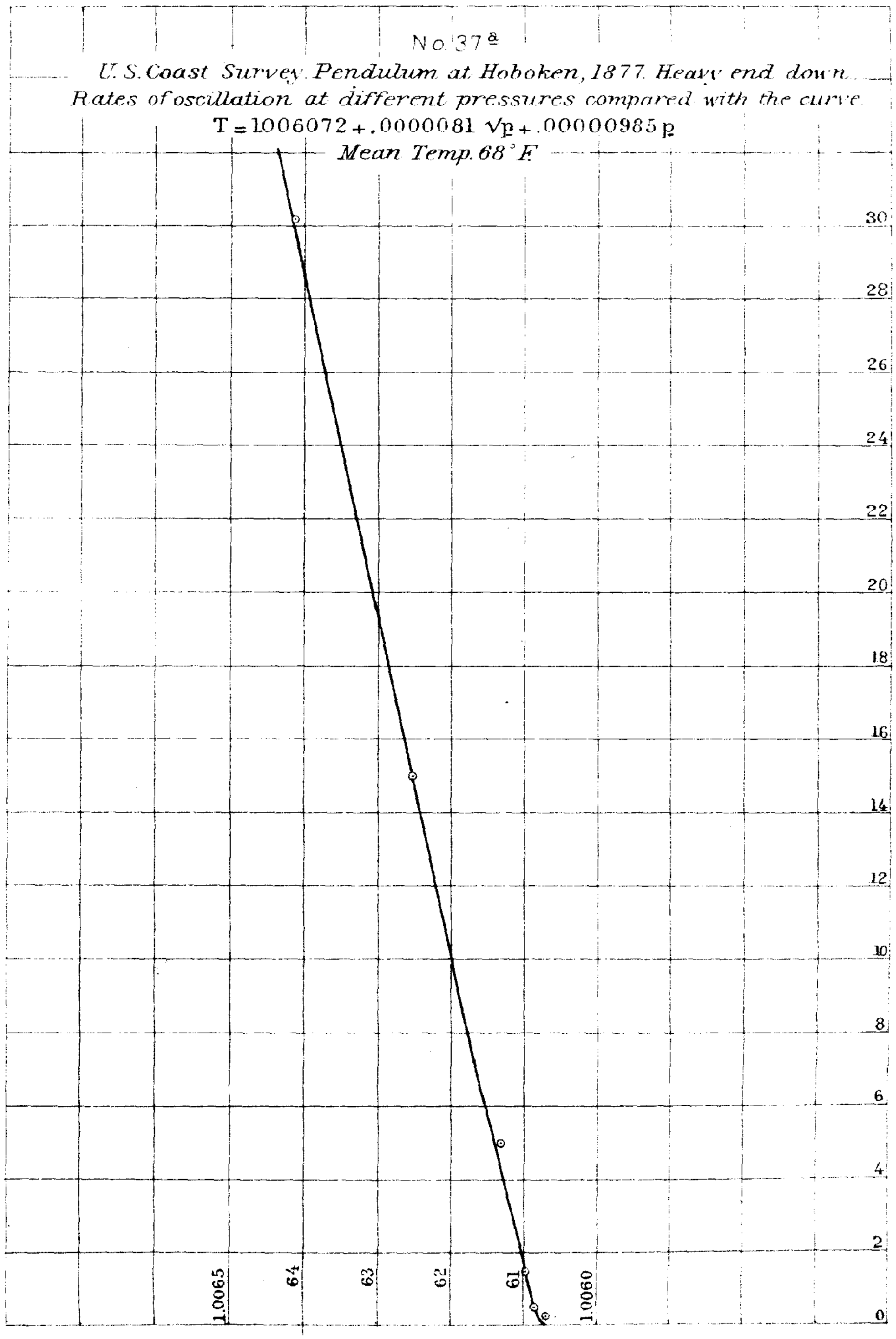
These values were used before the last calculations of the *à priori* values were completed, and it was not thought worth while to change them; but the *à priori* values are preferred.

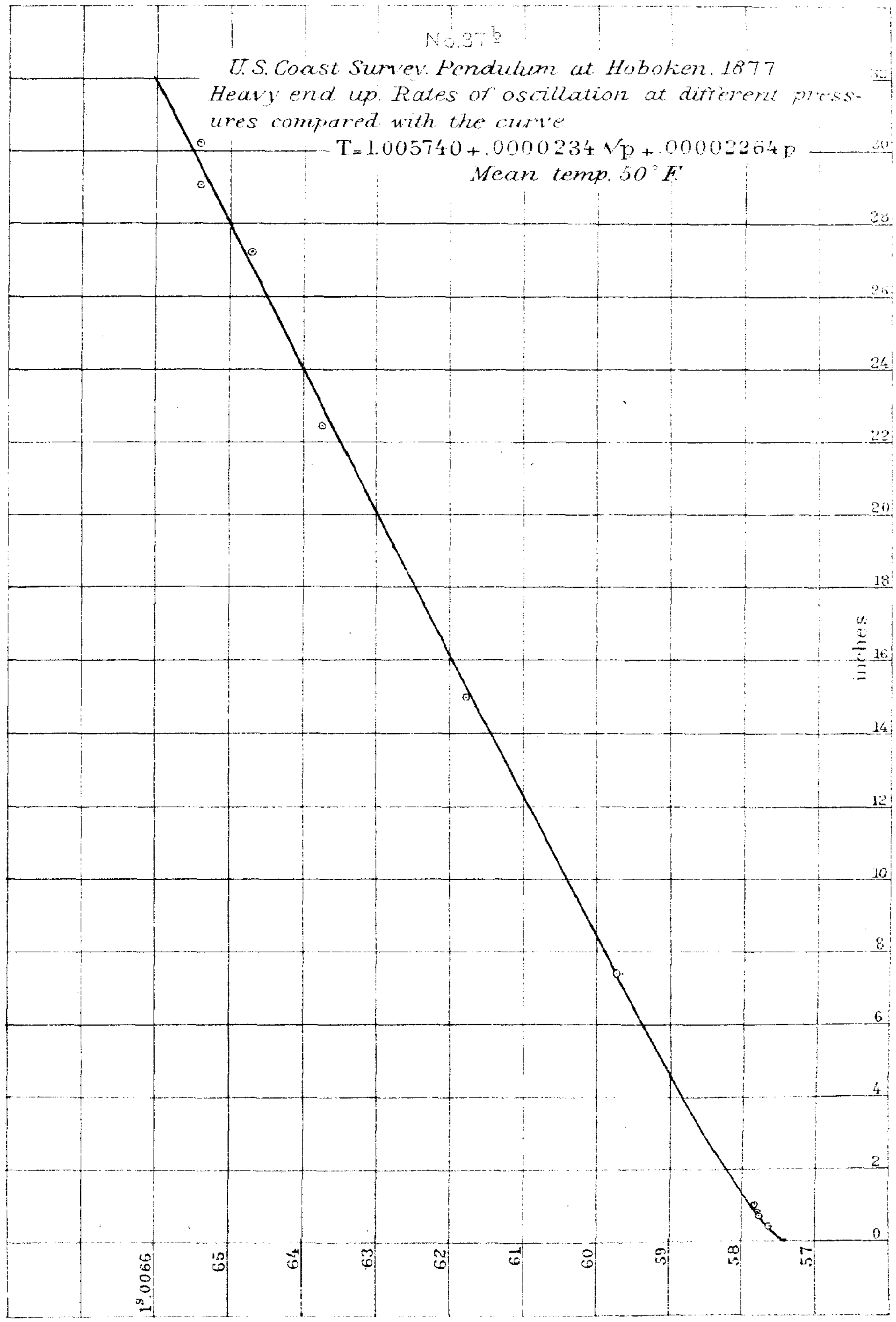
COEFFICIENT OF EXPANSION.

The coefficient of expansion of the pendulum has been determined, by comparing it directly with a meter obtained from the German Imperial Eichungsamt and there designated as Normal Meter No. 49, and also by assuming it to have the same coefficient as the pendulum-meter, the pendulum-meter having also been compared at different temperatures with No. 49. The coefficient of expansion of No. 49 has been absolutely determined by comparison with meter made for the purpose and marked "U. S. C. S.—C. S. P.—1878.—A." The comparisons have been made

Between No. 49 at 13° and A at 3°
Between No. 49 at 3° and A at 13°
Between No. 49 at 4° and A at 4°
and between No. 49 at 18° and A at 18°

The two meters were compared by means of the vertical comparator belonging to the reversible pendulum. They stood in two vertical brass tubes, 4½ cm. in diameter and 1¼ m. long, polished on the exterior and closed at the bottom by a foot terminating in a conical point. One of them rested in the step designed for the pendulum-meter, and was held at the top in a stirrup, movable upon a screw in such a way as to vary its distance from the microscope. The meter rested at the bottom of the tube in a species of trap, in which it was compressed just sufficiently to hold it in place. At about ¾ m. from the bottom it was lightly held, by springs on its four sides, into a frame which was capable of being moved in any direction, by means of four horizontal screws penetrating the walls of the brass tube. India-rubber washers and nuts kept the screw-holes water-tight. Opposite the lines at the top and bottom of the meter, two windows were inserted in the brass tube, setting into little sashes formed of brass casting. These windows were made of plate-glass, about 3 mm. thick, which was carefully selected with a view to the parallelism of its sides, and placed in the sash in such a way that the slope of the wedge should be horizontal. These glass windows were kept water-tight by rubber washers, having a brass washer over them screwed down by four thumb-


No. 37ª
U. S. Coast Survey Pendulum at Hoboken, 1877. Heavy end down.
Rates of oscillation at different pressures compared with the curve
T = 1.006072 + .0000081 √p + .00000985 p
Mean Temp. 68° F
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
1.0065
64
63
62
61
1.0060


No. 37b
U.S. Coast Survey. Pendulum at Hoboken. 1877
Heavy end up. Rates of oscillation at different pressures compared with the curve
T=1.005740+.0000234 √p +.00002264 p
Mean temp. 50° F.
inches
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
1s.0066
65
64
63
62
61
60
59
58
57

screws. At the opposite side of the tube, a little above the middle of the meter, was a third window through which a thermometer placed within the tube could be read. These tubes were furnished with stop-cocks at the bottom, and a rapid current of water was kept running through them during the measures and for at least an hour previous. The comparator, tubes, and meters having been put into perfect adjustment, the tubes were fixed tightly in their places by screws, and remained unmoved during the whole of the experiments. Particular care was taken that they should not turn on their axes, so as to alter in any degree the effect of refraction in the glass windows. The comparator was not fixed, since it was necessarily turned from one meter to the other, and also changed in length by a fraction of a millimeter, when the temperature was changed, a screw being provided for the purpose. The two meters were separated from one another and from the comparator by means of screens about 4 mm. thick, consisting of light wooden frames with tin-plate on the two sides, and loosely filled in the interior with cotton batting.

The coefficient of expansion of No. 49 is checked by comparisons at various temperatures with the platinum meter of the German Eichungsamt; for the coefficient of expansion of platinum has been accurately determined by Fizeau.

As one of the most prominent living metrologists has stated his conviction that the coefficient of expansion of a meter may be expected to be different in the vertical and horizontal positions it is proper to examine this question. Let the meter be of brass and let its cross-section be 2 $(\mathrm{cm})^2$. The solid contents of this meter will be 200 $(\mathrm{cm})^3$ and its mass may be taken at 1680 gr. The modulus of elasticity of brass, according to Wertheim, is $9 \times 10^{11} \frac{\mathrm{gr.}\,(\mathrm{s})^2}{(\mathrm{cm})}$. This is not the same at all temperatures, however, and judging from the analogy of copper we may suppose it $\frac{1}{11}$ part smaller at 100° than at 0°. If the meter be set up on end the mean pressure of its own weight is $\frac{1}{4}\, 1680 \times 981 \frac{\mathrm{gr.\ s}^2}{\mathrm{cm.}}$. Let the expansion from 0° to 100° be x' in the horizontal position. Let the length of the bar in the horizontal position at 0° C. be 1 meter. Let it be heated to 100° C. in this position; its length will then be $1 + x'$. Let it next be placed in the vertical position; then, its length will be reduced by its weight to

$$(1 + x')\,(1 - \tfrac{11}{10} \times 420 \times 109 \times 10^{-11}).$$

Let it, next, be cooled to 0° and its length will be

$$(1 + x')\,(1 - \tfrac{11}{10} \times 420 \times 109 \times 10^{-11})\,(1 - x).$$

Let it, next, be brought to the horizontal position, and its length will be

$$(1 + x')\,(1 - \tfrac{11}{10} \times 420 \times 109 \times 10^{-11})\,(1 - x)\,(1 + 420 \times 109 \times 10^{-11}).$$

But it is, now, in the original condition so that this length = 1. This gives the equation

$$x' - x \doteq 42 \times 109 \times 10^{-11},$$

a quantity too small to be detected.

The following is a summary of the results of the comparisons between meters A and 49:

Temp. of 49.	Temp. of A.	A — 49.
°	°	μ.
+13	+ 3	+130.5
3	13	−247.3
4	4	− 58.8
18	18	− 57.7

These results are all satisfied to the last place of decimals by taking the coefficient of expansion

$$\begin{aligned} &\quad\ \ \mu. \\ &\text{of A} = 18.95 \text{ for } 1^\circ \text{ C.} \\ &\text{of } 49 = 18.83 \text{ for } 1^\circ \text{ C., and} \\ &\text{A} - 49 \text{ at } 0^\circ \text{ C} = -59.3 \end{aligned}$$

The comparisons are given in detail in the subjoined table.

UNITED STATES COAST SURVEY.—COMPARISONS OF METERS "A" AND "49."

[t_A temperature Centigrade of A. t_{49} temperature of 49.]

Date.	Time.	$\frac{1}{2}(t_A+t_{49})$	$\frac{1}{2}(t_A-t_{49})$	A − 49	$\frac{1}{2}\Sigma$ to 8°	$\frac{1}{2}\Delta$ to +5°	Corrected A − 49	Diff. from mean (+130.5)
	h. m.	°	°	μ.	μ.	μ.	μ.	
May 14	....	7.57	+5.32	+138.7	0.0	−12.1	+126.6	−3.9
14	0 40	8.54	+4.68	+120.0	0.0	+12.1	+132.1	+1.6
14	....	8.65	+4.80	+123.6	−0.1	+ 7.6	+131.1	+0.6
14	....	8.30	+5.25	+142.1	0.0	− 9.5	+132.6	+2.1
14	....	8.36	+5.24	+140.8	0.0	− 9.1	+131.7	+1.2
14	....	8.38	+5.24	+141.5	0.0	− 9.1	+132.4	+1.9
14	....	8.79	+4.99	+132.5	−0.1	+ 0.4	+132.8	+2.3
14	....	8.91	+5.11	+137.8	−0.1	− 4.2	+133.5	+3.0
15	....	9.11	+4.33	+105.5	−0.1	+25.3	+130.7	+0.2
15	....	8.72	+4.80	+121.1	−0.1	+ 7.6	+128.6	−1.9
15	....	8.22	+5.30	+139.9	0.0	−11.3	+128.6	−1.9
15	....	8.26	+5.29	+139.8	0.0	−11.0	+128.8	−1.7
15	....	8.40	+5.10	+133.5	0.0	− 3.8	+129.7	−0.8
15	....	8.50	+5.05	+131.6	0.0	− 1.9	+129.7	−0.8
15	....	8.61	+5.06	+133.4	−0.1	− 2.3	+131.0	+0.5
15	9 00	8.94	+4.84	+121.8	−0.1	+ 6.0	+127.7	−2.8
					$\frac{1}{2}\Sigma$ to 8°	$\frac{1}{2}\Delta$ to −5°		(−247.3)
16	21 00	7.90	−4.16	−216.6	0.0	−31.8	−248.4	−1.1
16	22 00	7.28	−4.95	−245.3	+0.1	− 1.9	−247.1	+0.2
16	23 00	7.42	−5.02	−247.7	+0.1	+ 0.8	−246.8	+0.5
16	....	7.58	−5.09	−251.4	0.0	+ 3.4	−248.0	−0.7
16	0 30	7.72	−4.99	−247.9	0.0	− 0.4	−248.3	−1.0
16	3 30	8.94	−5.14	−251.5	−0.1	+ 5.3	−246.3	+1.0
16	....	8.92	−5.22	−257.4	−0.1	+ 8.3	−249.2	−1.9
16	....	8.85	−5.32	−258.4	−0.1	+12.1	−246.4	+0.9
16	8 30	9.84	−4.73	−235.2	−0.2	−10.2	−245.6	+1.7
					$\frac{1}{2}\Sigma$ to 4°	$\frac{1}{2}\Delta$ to 0°		(−58.8)
17	22 00	3.45	+0.02	− 59.2	+0.1	− 0.8	− 59.9	−1.1
17	22 30	4.04	−0.04	− 62.1	0.0	+ 1.5	− 60.6	−1.8
17	23 30	4.58	+0.04	− 55.5	−0.1	− 1.5	− 57.1	+1.7
17	....	6.00	0.00	− 61.0	−0.2	0.0	− 61.2	−2.4
17	....	6.46	+0.03	− 58.1	−0.3	− 1.1	− 59.5	−0.7
18	....	4.06	−0.02	− 58.5	0.0	+ 0.8	− 57.7	+1.1
18	3 15	4.05	−0.01	− 61.5	0.0	+ 0.4	− 61.1	−2.3
18	3 30	3.96	0.00	− 57.8	0.0	0.0	− 57.8	+1.0
18	3 50	4.04	−0.02	− 56.4	0.0	+ 0.8	− 55.6	+3.2
18	4 08	4.14	0.00	− 58.2	0.0	0.0	− 58.2	+0.6
18	4 20	4.22	0.00	− 60.4	0.0	0.0	− 60.4	−1.6
18	4 50	4.34	+0.02	− 56.9	0.0	− 0.8	− 57.7	+1.1
18	5 15	4.46	0.00	− 57.4	0.0	0.0	− 57.4	+1.4
					$\frac{1}{2}\Sigma$ to 18°	$\frac{1}{2}\Delta$ to 0°		(−57.2)
20	21 30	18.19	0.00	− 57.1	0.0	0.0	− 57.1	+0.1
20	21 45	18.26	+0.01	− 57.0	0.0	−0.4	− 57.4	−0.2
20	22 00	18.32	0.00	− 56.8	0.0	0.0	− 56.8	+0.4
20	22 20	18.45	0.00	− 57.4	0.0	0.0	− 57.4	−0.2
20	23 30	18.65	+0.01	− 55.3	−0.1	−0.4	− 55.8	+1.4
20	23 45	18.70	0.00	− 57.2	−0.1	0.0	− 57.3	−0.1
20	1 45	18.72	+0.01	− 56.8	−0.1	−0.4	− 57.3	−0.1
20	2 15	18.76	+0.02	− 56.1	−0.1	−0.8	− 57.0	+0.2
20	3 50	18.75	+0.03	− 57.1	−0.1	−1.1	− 58.3	−1.1

The comparisons made between the German Normal Meter No. 49 and the platinum meter of the Eichungsamt give

$$\text{No. } 49 - \text{Pl. M} = -24.4 + 10^{\mu}.09\ \tau_1,$$

where τ_1 denotes the temperature Centigrade; and the following table shows the agreement of this formula with the observations communicated by Professor Förster:

τ_1	No. 49 — Pl. M.	Same, calc.
°	μ	μ
+ 3.25	+ 8.2	+ 8.4
6.28	39.2	39.0
23.55	213.1	213.2

Fizeau's coefficient of expansion for platinum is at 0° C., $8^{\mu}.68$ for 1° Centigrade; at 11° C., the mean temperature of these comparisons, this coefficient becomes $8^{\mu}.72$. Hence the coefficient of expansion for Meter No. 49, as deducted from these comparisons, is $18^{\mu}.81$. This agrees very well with the absolute determination above given, which may therefore be adopted.

The comparisons between the pendulum-meter and No. 49 are not very satisfactory in their results, but they show that the coefficient of expansion of the former is certainly the smaller of the two. The following table shows the observed results as compared with the formula

$$\text{U. S. Pendulum-Meter} - \text{No. } 49 = -13^{\mu}.3 - 0^{\mu}.448\ \tau_1.$$

Date.	τ_1	U. S.— No. 49.	Same, calc.	Calc. — Obs.
1878.	°	μ	μ	μ
March 20	20.25	− 25.0	− 22.4	+ 2.6
21	19.68	21.2	22.1	− 0.9
22	20.14	22.9	22.3	+ 0.6
23	19.93	22.0	22.2	− 0.2
24	16.84	19.1	20.8	− 1.7
25	13.37	19.5	19.3	+ 0.2
26	8.11	17.1	16.9	+ 0.2
26	14.07	18.5	19.6	+ 0.9
27	10.58	16.6	18.0	− 1.4

The pendulum itself, compared with No. 49 at 35° C., and at 10° C., gives the following equation:

$$\text{Pendulum} - \text{No. } 49 = -191^{\mu}.5 - 0^{\mu}.67\ \tau_1.$$

The coefficient of expansion of the pendulum, therefore, as given by these comparisons, is

$$18^{\mu}.83 - 0^{\mu}.67 = 18.16.$$

The coefficient of the pendulum-meter, just found, is $18^{\mu}.83 - 0.45 = 18^{\mu}.38$. The mean of these two values is $18^{\mu}.27$.

The coefficient of expansion of the pendulum was also determined from its rate of oscillation at different temperatures, a special series of experiments at high temperatures being taken for the purpose. The results are given below:

	HEAVY END DOWN.		HEAVY END UP.	
	τ_1	T_d	τ_1	T_u
	°	s.	°	s.
Observed values {	35.8	1.006537	36.3	1.006719
	19.4	1.006400	10.2	1.006536
Differences	16.4	.000137	26.1	.000183
Correction for atmospheric temperature	....	+ 16	....	+ 58
		.000153		.000241
Effect for 1° C	......	.0000093	....	.0000092

These give as the coefficient of expansion—

From oscillations with heavy end down	$18^{\mu}.5$
From oscillations with heavy end up	18 .3

The value 18 .3 has been used in the reductions.

CORRECTIONS FOR THE WEARING DOWN AND ROUNDING OFF OF THE KNIFE-EDGES.

If 5 kilogrammes' weight be put upon an absolutely sharp knife-edge of steel hardened in oil and having a bearing length of 2 centimeters, the steel edge will be crushed until the breadth of the bearing surface is 1 micron. Accordingly, from the very beginning, a knife-edge will wear down and round off. The wearing down and the blunting will have very different effects upon the period of oscillation.

The removal of the point of support of a pendulum from its center of mass, will have an effect which is readily calculated, thus:

$$\mathrm{d}\,T^2 = \mathrm{d}\,.\frac{\mathbb{O}^2}{g}\left(\frac{\gamma^2}{h} + h\right) = \frac{\mathbb{O}^2}{g}\left(-\frac{\gamma^2}{h} + h\right)\frac{\mathrm{d}\,h}{h}.$$

For a reversible pendulum,

$$\gamma^2 = h_d\,h_u.$$

Hence, we have,

$$\mathrm{d}\,T_d^{\,2} = T_d^{\,2}\,\frac{h_d - h_u}{h_d + h_u}\,.\,\frac{\mathrm{d}\,h_d}{h_d}$$

$$\mathrm{d}\,T_u^{\,2} = -\,T_u^{\,2}\,\frac{h_d - h_u}{h_d + h_u}\,.\,\frac{\mathrm{d}\,h_u}{h_u}$$

If a pendulum rolls upon a cylindrical surface of radius ρ, the instantaneous axis of rotation is the instantaneous line of contact; and a velocity of rotation about this axis is equivalent to the same velocity of rotation about the line of contact in the equilibrium position of the pendulum combined with such a translation velocity along the length of the pendulum as is necessary to fix the instantaneous axis; this is $2\rho\,\sin\tfrac{1}{2}\varphi\,.\,\frac{\mathrm{d}\varphi}{\mathrm{d}\,t}$. It follows that the amount by which the *vis viva* of the pendulum is affected by a cylindricity of the knife-edge is of the order of $\rho^2\,\varphi^2$ and may consequently be neglected. The moment of gravity is, however, obviously the same as if the axis of the cylinder were the axis of rotation, that is, it is multiplied by $\left(1 + \frac{\rho}{h}\right)$. Hence we have

$$\delta\,T^2 = -\,T^2\,\frac{\rho}{h}.$$

If the section is not circular, then obviously some sort of a mean radius of curvature must replace ρ. If the section is flatter than a circle, that is, if the lower parts in repose have the greater radii of curvature, then the mean radius, and consequently the effect on the period of oscillation, will be greater for small arcs than for large ones; while if the section is somewhat pointed downwards the reverse will be the case.

We know too little of the laws of crushing and grinding to be able to calculate the radius of curvature from the amount worn off. In fact, the ratio would probably depend on the hardness of the material. Neither can the radius be measured directly with any accuracy. But it may obviously be very large. When the pendulum is first brought down to rest on the edge, why may not the blunted surface be nearly flat? If it were so, the small oscillation through 4° or 5° could not round the edges enough to make the ratio of the radius to the wearing down at all small. Under these circumstances it is a question deserving consideration and experimental examination whether it would not be better to substitute for knife-edges cylinders of measurable diameter—say of 5 millimeters.

Our own experiments always began with a half-amplitude of 2° and ended with a half-amplitude of $\frac{1}{2}$°; but the time of intermediate transits of the pendulum across the vertical was observed when the half-amplitude reached $1\frac{1}{2}$° and 1°. The intervals of time between the 2° and $1\frac{1}{2}$° observations were too short to found any conclusions upon; but if we compare the mean period of oscillation while the arc is descending from 2° to 1° with the mean period while the arc is descending from 1° to $\frac{1}{2}$°, we do not find, after the usual correction for arc, any decided difference between them. This is shown in the following table:

Station.	Position of heavy end.	Periods of oscillation. (Corrected for arc, pressure, temperature, and rate of time-keeper.) Arc. 2° to 1°.	Arc. 1° to $\frac{1}{2}$°.	Difference. Heavy end down.	Heavy end up.
		s.	s.		
Paris	Down	1.006053	1.006050	−3	
	Up	1.006195	1.006198		+3
Berlin	Down	1.005901	1.005897	−4	
	Up	1.006028	1.006033		+5
Kew	Down	1.005931	1.005930	−1	
	Up	1.006054	1.006056		+2
Hoboken	Down	1.006355	1.006358	+3	
	Up	1.006550	1.006548		−2

There are, it is true, slight indications here of a correction varying with the amplitude and different for the Repsold support, used at the European stations, and for the Geneva support, used at Hoboken; but nothing can be concluded with certainty.

The measures of length show, in each case, an increase in the distance between the knife-edges from station to station. Thus we have—

	μ
Increase of length from Paris to Berlin	4.1
Berlin to Kew	3.1
Kew to Hoboken	8.9

This increase cannot have been in any degree due to the wearing down of the knife-edges, for the reason that the measures were made between the centers of the edges, which portions do not bear upon the support and appear, even now, nearly sharp. The increase is most probably due to accumulations of cocoa-butter, etc., under the steel of the knives where they bear on the brass. The effect is, therefore, that of a wearing down without any blunting.

The only indication which we have as to the actual wearing down and blunting is derived from the difference between the periods of oscillation with heavy end down and up. The difference will be

$$C - \frac{20}{21}\,\delta h - \frac{2}{21}\,\rho.$$

It is highly objectionable to infer the values of δh and ρ from the numbers which are to be used to determine the force of gravity. It is fortunate, therefore, that the whole of the wearing and blunting which it is necessary to take account of for the European stations, took place, all at once, at Berlin, after just one-half of the work there had been done. The change is therefore deducible from the Berlin observations alone, and that from the difference of the first half and last half of them, independently of the mean of the whole. The differences of T, with heavy end up and heavy end down, at Berlin, are as follow:

	First four days.	Last four days.
	s.	*s.*
	0.000153	0.000123
	0.000136	0.000126
	0.000131	0.000131
	0.000140	0.000123
Mean......	0.000140	0.000126

When these means are corrected for the measured change of length from that at Paris, they become

0.000142	0.000128

Now, the same difference at Paris is—

	s.
First four days	0.000144
Second four days	0.000140
Mean	0.000142

which agrees exactly with the first four days at Berlin; and the same difference at Kew, after correction for the measured change of length from Paris, is—

	s.
First four days	0.000128
Second four days	0.000129
Mean	0.000128

which agrees exactly with the last four days at Berlin. We thus find that it is quite unnecessary to take account of any other blunting in Europe than that which took place at that time.* To separate the effects on T_d and T_u, we have

BERLIN.

	T_d	T_u
	s.	*s.*
First days	1.0058980	1.0060378
Last days	1.0058988	1.0060246
Changes	+08	−132

The negatives of half these changes are, therefore, to be applied to the observations at Kew, and only the first 4 days' observations at Berlin are available. The deduced values of the wearing down and of the radius of curvature of the section are

$$\delta h = 11^{\mu}.5 \quad \rho = 32^{\mu}.$$

These results agree well with direct observations of the edges, which have been made under a high-power microscope, with illumination through the objective.

The regular set of observations at Hoboken were made upon the Geneva support, instead of the Repsold support, which had been used before. Now, it is a matter of observation that the edges do not even yet, after all their wearing, rest over their whole length upon the Repsold support, but only near their ends. On the other hand, on the Geneva support, they rest nearer the middle. The consequence is that when the Geneva support was used, quite new and unblunted parts of the edges came into play, so that the edges should have been in the same state as at Paris; and this will be assumed to have been the case, though the differences of the supports in other respects prevent a very close comparison.

* It will be observed that this comparison of the difference of T_d and T_u at different stations could not have been made if the atmospheric corrections had not first been applied.

The two knives cannot be assumed to be alike, either in respect to the distances of the edges from the planes of the bearings of the steel on the brass, or in the figure of the edges; but inasmuch as they are interchanged and equal numbers of experiments made in the two positions, this inequality can have no effect on the final result. But to exhibit the agreement of single days' experiments, the inequality should first be allowed for. In the Paris experiments and the first experiments at Berlin, there was no perceptible difference between them. Thus, we find

Excess of time on oscillation on knife 1 over that on knife 2.

	Heavy end down.	Heavy end up.
	s.	*s.*
Paris	+0.000002	−0.000008
Berlin (first days)	−0.000002	+0.000014
Weighted mean	+0.000001	+0.000001

But for the last days at Berlin, and for Kew, the difference is quite perceptible.

Excess of time of oscillation on knife 1 over that on knife 2.

	s.	*s.*
Berlin (last days)	+0.000006	+0.000010
Kew	+0.000004	+0.000005
Weighted mean	+0.000004	+0.000006

Half these amounts will be applied with their appropriate signs in exhibiting the results of single days. At Hoboken this correction disappears again.

CORRECTION FOR THE SLIP OF THE KNIFE-EDGES.

There is, according to Bessel, another effect due the knife-edges, which has reverse signs, with heavy end up and down, and which is consequently not eliminated in the formula for the reversible pendulum, although it is eliminated when the periods of oscillation are combined by the formula suitable in considering it as two invariable pendulums, to wit:

$$T = \frac{T_d h_d + T_u h_u}{h_d + h_u}.$$

This action may be termed the slip of the knife-edge, as it is supposed to be due to a motion which the knife is compelled to make, and which no elastic force resists.

In order to detect the existence of such an effect, a stiff steel knitting-needle was slipped through the notch in the support of the knife-edge, while the pendulum was in position, and was then brought up into contact with the edge by copper wires fastened to its extremities and also to a frame mounted on the head of the pendulum-support. These wires had considerable length, so that the knitting-needle was free to move in the direction of its length, while the friction on the knife-edge was very great, so that if the edge had any slip the knitting-needle must oscillate in the direction of its length. In order to observe any such motion a bit of convex spectacle-lens of long focus was attached to one end of the needle, and a plane piece of glass was brought up nearly against it; with the aid of a lamp burning alcohol with salt, Newton's rings were produced and were observed with a microscope, according to the well-known method of Fizeau. The result of this experiment was that, with the largest oscillation of the pendulum, not the least slip could be detected in this way, so that it seemed that there could be no slip as great as $\frac{1}{20}$ of the wave-length of the D line; that is, none amounting to $\frac{1}{20}$ of $\frac{4}{10}$ of a micron.

As this result was unexpected, not to say surprising, the apparatus was critically studied; but it seemed impossible that any slip should occur at the point of contact of the edge with the needle without showing itself in this way.

It might be, however, that, instead of a true slip, the edge was turned at each oscillation, so as to produce a motion similar to a slip; for such an effect would not be detected by our experiment.

CORRECTION FOR SHORTER LENGTH WITH HEAVY END UP.

When the heavy end is down the pendulum is stretched by a greater length. Calculating from the known coefficient of elasticity of brass, this stretching is found to amount to $1^{\mu}.0$. In the measures of length this amount has accordingly uniformly been added to the length with heavy end up. There is, therefore, a correction of +10 in the seventh place to be added to T^2_u to bring it to what it would be if the pendulum were as long as the reduced measures make it.

THE CORRECTION FOR THE FLEXURE OF THE SUPPORT.

The work upon this correction has been so extensive that it is thought best to reserve the full account of it for a separate paper. It has been shown by the writer that if a horizontal force equal to the weight of the unit of mass deflects the point of support through a distance S, and if M is the mass of the pendulum, h the distance of the center of mass from the point of support, and l the length of the corresponding simple pendulum, then the effect of the swaying of the stand with the movement of the pendulum is to lengthen the square of the period of oscillation by $T^2 \, M \, \frac{S\,h}{l^2}$; and the effect on λ the length of the seconds' pendulum found with the reversible pendulum is to make it too short by $M\,S\,\frac{\lambda}{l}$. This supposes the support to be perfectly elastic, and without any difference between statical and dynamical elasticity.

The quantity S has been found from a long series of experiments to be equal to $0^{mm}.0340$ for the Repsold support. At Paris a larger value ($0^{mm}.0371$) was found, but the larger value has not been used in the reductions. Possibly it should have been used.

When the Repsold tripod rests on a flexible support of any kind the value of S is of course increased. At Hoboken there was no such increase. It is not believed that this increase was a considerable quantity at either Berlin or Kew, but the first occasion will be seized of measuring it. At Paris it may account for the larger value of S there obtained. At Geneva the pendulum was swung on a wooden support. The effect of which may be estimated as follows: Professor Plantamour finds that the correction $M\,S\,\frac{\lambda}{l}$ for his pendulum swinging on this wooden support at Geneva was $0^{mm}.1724$, and at Berlin on a pier similar to that on which the Coast Survey pendulum tripod rested the same correction was $0^{mm}.1357$. The difference, or $0^{mm}.036$, may be taken as the effect of the flexure of the wooden stand. Now, the following are the data concerned, which differ for his experiments and for ours:

	Swiss pendulum.	American pendulum.
Mass of pendulum	3050 gr.	6308 gr.
Ratio of seconds' pendulum to length between knives	1.77	1
Height of edge above feet	1 m.	1.3 m.

It follows that the effect of the flexibility of the wooden table would be greater for the Coast Survey pendulum in the ratio of $\frac{6308}{3050} \times \frac{1}{1.77} \times (-1.3) = 1.98$; so that the correction for our pendulum swinging on this table will be $0^{mm}.217 + 0^{mm}.071 = 0^{mm}.287$.

For the Geneva support, set up as it was at Hoboken, the total value of S (for metallic part and piers) was found to be $0^{mm}.00405$. For the stiffest support the total value was $0^{mm}.0031$.

M. Plantamour has observed a phenomenon which he supposes to be due to the pendulum-support not yielding so much under the oscillations of the pendulum as the amount calculated from the statical flexure. For a wooden support the hypothesis is certainly admissible. For a metallic support it should only be admitted with extreme caution. Elaborate experiments at Hoboken seem to show the ratio of statical to dynamical flexure to be as 263 to 257. Whether or not, supposing the difference to exist, the statical or dynamical flexure ought to be used in calculating the correction has not been made out by any mathematical analysis. In the corrections applied in this research the statical value has been used.

LENGTH OF THE PENDULUM.

The distance between the knife-edges of the pendulum depends upon the comparisons shown in the following scheme:

Pendulum.

United States Pendulum Meter.

German Pendulum Meter. German Normal Meter No. 49.

German Normal Meter No. 1605. German Platinum Meter.

The comparisons between the German meters were made in the Imperial Eichungsamt at Berlin, and the results communicated by Professor Förster. In addition, Bruhns's Report on Professor Albrecht's Pendulum Experiments gives a comparison between the German pendulum meter and what he calls "Der Normalmeter der Eichungskommission," also made at the Eichungsamt in Berlin; but as it is impossible to tell specifically what he means by this expression, as the German Eichungskommission states that it never had any intention of establishing an independent standard of its own, and as the result given by Bruhns disagrees entirely with the results of other comparisons, this comparison must be excluded. It is possible that the German pendulum meter received some injury between the time of Bruhns's comparison and that with No. 1605, communicated by Professor Förster. If such was the case, the injury probably occurred before the first comparison between it and the United States pendulum meter, for, as then noticed, when the position had been adjusted by means of the spirit-level, the scales at the top and bottom of the tube were not in the same vertical line.

One set of comparisons between the two pendulum meters was made in 1875, and another in 1877, by means of the vertical comparator belonging to the United States apparatus.

Meter No. 49 in its comparison with the pendulum meter, and with the pendulum, was supported at the bottom on a foot made for the purpose.

Comparisons between the pendulum and pendulum meter were made before and after every swing, at first; but at Kew and Hoboken it was thought sufficient to make one comparison before and one after each change of knife-edges, in addition to those made at the beginning and end of the series.

The order in which micrometrical readings were taken in these comparisons was as follows:

1. Metallic thermometer.
2. Meter below.
3. Meter above.
4. Pendulum above, bright edge.
5. Pendulum above, dark edge.
6. Pendulum below, bright edge.
7. Pendulum below, dark edge.
8. Meter above.
9. Meter below.
10. Metallic thermometer.

The method of adjustment of the stand and comparator has been already described in the first part of this report, under the head of "Instruments." The length of the comparator was made nearly a mean between those of the pendulum and pendulum-meter, and its middle brought to a level with the middle of the pendulum. The middle of the meter was then raised to the same height, its foot moved until the vertical lines at its two ends were simultaneously in coincidence with the vertical webs of the two microscopes, and both ends brought into perfect focus. The adjustment of the comparator, once made, was never disturbed; that of the focus on the standard was often repeated.

The lines of the metallic thermometer observed on were the three nearest the three lines one-tenth of a millimeter apart at the beginning of the meter-scale. Care was taken, however, to observe on the same three lines in every comparison at the same station. When the metallic thermometer was below, the reading of the meter taken in conjunction with it was not repeated; when it was above, at Berlin and Paris, the meter-scale above was read twice. At Kew, the meter-scale below was read first and last, and the scale above with the thermometer but once. A calibrated mercurial thermometer was read to hundredths of a degree Centigrade before and after each observation of the metallic thermometer. The corrections of this thermometor will be given in another place. The mark at the end of one meter, and the two lines at the distance of one hundredth of a centimeter on each side, were observed on at Paris and Berlin; at Kew and Hoboken the lines observed on were those at distances of $99^{cm}.97$, $99^{cm}.98$, and $99^{cm}.99$ from the zero of the scale. Three readings on each line of the meter-scale, or nine in all, were taken at Paris; two readings on each line at Berlin and Kew, and but one at Hoboken, it being found advisable to finish the comparison as quickly as possible, that the apparatus might not be affected by the heat of the body. In the readings on the pendulum, the method followed at Geneva was used at all other stations. The order given above, of observing on edge bright and edge dark, was not at first strictly observed.

In reducing observations of length, account must be taken of the value of a revolution of the micrometer-screw, of the distance between the lines on the meter-scales (if the same ones were not always observed on), and of the varying compression of the scales compared when they supported varying weights.

In calculating the last-mentioned allowance, the coefficient of elasticity of brass given by Wertheim, namely,

$$927100000$$

for one gramme of weight supported and one square cm. of cross-section, was used.

Measures of the pendulum give as the cross-section of its tube $2.35(\text{cm})^2$. The additional weight carried with heavy end down is 2095 g; hence the pendulum is then longer by

$$\frac{2095}{2.35 \times 927100000} = 1^{\mu}.0.$$

Taking the same cross-section for the tube of the pendulum meter, and the weight of the metallic thermometer equal to that given by Bruhns for the thermometer of the German meter, 2213 g, its compression, when the thermometer is above, is $1^{\mu}.0$.

The cross-section of meter No. 49 is $1.80(\text{cm})^2$ and half its weight 757 g; hence it is longer when measured horizontally than when measured vertically by $0^{\mu}.5$.

ON THE LENGTHS OF VARIOUS MICROMETRIC SCALES EMPLOYED IN THIS RESEARCH.

1. *The United States glass decimeter-scale of centimeters.*—This scale is upon a piece of glass 0.29 cm. thick and roughly cut into a rectangular shape of 14.3 cm. by 5.3 cm. The lines are about 0.34 cm. long and are inclosed between two longitudinal lines which run the whole length of the plate. Lines 0, 5, 10 are longer than the others. The lines are etched, are 20 microns in thickness, and are excessively bad. They are either filled or stained by the etching acid, but the filling (or whichever sort it be) is all on one side.

I have applied to Assistant Hilgard, by whom this scale was made, for information in regard to its corrections. Professor Hilgard replies that the decimeter is correct to $\frac{1}{100000}$ part, but he does not state at what temperature.

The different centimeters of this scale have been compared together by Assistant C. A. Schott. I copy the following from a duplicate record found in the archives of the Coast Survey:

FEBRUARY 21, 1872.

Adjusted micrometer and direction of scale, measured with lower screw,* in order to obtain measures on the same part of the screw for the whole range of the scale:

Scale, 0 to 1.

Lower guide-line, upper screw 80.202 or A.

Upper 73.73

Set upper screw to 76.96

Temperature, 73°.

	0.	1.
	74.507	55.581
	516	583
	516	583
	521	593
	516	588
	513	585
	510	582
	514	588
	508	583
	508	584
Mean..	74.513	55.585
Difference	18.928	

1 to 2.

Upper scale 80.33

Lower 73.85

Middle 77.09

Temperature, 73°.8.

	1.	2.
	74.431	55.506
	429	498
	428	494
	432	500
	430	498
	429	500
	429	500
	430	499
	429	496
	431	498
Mean..	74.430	55.499
Difference	18.931	

2 to 3.

Upper scale 80.34

Lower 73.84

77.09

Temperature, 73°.5.

	2.	3.
	74.604	55.679
	610	686
	604	682
	602	680
	604	679
	606	680
	606	679
	602	678
	606	675
	601	677
Mean..	74.605	55.680
Difference	18.925	

3 to 4.

Upper scale 80.24

Lower 73.76

77.00

Temperature, 73°.5.

	3.	4.
	74.255	55.322
	257	317
	259	327
	266	320
	264	322
	266	318
	260	322
	261	326
	262	322
Mean..	74.261	55.321
Difference	18.940	

* Called O formerly.

4 to 5.

Upper scale	79.83
Lower scale	73.42
	76.63

Temperature, 73°.5.

	4.	5.
	74.502	55.556
	495	556
	493	550
	490	557
	497	558
	494	561
	493	558
	497	554
	497	557
	493	556
Mean..	74.495	55.556
Difference	18.939	

5 to 6.

Upper scale	79.80
Lower scale	73.34
	76.57

Temperature, 73°.3.

	5.	6.
	74.577	55.682
	575	683
	581	683
	577	686
	578	685
	577	686
	576	678
	576	679
	578	679
	576	678
Mean..	74.577	55.682
Difference	18.895	

6 to 7.

Upper scale	79.79
Lower scale	73.32
	76.56

Temperature, 73°.4.

	6.	7.
	74.425	55.488
	424	484
	422	488
	424	486
	425	490
	426	485
	422	480
	426	484
	426	489
	427	483
Mean..	74.425	55.486
Difference	18.939	

7 to 8.

Upper scale	79.79
Lower scale	73.33
	76.56

Temperature, 73°.4.

	7.	8.
	74.498	55.578
	494	575
	496	576
	498	578
	502	571
	499	572
	496	579
	496	580
	498	577
	499	585
Mean..	74.498	55.577
Difference	18.921	

8 to 9.

Upper scale	79.78
Lower scale	73.32
	76.55

Temperature, 73°.0.

	8.		9.
	74.424		55.498
	423		494
	421		493
	424		492
	420		492
	421		492
	421		496
	420		497
	426		491
	424		496
Mean..	74.422		55.494
Difference		18.928	

9 to 10.

Upper scale	79.78
Lower scale	73.30
	76.54

Temperature, 73°.5.

	9.		10.
	74.466		55.547
	458		545
	458		543
	463		547
	468		550
	458		547
	459		549
	462		549
	463		551
	463		544
Mean..	74.462		55.547
Difference		18.915	

Repetition of measure 5 to 6.

Upper scale	79.77
Lower scale	73.15
	76.46

Temperature, 73°.8.

	5.		6.
	74.474		55.577
	474		575
	468		574
	472		572
	475		572
	476		573
	477		576
	474		576
	475		578
	469		573
Mean..	74.473		55.575
Difference		18.898	
First difference		18.895	
Mean.........		18.897	

The probable errors of these measures, as calculated from the sums of the residuals, and from the sums of their squares, are shown in the following table:

Line.	Probable error of one pointing in $\frac{1}{1000}$s of rev.		
	From [Δ]	From [$Δ^2$]	Mean.
0	2.4	3.1	2.8
1	2.5	2.5	2.5
1	1.0	0.8	0.9
2	1.9	2.0	2.0
2	1.9	1.8	1.8
3	1.9	2.0	2.0
3	2.8	2.5	2.6
4	2.0	2.2	2.1
4	2.2	2.2	2.2
5	1.7	1.9	1.8
5	1.1	1.2	1.2
5	2.2	2.0	2.1
6	2.4	2.2	2.3
6	1.2	1.2	1.2
6	1.6	1.5	1.5
7	2.2	2.1	2.1
7	1.4	1.5	1.5
8	2.6	2.7	2.7
8	1.6	1.4	1.5
9	1.9	1.7	1.8
9	2.9	2.6	2.7
10	1.8	1.8	1.8
Mean...	1.96	1.95	1.95

It appears from this that the probable error of the measure of one centimeter by 10 pointings on each of its limiting lines is only $\pm 1.95 \sqrt{\frac{2}{10}}$ thousandths of a revolution or 0.46 micron. The values of the different centimeters of the scale, according to Mr. Schott's measures, are as follows:

Centimeter.	Length in—	
	Revolutions.	True centimeters.
0 to 1	18.928	1.0001
1 to 2	18.931	1.0003
2 to 3	18.925	0.9999
3 to 4	18.939	1.0007
4 to 5	18.939	1.0007
5 to 6	18.897	0.9985
6 to 7	18.939	1.0007
7 to 8	18.921	0.9997
8 to 9	18.928	1.0001
9 to 10	18.915	0.9994
Mean	18.926	....

2. *Glass centimeter, No. 1, and other scales ruled upon Rutherfurd's machine.*—This centimeter was ruled upon the best ruling-machine of L. M. Rutherfurd. It is upon a piece of glass 0.20 cm. thick, 3.5 cm. broad, and 3.6 cm. long. Though used January 16, 1878, it is marked "Jan. 18, 1878, No. 1, $18\frac{329\frac{1}{2}}{360}$ rev." The lines upon this scale are about 2 cm. long and one micron broad. They are filled with black lead and varnished over. The limiting lines of the centimeter are each midway between two bundles of 10 lines each, distant from one another by $\frac{5}{360}$ rev. The limiting lines are distant $\frac{45}{360}$ rev. from the nearest lines of the bundles. The extremities of the limiting lines are marked by crosses roughly cut by hand. The measures are to be made over a longitudinal line, which is marked by crosses at the two ends.

On January 16, 1878, this centimeter was superposed face to face, upon the centimeter 5 to 6 on the upper scale glass decimeter-scale of centimeters. The difference was measured upon a Rutherfurd screw-micrometer.

Beginning of cm.:

Cm. No. 1.	U. S. 5–6.		
r	r		r
83.405	83.336		
.406	.334		
.404	.334		
83.405	83.335	Diff.	= 0.070

End of cm.:

64.495	64.451		
.494	.447		
.491	.449		
64.493	64.449	Diff.	= 0.044

Cm. No. 1 longer than 5–6	0.026
Cm. U. S. 5–6, too short by	0.029
Cm. No. 1 too short by	0.003
Cm. No. 1	= 0.9998 cm.

Repetition of the same comparison.

Beginning of cm.:

Cm. No. 1.	U. S. 5–6.		
r	r		r
64.679	64.630		
.681	.634		
.686	.634		
64.682	64.633	Diff.	= 0.049

End of cm.:

83.596	83.518		
.597	.517		
.595	.516		
83.596	83.517	Diff.	= 0.079

Cm. No. 1 longer than 5–6	0.030
Cm. U. S. 5–6, too short by	0.029
Cm. No. 1 too long by	0.001
Cm. No. 1	= 1.0001
Mean of two comparisons	0.9999

Comparison of Cm. No. 1 with U. S. 3–4.

Beginning of cm.:

U. S. 3–4.	Cm. No. 1.		
r	r		
.476	.433		
.475	.433		
.474	.432		r
.475	.433	Diff.	= 0.042

End of cm.:

.552	.516		
.552	.517		
.553	.518		
.552	.517	Diff.	= 0.035

Cm. No. 1 shorter than U. S. 3–4	0.007
U. S. 3–4 too long	0.013
Cm. No. 1 too long	0.006
Cm. No. 1	= 1.0003

Repetition of comparison, with low power.

Beginning of cm.:

U. S. 3–4.	Cm. No. 1.		
r	r		
.530	.042		
.532	.040		
.532	.040		r
.531	.041	Diff.	= 0.490

End of cm.:

.452	.956		
.452	.958		
.453	.955		
.452	.955	Diff.	= 0.496

Cm. No. 1 shorter than U. S. 3–4	0.006
U. S. 3–4 too long	0.013
Cm. No. 1 too long	0.007
Cm. No. 1	= 1.0004
Mean of comparisons with U. S. 3–4	1.0003
Mean of comparisons with U. S. 5–6	.9999
Mean of both cm. No. 1	= 1.0001

On 1878, January 17, measures were made on the same Rutherfurd screw-micrometer, using revolutions $55\frac{1}{2}$ to $150\frac{1}{2}$, of the U. S. 5 centimeters from 0 to 5, the 5 centimeters from 5 to 10, and also five measures of the Cm. No. 1, so as to cover the same part of the screw.

U. S. ½ *decimeter* 5 to 10.

5.	10.	5.	
r.	*r.*	*r.*	
150.180	55.652	150.183	
.181	.653	.184	
.181	.654	.185	
150.181	55.653	150.184	Difference 94.529

U. S. ½ *decimeter* 5 to 0.

5.	0.	5.	
r.	*r.*	*r.*	
150.646	56.054	150.638	
.647	.054	.635	
.645	.054	.635	
150.646	56.054	150.636	Difference 94.587

After a complete readjustment the measures were repeated. "Filar 10" and "Filar 8" signify two different positions of the cross-wire.

U. S. ½ *decimeter* 5 to 10.

Filar 10.

10.	5.	10.	
r.	*r.*	*r.*	
55.663	150.181	55.659	
.666	.180	.662	
.667	.180	.662	
55.665	150.180	55.661	Difference 94.517

Filar 8.

r.	*r.*	*r.*	
56.054	150.574	56.052	
.053	.571	.052	
.053	.572	.051	
56.053	150.572	56.052	Difference 94.519
			Mean 94.518

U. S. ½ *decimeter* 0 to 5.

Filar 10.

0.	5.	0.	
r.	*r.*	*r.*	
55.647	150.231	55.655	
.653	.229	.655	
.656	.228	.657	
55.652	150.229	55.656	Difference 94.575

Filar 8.

r.	*r.*	*r.*	
56.041	150.615	56.044	
.038	.614	.044	
.041	.617	.044	
56.040	150.615	56.044	Difference 94.573
			Mean 94.574

	First measure.	Second measure.	Mean.
	r.	*r.*	*r.*
0 to 5..........	94.587	94.574	94.580
5 to 10..........	94.529	94.518	94.524
Difference	.058	.056	
Diff. in microns..	31	30	

Diff. microns, according to Schott, 33

Cm. No. 1.

	r.		*r.*
Beginning....	150.754		
End	131.841	Diff.	18.913
Beginning....	131.485		
End	112.570	Diff.	18.915
Beginning....	112.684		
End	93.778	Diff.	18.906
Beginning....	93.824		
End	74.913	Diff.	18.911
Beginning....	74.756		
End	55.842	Diff.	18.914

Mean.....................	18.9118
Mean of decimeter........	18.9104
	cm.
Length of centimeter...	1.0001
By measures of January 16	1.0001
Concluded value.................	1.0001

This value has a probable error of less than $\frac{1}{10000}$.

This centimeter is equal to 6809½ teeth of Rutherfurd's machine, and as it is $\frac{1}{10000}$ too long, we conclude that 6809 teeth make a centimeter at ordinary temperatures, say about 18° C.

From an investigation which will be published elsewhere, it has been ascertained that the accidental error of position of a line ruled upon Rutherfurd's machine is only $\pm 0^{\mu}.01$; the periodic error differs greatly on different plates, but never amounts nearly to $0^{\mu}.1$. We, therefore, adopt the following values:

$$1 \text{ mm. of } 681 \text{ teeth} = 1^{mm}.0002$$
$$\tfrac{1}{10} \text{ mm. of } 68 \text{ teeth} = 99^{\mu}.87$$
$$\tfrac{1}{100} \text{ mm. of } 7 \text{ teeth} = 10^{\mu}.28$$

ON THE TENTHS OF MILLIMETERS AT THE ENDS OF THE U. S. C. S. PENDULUM-METER, AND ON THE SCREW-REVOLUTIONS OF THE REPSOLD VERTICAL COMPARATOR.

The U. S. C. S. pendulum-meter is similar to the German pendulum-meter described in the publication of the Royal Prussian Geodetical Institute for 1870. At one end, the meter begins with three lines ruled at distances of 10ths of millimeters. At the other end there is a scale of similar lines. The following tables show the comparison of a part of this scale with a millimeter scale of tenths (No. 3) ruled upon Mr. Rutherfurd's machine.

The figures in parenthesis indicate the order of the pointings.

1878, *February* 4.

GLASS SCALE.

	mm. 0.5		*mm.* 0.4		*mm.* 0.3		*mm.* 0.2		*mm.* 0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.023	(2)	1.001	(3)	1.987	(4)	2.983	(5)	3.981
(10)	0.017	(9)	1.007	(8)	1.990	(7)	2.986	(6)	3.990
(31)	0.004	(32)	0.999	(33)	1.980	(34)	2.981	(35)	3.969
(40)	0.007	(39)	1.000	(38)	1.976	(37)	2.972	(36)	3.968
	0.013		1.002		1.983		2.980½		3.977

METAL SCALE.

	m. 1.0002		m. 1.0001		m. 1.0000		m. 0.9999		m. 0.9998
	r.		r.		r.		r.		r.
(11)	$\bar{1}$.988	(12)	0.985	(13)	1.983	(14)	2.973	(15)	3.976
(20)	$\bar{1}$.995	(19)	0.988	(18)	1.978	(17)	2.984	(16)	3.971
(21)	0.000	(22)	0.982	(23)	1.990	(24)	2.986	(25)	3.968
(30)	1.999	(29)	0.992	(28)	1.985	(27)	2.986	(26)	3.975
	1.994½		0.987		1.984		2.982		3.971

SECOND SERIES.

GLASS SCALE.

	mm. 0.5		mm. 0.4		mm. 0.3		mm. 0.2		mm. 0.1
	r.		r.		r.		r.		r.
(1)	0.045	(2)	1.024	(3)	2.030	(4)	3.037	(5)	4.023
(10)	0.061	(9)	1.042	(8)	2.039	(7)	3.030	(6)	4.024
(31)	0.011	(32)	1.019	(33)	2.003	(34)	3.010	(35)	4.001
(40)	0.041	(39)	1.020	(38)	2.011	(37)	3.009	(36)	4.000
	0.039½		1.026		2.021		3.021½		4.012

METAL SCALE.

	m. 1.0002		m. 1.0001		m. 1.0000		m. 0.9999		m. 0.9998
	r.		r.		r.		r.		r.
(11)	$\bar{1}$.969	(12)	0.973	(13)	1.970	(14)	2.972	(15)	3.962
(20)	1.989	(19)	0.990	(15)	1.974	(17)	2.967	(16)	3.054
(21)	1.987	(22)	0.986	(23)	1.980	(24)	2.976	(25)	3.969
(30)	$\bar{1}$.980	(29)	0.980	(28)	1.969	(27)	2.974	(26)	3.065
	1.981		0.982		1.973		2.972		3.962½

February 5.

THIRD SERIES.

GLASS SCALE.

	mm. 0.5		mm. 0.4		mm. 0.3		mm. 0.2		mm. 0.1
	r.		r.		r.		r.		r.
(1)	0.005	(2)	0.991	(3)	1.979	(4)	2.990	(5)	3.994
(10)	$\bar{1}$.995	(9)	0.995	(8)	1.993	(7)	2.990	(6)	3.971
(31)	1.981	(32)	0.980	(33)	1.969	(34)	2.965	(35)	3.978
(40)	1.989	(39)	0.986	(38)	1 974	(37)	2.971	(36)	3.972
	$\bar{1}$.992½		0.988		1.979		2.979		3.979

METAL SCALE.

	m. 1.0002		m. 1.0001		m. 1.0000		m. 0.9999		m. 0.9998
	r.		r.		r.		r.		r.
(11)	1.982	(12)	0.992	(13)	1.978	(14)	2.980	(15)	3.974
(20)	0.001	(19)	1.000	(18)	1.990	(17)	2.990	(16)	3.972
(21)	0.001	(22)	1.006	(23)	1.992	(24)	2.988	(25)	3.973
(30)	0.004	(29)	0.997	(28)	1.991	(27)	2.986	(26)	3.977
	$\bar{1}$.997		0.999		1.988		2.986		3.974

FOURTH SERIES.

GLASS SCALE.

	mm. 0.5		mm. 0.4		mm. 0.3		mm. 0.2		mm. 0.1
	r.		r.		r.		r.		r.
(1)	0.036	(2)	1.049	(3)	2.020	(4)	3.011	(5)	4.019
(10)	0.037	(9)	1.028	(8)	2.012	(7)	3.016	(6)	4.024
(31)	0.021	(32)	1.017	(33)	2.000	(34)	3.000	(35)	4.008
(40)	0.037	(39)	1.023	(38)	2.000	(37)	2.999	(36)	3.996
	0.033		1.029		2.008		3.006½		4.012

METAL SCALE.

	m. 1.0002		m. 1.0001		m. 1.0000		m. 0.9999		m. 0.9998
	r.		r.		r.		r.		r.
(11)	$\bar{1}$.992	(12)	0.990	(13)	1.992	(14)	2.982	(15)	3.978
(20)	$\bar{1}$.994	(19)	0.996	(18)	1.986	(17)	2.982	(16)	3.973
(21)	$\bar{1}$.999	(22)	0.995	(23)	1.985	(24)	2.980	(25)	3.974
(30)	$\bar{1}$.999	(29)	1.000	(28)	1.993	(27)	2.985	(26)	3.974
	$\bar{1}$.996		0.995		1.989		2.982		3.975

February 6.

FIFTH SERIES.

GLASS SCALE.

	mm. 0.5		mm. 0.4		mm. 0.3		mm. 0.2		mm. 0.1
	r.		r.		r.		r.		r.
(1)	$\bar{1}$.990	(2)	0.983	(3)	1.981	(4)	2.990	(5)	3.987
(10)	$\bar{1}$.989	(9)	0.986	(8)	1.980	(7)	2.990	(6)	3.980
(31)	$\bar{1}$.953	(32)	0.969	(33)	1.968	(34)	2.977	(35)	3.969
(40)	$\bar{1}$.970	(39)	0.970	(38)	1.951	(37)	2.962	(36)	3.973
	$\bar{1}$.975½		0.977		1.970		2.980		3.977

METAL SCALE.

	m. 1.0002		m. 1.0001		m. 1.0000		m. 0.9999		m. 0.9998
	r.		r.		r.		r.		r.
(11)	$\bar{1}$.982	(12)	0.985	(13)	1.984	(14)	2.978	(15)	3.972
(20)	$\bar{1}$.994	(19)	0.990	(18)	1.982	(17)	2.980	(16)	3.970
(21)	$\bar{1}$.992	(22)	0.990	(23)	1.985	(24)	2.981	(25)	3.972
(30)	$\bar{1}$.996	(29)	0.994	(28)	1.989	(27)	2.981	(26)	3.972
	$\bar{1}$.991		0.990		1.985		2.980		3.971½

SIXTH SERIES.

GLASS SCALE.

	mm. 0.5		mm. 0.4		mm. 0.3		mm. 0.2		mm. 0.1
	r.		r.		r.		r.		r.
(1)	0.020	(2)	1.007	(3)	1.999	(4)	2.990	(5)	3.986
(10)	0.024	(9)	1.010	(8)	1.998	(7)	2.991	(6)	4.000
(31)	$\bar{1}$.985	(32)	0.979	(33)	1.976	(34)	2.973	(35)	3.970
(40)	0.002	(39)	0.990	(38)	1.968	(37)	2.982	(36)	3.980
	0.008		0.996½		1.985		2.984		3.984

SIXTH SERIES—Continued.

METAL SCALE.

	m. 1.0001		*m.* 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.001	(12)	0.993	(13)	1.996	(14)	2.990	(15)	4.010
(20)	0.012	(19)	1.001	(18)	1.996	(17)	2.990	(16)	4.019
(21)	0.009	(22)	1.000	(23)	1.995	(24)	2.984	(25)	4.023
(30)	0.008	(29)	0.997	(28)	1.997	(27)	2.984	(26)	4.019
	0.007½		0.998		1.996		2.987		4.018

February 7.

SEVENTH SERIES.

GLASS SCALE.

	mm. 0.5		*mm.* 0.4		*mm.* 0.3		*mm.* 0.2		*mm.* 0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.026	(2)	1.022	(3)	2.016	(4)	3.005	(5)	4.013
(10)	0.032	(9)	1.020	(8)	2.004	(7)	3.023	(6)	4.010
(31)	$\bar{1}$.982	(32)	0.972	(33)	1.962	(34)	2.962	(35)	3.960
(40)	$\bar{1}$.990	(39)	0.981	(38)	1.965	(37)	2.969	(36)	3.980
	0.007½		1.019		1.987		2.990		3.991

METAL SCALE.

	m. 1.0001		*m.* 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.018	(12)	1.021	(13)	2.018	(14)	3.008	(15)	4.036
(20)	0.023	(19)	1.030	(18)	2.020	(17)	3.007	(16)	4.035
(21)	0.021	(22)	1.023	(23)	2.015	(24)	3.012	(25)	4.039
(30)	0.019	(29)	1.025	(28)	2.011	(27)	3.003	(26)	4.031
	0.020		1.025		2.016		3.007½		4.035

EIGHTH SERIES.

GLASS SCALE.

	mm. 0.5		*mm.* 0.4		*mm.* 0.3		*mm.* 0.2		*mm.* 0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.020	(2)	1.014	(3)	1.990	(4)	2.995	(5)	3.997
(10)	0.011	(9)	1.004	(8)	1.994	(7)	2.996	(6)	3.995
(31)	$\bar{1}$.983	(32)	0.980	(33)	1.977	(34)	2.976	(35)	3.976
(40)	0.003	(39)	1.000	(38)	1.980	(37)	2.978	(36)	3.976
	0.004		0.999½		1.985		2.986		3.986

METAL SCALE.

	m. 1.0001		*m.* 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.005	(12)	1.013	(13)	2.000	(14)	2.995	(15)	4.019
(20)	0.011	(19)	1.016	(18)	2.010	(17)	2.995	(16)	4.019
(21)	0.010	(22)	1.013	(23)	2.004	(24)	2.997	(25)	4.019
(30)	0.013	(29)	1.007	(28)	2.007	(27)	3.003	(26)	4.021
	0.010		1.012		2.005		2.997½		4.019½

February 8.

NINTH SERIES.

GLASS SCALE.

	mm.		*mm.*		*mm.*		*mm.*		*mm.*
	0.5		0.4		0.3		0.2		0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.034	(2)	1.034	(3)	2.033	(4)	3.025	(5)	4.011
(10)	0.038	(9)	1.022	(8)	2.021	(7)	3.028	(6)	4.016
(31)	0.006	(32)	1.003	(33)	1.990	(34)	2.983	(35)	3.993
(40)	0.009	(39)	1.000	(38)	1.995	(37)	2.990	(36)	3.993
	0.022		1.015		2.010		3.006½		4.003

METAL SCALE.

	m.		*m.*		*m.*		*m.*		*m.*
	1.0001		1.0000		0.9999		0.9998		0.9997
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.020	(12)	1.020	(13)	2.014	(14)	3.011	(15)	4.036
(20)	0.029	(19)	1.023	(18)	2.019	(17)	3.012	(16)	4.036
(21)	0.030	(22)	1.024	(23)	2.016	(24)	3.010	(25)	4.031
(30)	0.018	(29)	1.024	(28)	2.010	(27)	3.011	(26)	4.031
	0.024		1.023		2.015		3.011		4.033½

February 10.

TENTH SERIES.

GLASS SCALE.

	mm.		*mm.*		*mm.*		*mm.*		*mm.*
	0.5		0.4		0.3		0.2		0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.006	(2)	1.001	(3)	1.990	(4)	2.989	(5)	4.007
(10)	0.029	(9)	1.010	(8)	2.009	(7)	2.993	(6)	4.012
(31)	0.000	(32)	1.010	(33)	2.003	(34)	3.000	(35)	4.013
(40)	0.026	(39)	1.018	(38)	2.009	(37)	3.004	(36)	4.018
	0.015		1.010		2.003		2.996½		4.012½

METAL SCALE.

	m.		*m.*		*m.*		*m.*		*m.*
	1.0001		1.0000		0.9999		0.9998		0.9997
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.046	(12)	1.041	(13)	2.042	(14)	3.023	(15)	4.054
(20)	0.050	(19)	1.041	(18)	2.041	(17)	3.023	(16)	4.055
(21)	0.044	(22)	1.043	(23)	2.040	(24)	3.032	(25)	4.052
(30)	0.043	(29)	1.049	(28)	2.036	(27)	3.026	(26)	4.059
	0.046		1.043½		2.040		3.026		4.055

February 11.

ELEVENTH SERIES.

GLASS SCALE.

	mm.		*mm.*		*mm.*		*mm.*		*mm.*
	0.5		0.4		0.3		0.2		0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.034	(2)	1.024	(3)	2.003	(4)	3.010	(5)	4.022
(10)	0.024	(9)	1.018	(8)	2.006	(7)	3.010	(6)	4.014
(31)	1.969	(32)	0.974	(33)	1.964	(34)	2.964	(35)	3.976
(40)	1.985	(39)	0.972	(38)	1.964	(37)	2.974	(36)	3.980
	0.003		0.997		1.984		2.989½		3.998

ELEVENTH SERIES—Continued.

METAL SCALE.

	m. 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997		*m.* 0.9996
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.021	(12)	1.025	(13)	2.011	(14)	3.041	(15)	4.028
(20)	0.034	(19)	1.019	(18)	2.019	(17)	3.041	(16)	4.016
(21)	0.034	(22)	1.025	(23)	2.015	(24)	3.037	(25)	4.015
(30)	0.032	(29)	1.019	(28)	2.014	(27)	3.030	(26)	4.020
	0.030		1.022		2.015		3.037		4.020

TWELFTH SERIES.

GLASS SCALE.

	mm. 0.5		*mm.* 0.4		*mm.* 0.3		*mm.* 0.2		*mm.* 0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.010	(2)	1.001	(3)	1.985	(4)	2.977	(5)	3.975
(10)	0.028	(9)	1.020	(8)	1.992	(7)	2.987	(6)	3.992
(31)	0.000	(32)	0.980	(33)	1.964	(34)	2.960	(35)	3.970
(40)	0.000	(39)	0.993	(38)	1.970	(37)	2.952	(36)	3.970
	0.009½		0.998½		1.978		2.969		3.977

METAL SCALE.

	m. 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997		*m.* 0.9996
	r.		*r.*		*r.*		*r.*		*r.*
(11)	$\bar{1}$.979	(12)	0.979	(13)	1.965	(14)	2.995	(15)	3.960
(20)	$\bar{1}$.996	(19)	0.991	(18)	1.974	(17)	2.995	(16)	3.970
(21)	$\bar{1}$.993	(22)	0.985	(23)	1.976	(24)	2.991	(25)	3.965
(30)	$\bar{1}$.990	(29)	0.983	(28)	1.964	(27)	2.991	(26)	3.974
	$\bar{1}$.989½		0.984½		1.970		2.993		3.967

THIRTEENTH SERIES.

GLASS SCALE.

	mm. 0.5		*mm.* 0.4		*mm.* 0.3		*mm.* 0.2		*mm.* 0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	$\bar{1}$.993	(2)	0.982	(3)	1.964	(4)	2.967	(5)	3.961
(10)	0.010	(9)	0.984	(8)	1.970	(7)	2.970	(6)	3.975
(31)	$\bar{1}$.970	(32)	0.974	(33)	1.951	(34)	2.953	(35)	3.966
(40)	$\bar{1}$.993	(39)	0.982	(38)	1.968	(37)	2.960	(36)	3.967
	$\bar{1}$.991½		0.980½		1.963		2.962½		3.967

METAL SCALE.

	m. 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997		*m.* 0.9996
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.041	(12)	1.030	(13)	2.012	(14)	3.037	(15)	4.010
(20)	0.043	(19)	1.030	(18)	2.020	(17)	3.040	(16)	4.010
(21)	0.039	(22)	1.028	(23)	2.013	(24)	3.029	(25)	4.010
(30)	0.039	(29)	1.030	(28)	2.008	(27)	3.030	(26)	4.017
	0.040½		1.029½		2.013		3.034		4.012

February 12.

FOURTEENTH SERIES.

GLASS SCALE.

	mm. 0.5		*mm.* 0.4		*mm.* 0.3		*mm.* 0.2		*mm.* 0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.060	(2)	1.040	(3)	2.026	(4)	3.040	(5)	4.054
(10)	0.063	(9)	1.060	(8)	2.037	(7)	3.039	(6)	4.051
(31)	0.020	(32)	1.014	(33)	2.007	(34)	3.010	(35)	4.014
(40)	0.033	(39)	1.028	(38)	2.008	(37)	3.011	(36)	4.025
	0.044		1.035½		2.019½		3.025		4.036

METAL SCALE.

	m. 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997		*m.* 0.9996
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.021	(12)	1.016	(13)	2.000	(14)	3.028	(15)	4.006
(20)	0.019	(19)	1.029	(18)	2.010	(17)	3.030	(16)	3.996
(21)	0.036	(22)	1.022	(23)	2.013	(24)	3.036	(25)	4.003
(30)	0.031	(29)	1.019	(28)	1.996	(27)	3.020	(26)	4.006
	0.027		1.021½		2.005		3.028½		4.003

FIFTEENTH SERIES.

GLASS SCALE.

	mm. 0.5		*mm.* 0.4		*mm.* 0.3		*mm.* 0.2		*mm.* 0.1
	r.		*r.*		*r.*		*r.*		*r.*
(1)	0.020	(2)	1.002	(3)	2.003	(4)	3.008	(5)	3.997
(10)	0.018	(9)	1.019	(8)	2.001	(7)	3.004	(6)	3.992
(31)	1.989	(32)	0.980	(33)	1.984	(34)	2.982	(35)	3.976
(40)	0.012	(39)	0.999	(38)	1.974	(37)	2.980	(36)	3.979
	0.010		1.000		1.990½		2.993½		3.986

METAL SCALE.

	m. 1.0000		*m.* 0.9999		*m.* 0.9998		*m.* 0.9997		*m.* 0.9996
	r.		*r.*		*r.*		*r.*		*r.*
(11)	0.019	(12)	1.016	(13)	1.994	(14)	3.025	(15)	4.004
(20)	0.028	(19)	1.027	(18)	2.006	(17)	3.035	(16)	4.005
(21)	0.030	(22)	1.022	(23)	2.013	(24)	3.027	(25)	4.004
(30)	0.026	(29)	1.024	(28)	2.010	(27)	3.024	(26)	4.006
	0.026		1.022		2.006		3.028		4.005

The following tables exhibit the results of all these measures:

GLASS SCALE.

	mm. *mm.* 0.5 to 0.4	*mm.* *mm.* 0.4 to 0.3	*mm.* *mm.* 0.3 to 0.2	*mm.* *mm.* 0.2 to 0.1
	r.	*r.*	*r.*	*r.*
First series	0.989	0.981½	0.997	0.996½
Second series	0.987	0.994½	1.001	0.990½
Third series	0.985½	0.991	1.000	1.000
Fourth series	0.986½	0.979	0.998½	1.005
Fifth series	1.001½	0.993	1.010	1.000
Mean	0.994	0.988	1.001	0.998

GLASS SCALE—Continued.

	mm. mm. 0.5 to 0.4	mm. mm. 0.4 to 0.3	mm. mm. 0.3 to 0.2	mm. mm. 0.2 to 0.1
	r.	r.	r.	r.
Sixth series	0.989	0.989	0.999	1.000
Seventh series	0.986	0.988	1.003	1.001
Eighth series	0.995	0.986	1.001	1.000
Ninth series	0.993½	0.995	0.997	0.997
Tenth series	0.994½	0.993	0.994	1.016
Mean	0.992	0.990	0.999	1.003
	r.	r.	r.	r.
Eleventh series	0.994	0.987	1.005	1.008½
Twelfth series	0.989	0.979	0.991	1.008
Thirteenth series	0.989	0.982½	0.999	1.005
Fourteenth series	0.991½	0.984	1.005½	1.006
Fifteenth series	0.990	0.990½	1.003	0.992½
Mean	0.991	0.985	1.001	1.004
Mean of 15	0.992	0.988	1.000	1.002

Probable error of a mean of 15 = ± 0r.001.

METAL SCALE.

	m. 1.0002 to 1.0001	m. 1.0001 to 1.0000	m. 1.0000 to 0.9999	m. 0.9999 to 0.9998
	r.	r.	r.	r.
First series	0.989	0.997	0.998	0.990
Second series	1.001	0.991	0.999	0.990
Third series	1.002	0.989	0.998	0.988
Fourth series	0.999	0.994	0.993	0.992½
Fifth series	0.999	0.995	0.995	0.991½
Mean	0.998	0.993	0.997	0.990

	m. 1.0001 to 1.0000	m. 1.0000 to 0.9999	m. 0.9999 to 0.9998	m. 0.9998 to 0.9997
	r.	r.	r.	r.
Sixth series	0.990	0.998	0.991	1.031
Seventh series	1.004½	0.991	0.991½	1.027½
Eighth series	1.002½	0.993	0.992	1.022
Ninth series	1.001½	0.992	0.996	1.022½
Tenth series	0.998	0.996	0.986	1.029
Mean	0.999	0.994	0.991	1.026

METAL SCALE—Continued.

	m. 1.0000 to 0.9999	*m.* 0.9999 to 0.9998	*m.* 0.9998 to 0.9997	*m.* 0.9997 to 0.9996
	r.	*r.*	*r.*	*r.*
Eleventh series................	0.992	0.993	1.022½	0.982½
Twelfth series.................	0.995	0.985	1.023	0.978
Thirteenth series...............	0.989	0.984	1.021	0.978
Fourteenth series	0.992	0.986	1.024	0.974
Fifteenth series	0.996½	0.983	1.022	0.977
Mean.	0.993	0.986	1.022½	0.978

Probable error of a mean of 15 $= 0^r.0006$.

The following table shows the apparent inequalities of different revolutions of the screws:

	Excess of measure on 1^r to 2^r / Over measure on 0^r to 1^r	Excess of measure on 2^r to 3^r / Over measure on 1^r to 2^r	Excess of measure on 3^r to 4^r / Over measure on 2^r to 3^r
m. *m.*	*r.*		
1.0001 to 1.0000	− 0.006	*r.*	
1.0000 to 0.9999	+ 0.001	+ 0.003	*r.*
0.9999 to 0.9998		+ 0.005	− 0.001
0.9999 to 0.9997			+ 0.003½
Mean..........	− 0.002½	+ 0.004	+ 0.001
By glass scale..	− 0.004	+ 0.012	+ 0.002

These results accord in showing certain inequalities in different parts of the screw; but I shall neglect these as being of no importance for the measures of the pendulum. There would seem, from these measures, to be an inequality in the spaces on the glass scale; but measures made with a higher power have proved that these are simply due to constant errors of judgment in bisecting the lines on glass.

The mean value of the divisions of the glass scale (giving the middle two double weight, according to the requirement of the theory of least squares) is $0^r.995$, whence we may assume

$$\tfrac{1}{10} \text{ millimeter} = 0^r.996.$$

This gives us for the spaces on the metallic scale

m. 1.0002 to 1.0001	*m.* 1.0001 to 1.0000	*m.* 1.0000 to 0.9999	*m.* 0.9999 to 0.9998	*m.* 0.9998 to 0.9997	*m.* 0.9997 to 0.9996
mm.	*mm.*	*mm.*	*mm.*		
0.1002	0.0997	0.1001	0.0994	*mm.*	
	0.1003	0.0998	0.0995	0.1030	*mm.*
		0.0997	0.0990	0.1026	0.0982
0.1002	0.1000	0.0999	0.0993	0.1028	0.0982

The following are comparisons of the spaces between the three lines at the beginning of the meter with those between the lines $0^{mm}.1$, $0^{mm}.2$, $0^{mm}.3$ of the same glass scale:

First comparison.

	GLASS SCALE.		
	0.3	0.2	0.1
	r.	r.	r.
	0.942	1.932	2.921
	0.944	1.940	2.922
Mean....	0.943	1.936	2.921
	METAL SCALE.		
	−0.0	0.0	+0.1
	0.978	1.970	2.967
	1.002	1.992	2.970
Mean....	0.990	1.981	2.969
	0.992	1.997	2.980
	1.005	1.990	2.970
Mean....	0.998	1.993	2.975
	GLASS SCALE.		
	0.911	1.888	2.907
	0.907	1.910	2.891
Mean...	0.909	1.899	2.899

Second comparison. Temp., 13°.5 C.

	GLASS SCALE.		
	0.3	0.2	0.1
	r.	r.	r.
	1.021	2.019	3.016
	1.005	2.005	3.016
Mean....	1.013	2.012	3.016
	METAL SCALE.		
	−0.0	0.0	+0.1
	0.990	2.022	3.019
	1.020	2.016	3.026
Mean....	1.005	2.019	3.022
	1.013	2.016	3.015
	1.019	2.009	3.005
Mean....	1.016	2.013	3.010
	GLASS SCALE.		
	0.994	2.000	3.009
	0.980	1.995	3.004
Mean....	0.987	1.998	3.006

The above measures were made on the part of the metallic lines usually used. The following were taken somewhat nearer the line separating the meter-scale from that of the metallic thermometer:

Third comparison. 13° C.

	GLASS SCALE.		
	0.3	0.2	0.1
	r.	r.	r.
	1.032	2.050	3.017
	1.040	2.052	3.028
Mean..	1.036	2.051	3.023
	METAL SCALE.		
	−0.1	0.0	+0.1
	1.060	2.051	3.053
	1.066	2.050	3.072
Mean..	1.063	2.051	3.063
	1.079	2.048	3.036
	1.052	2.050	3.042
Mean..	1.065	2.049	3.039
	GLASS SCALE.		
	1.010	2.026	3.014
	1.038	2.013	3.024
Mean..	1.024	2.019	3.019

Fourth comparison.

	GLASS SCALE.		
	0.3	0.2	0.1
	r.	r.	r.
	1.012	2.018	3.003
	1.020	2.001	3.006
Mean..	1.016	2.010	3.005
	METAL SCALE.		
	−0.1	0.0	+0.1
	1.010	1.989	3.014
	1.005	1.990	3.019
Mean..	1.008	1.989	3.016
	1.011	1.990	3.002
	1.009	1.981	3.006
Mean..	1.010	1.985	3.004
	GLASS SCALE.		
	0.990	1.990	2.995
	1.013	2.010	3.016
Mean..	1.002	2.000	3.005

Fifth comparison.

	GLASS SCALE.		
	0.3	0.2	0.1
	r.	r.	r.
	1.018	2.019	3.012
	1.031	2.025	3.002
Mean..	1.024	2.022	3.007
	METAL SCALE.		
	−0.1	0.0	+0.1
	1.008	1.992	2.988
	1.008	1.997	3.008
Mean..	1.008	1.995	2.998
	1.010	1.988	2.999
	1.007	1.987	3.008
Mean..	1.008	1.987	3.004
	GLASS SCALE.		
	0.974	1.990	2.986
	0.970	1.979	2.965
Mean..	0.972	1.984	2.975

The following are the results of these comparisons:

GLASS SCALE.

	0.3 to 0.2	0.2 to 0.1
	r.	r.
First comparison	0.991	0.993
Second comparison	1.005	1.007
Third comparison	1.005	0.985
Fourth comparison	0.996	1.000
Fifth comparison	1.000	0.988
	0.999	0.995

METAL SCALE.

	−0.1 to 0.0	0.0 to +0.1
First comparison	0.993	0.984
Second comparison	1.005	1.001
Third comparison	0.985	1.001
Fourth comparison	0.979	1.023
Fifth comparison	0.983	1.010
	0.989	1.004

From this it appears that the values of the spaces on the metallic scale are

−0.1 to 0.0	0.0 to +0.1
$98^{\mu}.9$	$100^{\mu}.8$

ON THE VALUE OF THE SCREW-REVOLUTIONS OF THE UPPER MICROSCOPE OF THE REPSOLD VERTICAL COMPARATOR.

On 1878, January 21, a glass scale of 68 teeth of Rutherfurd's ruling-machine per division was measured by this micrometer. The following are the results:

	Line 1 to line 2. (0^r to 1^r)	Line 2 to line 3. (1^r to 2^r)	Line 3 to line 4. (2^r to 3^r)	Line 4 to line 5. (3^r to 4^r)
	r.	r.	r.	r.
	1.000	0.994	0.999	0.997
	1.000	0.992	0.999	0.993
	1.005	1.000	0.996	0.993
	1.008	0.994	1.003	0.993
Means....	1.003	0.995	0.999	0.994

The probable error of each mean is $\pm 0^r.001$. We, therefore, have for the corrections of tenths of millimeters measured on these parts of the screw in order to bring them to the mean value between 1^r and 2^r

r. r.	r.
0 to 1	−0.006
1 to 2	+0.002
2 to 3	−0.002
3 to 4	+0.003

The corrections to bring the means of three lines measured on three parts of the screw to the middle are:

Correction to mean of 0^r, 1^r, and 2^r, to bring to 2^r $= + 1^r.002$
Correction to mean of 2^r, 3^r, and 4^r, $= - 0^r.999$
Correction to mean of 1^r, 2^r, and 3^r, $= + 0^r.0013$

The values of the two revolutions from 1^r to 3^r are best obtained from the observations made in comparing the pendulum with the standard at different stations.

	m.	m.		r.	r.
At Geneva,	0.9999 to	1.0001	on pendulum-meter measured	1.994	± 0.001
At Paris,	− 0.0001 to	+ 0.0001	on same meter measured	1.991	± 0.001
At Berlin,	− 0.0001 to	+ 0.0001	on same meter measured	1.996	± 0.001
At Kew,	− 0.0001 to	+ 0.0001	on same meter measured	2.001	± 0.001
At Hoboken,	0.9997 to	0.9999	on same meter measured	2.022	

The known values of these intervals give for the measure of $\frac{2}{10}$ of a millimeter

	r.
From Geneva observations	1.995
From Paris observations	1.994
From Berlin observations	1.999
From Kew observations	2.004
From Hoboken observations	2.001

As the focalization of this microscope is made with difficulty, it is not surprising that its magnifying power should differ by some thousandths of itself. The focalization at Berlin was particularly good, and $1^r.999$ may be taken as the true value, giving for the value of one revolution between 1^r and 3^r

$$0^{mm}.10005$$

VALUE OF SCREW REVOLUTIONS OF THE LOWER MICROSCOPE.

Corresponding distances measured with the lower microscope, between marks on the pendulum-meter, were obtained from the observations at the different stations, as follows:

	m.	m.		r.
At Geneva,	− 0.0001 to	+ 0.0001	measured	1.987
At Paris,	0.9999 to	1.0001	measured	1.991
At Berlin,	0.9999 to	1.0001	measured	1.991
At Kew,	0.9997 to	0.9999	measured	2.014
At Hoboken	− 0.0001 to	+ 0.0001	measured	2.002

Hence, by substituting the values of the intervals, we obtain as the value of $\frac{2}{10}$mm. in revolutions of the microscope screw:

	r.
Geneva observations	1.990
Paris observations	1.992
Berlin observations	1.992
Kew observations	1.993
Hoboken observations	2.005

Adopting, as before, the Berlin value, we obtain as the value of one revolution,

$$0^{mm}.1004$$

RESULTS OF OBSERVATIONS OF LENGTH.

Professor Förster gives as the length of the German pendulum-meter, compared with No. 1605, in a letter dated June 24, 1878:

$$\text{B. P.} - 1605 = - 103^{\mu}.5 - 0^{\mu}.2\,(\tau' - 17^{\circ}.0).$$

The following is the translation of a communication from Professor Förster, giving the length and distance between the lines at the end of meter No. 49:

BERLIN, *March* 29, 1878.

Summary of the results of comparisons of lengths between the standard meter No. 49 and others.

No. 49 is a brass bar with a shoulder at each end. The surfaces of these shoulders are in a horizontal plane passing through the axis of the bar. Let into surface of each shoulder is a silver plate having three lines ruled on it. The contact-cylinders used for comparisons between this bar (which is a line measure) and end measures may be considered a part of No. 49.

Standard No. 49, described above, was carefully compared at different temperatures with standard meter No. 1605, belonging to the "Normal Standards Commission." No. 1605 is a line measure of brass, entirely prismatic in form. The defining lines are ruled on silver plates let into the surface of the bar. In these comparisons, both bars were supported on a plane surface of brass. No 1605 had repeatedly been compared with the platinum meter belonging to the commission. The length of the platinum meter was derived from comparison made in 1860 between it and the mètre du Conservatoire and the mètre des Archives. But the direct comparisons between No. 1605 and the platinum meter were not as accurate as the direct comparisons between the platinum meter and No. 49, the results of which will be given farther on.

The absolute length of our platinum meter is not as yet known with sufficient accuracy, because the comparison between it and the mètre des Archives gave a result differing nearly 0.02^{mm} from the result of comparisons between it and the mètre du Conservatoire, while the actual difference between these Paris meters is said not to exceed 0.003^{mm}.

At the same time, neither the direct nor the indirect comparisons between No. 49 and the platinum meter have as much value as the more accurate comparisons between No. 49 with No. 1605, because in these last comparisons the errors due to our former defective arrangements for securing uniformity of temperatures are less sensible on account of the similarity of the material of which both bars are composed, than in comparisons between brass and platinum, and also because it will soon be possible to compare, by the aid of the most refined means, No. 1605 with the new prototype.

The results of all comparisons referring to No. 49 will be given in the following pages, and will serve as data for a final reduction at some future time. We begin with the most important, namely, the comparisons between No. 49 and No. 1605.

The lines defining the length of No. 49 are each midway between two other lines distant from them about 7^{mm}. The latter are used in connection with the contact-cylinder in comparisons between No. 49 and end measures. Therefore, No. 1605 could be compared with three sets of meter-defining lines on No. 49, to wit:

With the meter defined by the central lines of each group of 3 near the ends of the bar.
With the meter defined by the outside line on the left hand and the inside line on the right.
With the meter defined by the inside line on the left and the inside line on the right.

Of course, all these measurements refer to that part of the transverse lines lying midway between the two longitudinal ones.

Very accurate micrometric measurements give the following values for the four spaces between the transverse lines of No. 49, at a temperature of 0° C. Beginning on the outside of one end of the bar the several lines are numbered consecutively, 1, 2, 3, and beginning on the inside of the other end, 4, 5, 6:

		mm.		*mm.*
Space 1–2	=	7.3796	±	0.00016
Space 2–3	=	7.3761	±	0.00022
Space 4–5	=	7.3782	±	0.00023
Space 5–6	=	7.3774	±	0.00018

Putting .001 millimeter = μ, we find from the foregoing measurements that the meter defined by the lines 1–4 is longer than that defined by the middle lines 2–5, by $1^{\mu}.4$, and also that the meter between the lines 3–6 is longer than the meter between the lines 2–5, by $1^{\mu}.3$.

By making use of the values thus found and reducing all comparisons between No. 1605 and No. 49 to the meter defined on the latter by the middle or principal lines 2–5, we get:

Temperature (C.)	No. 1605 — No. 49
°	μ
4.07	— 39.5
21.53	— 42.0
21.61	— 42.1
23.42	— 41.4
23.44	— 41.55
23.47	— 40.65
23.49	— 41.45
23.63	— 41.60
23.80	— 42.45
25.37	— 42.45
25.48	— 41.65
25.59	— 41.95

Reducing these results by least squares we deduce the following formula for the difference of length 1605 — 49:

$$1605 - 49 = \left\{ \begin{array}{rr} \overset{\mu}{-\ 39.08} & -\ 0.110\,t \\ \pm\ \ 0.42 & \pm\ \ .019\,t \end{array} \right.$$

where t stands for temperature in terms of the Centigrade scale. The probable errors of the numerical quantities are denoted by the figures having the signs ± prefixed, and these will throughout this paper be written below or next to the quantities to which they refer. The following table, calculated by the above formula, gives the differences in length (1605 — 49) and their probable errors at the given temperatures:

Temperature.	1605 — 49	
°	μ	μ
0	— 39.1	± 0.4
+ 5	39.6	0.3
+ 10	40.2	0.2
+ 15	40.7	0.1
+ 20	41.3	0.1
+ 25	41.8	0.1

In regard to the comparisons between No. 49 and the platinum meter (end measure), it is necessary to explain the manner in which the contact-cylinders were used. The contact ends of the two cylinders I and II are spherical surfaces. A glass scale, which can be distinctly seen by microscopes supplied with the necessary reflecting apparatus, with transverse and two longitudinal lines, is let into the axis of each cylinder in such a manner that the point on the sixth transverse line midway between the two longitudinal lines is in the center of the sphere of which the spherical contact surface is a part. The transverse lines are reckoned from the contact surface along the axis of the cylinder. When the two cylinders are in contact the relation between the sum of the radii of the two spherical abutting surfaces and the distance between the transverse lines before mentioned is independent of small variations in the inclination of the axes of the cylinders to each other. We can therefore deduce with perfect accuracy the linear value of the sum of the two radii by measuring the distance between the specified transverse lines when the cylinders are in contact, and can thus reduce the pointings on the centers of the spherical surfaces to their point of contact; a reduction which is necessary in the comparison of the end measure with the line measure. Because the reference-points of the scales are in the centers of the spheres of which the abutting surfaces are part, no sensible error need be apprehended from changes in the relative position of the axes of the cylinders when they are in contact with the abutting surfaces of the end measure. The length of the sum of the two radii of the two cylinders in contact, that is, the distance between the specified transverse lines, was measured on a good transverse comparator, and was referred to the same standard scale used in measuring the auxiliary spaces on Standard No. 49. Thus the reduction of the distance between the defining lines of the cylinders when in contact with the platinum meter to the length of the platinum meter itself, can be referred to the same standard scale, and will be liable to the same errors as the reduction of the extreme lines on No. 49 to the middle or standard lines. As far as the use of the auxiliary distances measured on the transverse comparator are concerned the effect of errors of graduation of the standard scale to which they are referred might have been eliminated from the comparison between the line and end measure. Full attention was not paid to this consideration in these comparisons. However, only the differences of the errors of graduation of adjacent lines of the standard scale came into account, and these were determined with extraordinary precision, and were carefully taken into account in the computations.

The distance between the defining lines of the glass scales in the axes of the cylinders when the latter were in carefully adjusted contact is given by the following expression:

$$C = \begin{cases} \overset{mm.}{14.7432} + \overset{mm.}{0.000162}\, t. \\ \pm\ 0.00026 \end{cases}$$

For the sum of the auxiliary spaces 1–2 and 5–6 on No. 49, we get in conformity with the values previously given:

$$S = \begin{cases} \overset{mm.}{14.7570} + \overset{mm.}{0.000265} \times t. \\ \pm\ 0.00024 \end{cases}$$

At any temperature t°, the excess of the sum of the outside auxiliary spaces on No. 49, over the sum of the spaces between the point of contact of the cylinders and the defining lines of their scales, is therefore

$$S - C = \begin{cases} + \overset{\mu}{13.8} + \overset{\mu}{0.103}\, t. \\ \pm\ .35 \end{cases}$$

This value represents the correction which must be applied to the difference in length between the extreme lines on No. 49 and the distance between the defining lines in the cylinders when the latter are in contact with the abutting surfaces of the platinum meter.

Our comparisons give directly:

$$\lambda = (Pl + C) - (1605 + S),$$

and, therefore,

$$Pl - 1605 = \lambda + S - C,$$

and substituting the foregoing numerical values,

$$Pl - 1605 = \lambda + \overset{\mu}{13.8} + \overset{\mu}{0.103}\, t.$$

In this way were obtained the following differences of length between the platinum meter, defined by the distance between the middles of its end-surfaces, and standard No. 49, defined by the distance between the middle transverse line near each end, measured midway between the longitudinal lines:

Temperature.	Pl — 49	Calc. — Obs.
	μ	μ
+ 3.25	— 8.2	— 0.2
+ 6.28	— 39.2	+ 0.2
+ 23.55	— 213.1	— 0.1

From these values we deduce the following expression by the method of least squares:

$$Pl - 49 = \begin{cases} + \overset{\mu}{24.4} - \overset{\mu}{10.086}\, t \\ \pm\ 0.20 \pm 0.014\, t \end{cases}$$

by means of which the following tabulated values are computed:

Temperature.	Pl — 49
°	μ μ
0	+ 24.4 ± 0.20
5	− 26.0 ± 0.15
10	− 76.5 ± 0.12
15	− 126.9 ± 0.13
20	− 177.3 ± 0.17
25	− 227.8 ± 0.23

The uncertainty pertaining to these values may be much larger than the probable errors indicate because of the uncertainty of the relative expansion and of the imperfect methods of determining the temperatures, and because the observations are not sufficiently numerous.

If we take the result of the comparison made in Paris between our platinum meter and the mètre des Archives, namely: Mètre des A. − Pl = − $3^{\mu}.01$, we obtain the equation:

$$\text{M. d. A.} - \text{No. } 49 = \{ + 21^{\mu}.4 - 10^{\mu}.086\ t \}.$$

This equation, however, is subject to doubt on account of the imperfection of the comparisons at Paris, previously mentioned, a doubt which can only be removed at some future time with the assistance of the more accurate equation established between 1605 and 49. The direct comparison between our platinum meter and the mètre des Archives (A) appears much less complete than the indirect comparison through the medium of the mètre du Conservatoire (C). On the assumption made at Paris that the coefficient of expansion of A, C, and Pl are nearly equal, the following results were obtained for the indirect comparisons. Direct observations between C and Pl gave:

$$\text{C} - \text{Pl} = - 16^{\mu}.2$$

But according to subsequent determinations (Procès-Verbaux de la Section Francaise, 1870), A − C = − $3^{\mu}.2$, it would follow that A − Pl = − $19^{\mu}.2$, and using this last equation, we get:

$$\text{A} - 49 = + 5^{\mu}.2 - 10.086\ t,$$

and if for 1° Centigrade we take the expansion = $8^{\mu}.60$, we obtain:

$$\text{No. } 49 = \overset{mm.}{999.9948} + \overset{mm.}{0.01869}\ t,$$

a value subject to a much greater uncertainty than is indicated by the probable errors given above on account of the imperfections of the Paris results.

The results of the scale values of the cylinders are given in conclusion (the lines designated by 1 are those nearest the contact ends):

Space.	Cylinder I.		Cylinder II.	
	μ		μ	
Line 1 to 2	54.2		49.4	
1 to 3	104.3	50.1	100.7	51.3
1 to 4	154.5	50.2	148.7	48.0
1 to 5	207.8	53.3	199.4	50.7
1 to 6	252.8	45.0	249.7	50.3
1 to 7	305.5	52.7	303.4	53.7
1 to 8	355.8	50.3	352.7	49.3
1 to 9	407.8	52.0	401.3	48.6
1 to 10	454.0	46.2	452.9	51.6
1 to 11	504.4	50.4	503.6	50.7

For the Imperial Commission on Weights and Measures.

[Signed] FOERSTER.

The comparisons made in 1875 between the U. S. and German pendulum-meters give:

$$\text{U. S. meter} - \text{German meter} = + 131^{\mu}.9,$$

and those made in 1877 give:

$$\text{U. S. meter} - \text{German meter} = + 131^{\mu}.3;$$

The mean of these values, or + $131^{\mu}.6$, is adopted. The comparisons in detail are given in the following tables. The unit is one-thousandth of the revolution of the micrometer-screws.

UNITED STATES COAST SURVEY.—COMPARISONS OF PRUSSIAN AND U. S. PENDULUM STANDARDS, 1875.

Date.	Time.	Which first.	Prussian. Below.	Prussian. Above.	U. S. Below.	U. S. Above.	Prussian *minus* comparator.	U. S. *minus* comparator.	U. S. *minus* Prussian.	Mean mercurial ther.	Prussian met. ther.	U. S. met. ther.	U. S. − Pr. ther.	Select values for mean.
BOTH THERMOMETER SCALES DOWN.														
										°				
June 17	0.10 p. m.	{ Pr. ?	1362	1263	1125	1309	− 99	+184	+ 283	20. 1	2762	503	......	......
17	0.10 p. m.	{ U. S. ?	2041	1025	2058	2303	−1016	+245	+1261	19. 8	2900	556	......	......
17	6.24 p. m.	....	2041	1043	1037	2332	− 998	+295	+1293	19. 6	2932	629	......	......
18	10.20 a. m.	....	2119	894	2220	2250	−1225	+ 30	+1255	19. 7	2926	427	......	......
18	10.50 a. m.	....	2145	939	2195	2295	−1206	+100	+1306	19. 4	2906	427	......	......
18	4.15 p. m.	....	2124	941	2193	2268	−1183	+ 75	+1258	19. 4	2981	368	......	......
19	11.00 a. m.	....	2123	920	2212	2303	−1203	+ 91	+1294	19. 3	2973	358	......	......
19	1.30 p. m.	....	2556	1224	2408	2541	−1332	+133	+1465	....	2794	687	......	......
21	10.15 a. m.	U. S.	2552	1107	2457	2367	−1445	− 90	+1355	18. 8	+ 81	−287	−368	......
21	11.10 a. m.	Pr. ?	2545	1086	2454	2337	−1459	−117	+1342	19. 2	+ 16	−371	−387	......
21	2.50 p. m.	....	2551	1118	2435	2358	−1433	− 77	+1356	19. 0	+ 65	−302	−367	......
22	10.00 a. m.	{ U. S.	2543	1116	2452	2359	−1427	− 93	+1334	18. 8	+ 45	−318	−363	}
22	11.00 a. m.	{ Pr.	2546	1117	2453	2351	−1429	−102	+1327	19. 7	− 12	−382	−370	} −368
22	11.45 a. m.	U. S.	2552	1089	2456	2334	−1417	−122	+1341	20. 0	− 40	−407	−367	}
22	3.10 p. m.	U. S.	2531	1086	2449	2353	−1445	− 96	+1349	19. 1	− 5	−377	−372	}
July 2	9.45 a. m.	U. S.	2548	1077	2759	2642	−1471	−117	+1354	20. 2	− 322	−684	−362	} −354
2	10.00 a. m.	U. S.	2549	1057	2755	2616	−1492	−139	+1353	21. 5	− 349	−695	−346	}
														−361
BOTH THERMOMETER SCALES UP.														
June 25	10.00 a. m.	Pr.	2877	1458	2871	2755	−1419	−116	+1303	19. 6	− 31	+343	−374	}
25	10.35 a. m.	U. S.	2869	1415	2872	2754	−1454	−118	+1336	19. 7	+ 2	+362	−360	} −371
25	4.05 p. m.	Pr.	2838	1401	2933	2789	−1437	−144	+1293	19. 4	− 26	+342	−368	}
26	9.55 a. m.	{ Pr.	2833	1427	2951	2813	−1406	−138	+1268	19. 3	− 91	+286	−377	......
26	10.30 a. m.	{ U. S.	2831	1350	2939	2762	−1481	−177	+1304	19. 5	− 36	+322	−358	......
June 29	10.00 a. m.	{ U. S.	2838	1391	1829	1707	−1447	−122	+1325	19. 9	+ 87	+452	−365	......
29	10.20 a. m.	{ Pr.	2841	1387	1829	1694	−1454	−135	+1319	20. 1	+ 115	+495	−380	......
29	10.35 a. m.	{ U. S.	2838	1378	1824	1696	−1460	−128	+1332	20. 2	+ 125	+497	−372	} −373
29	10.55 a. m.	{ Pr.	2835	1377	1820	1689	−1458	−131	+1327	20. 2	+ 138	+512	−374	}
29	4.15 p. m.	U. S.	2842	1422	1823	1727	−1420	− 96	+1324	19. 7	+ 51	+435	−384	......
														−372
PRUSSIAN THERMOMETER SCALE UP, U. S. DOWN.														
June 23	10.30 a. m.	....	2887	1182	2788	2445	−1705	−343	+1362	20. 1	+ 111	−513	−402	} −402
23	2.45 p. m.	....	2878	1212	2799	2478	−1666	−321	+1345	19. 8	+ 70	−472	−402	}
24	10.20 a. m.	{ U. S.	2879	1204	2789	2479	−1675	−310	+1365	19. 7	+ 51	−429	−378	......
24	10.50 a. m.	{ Pr.	2879	1205	2786	2445	−1674	−341	+1333	19. 8	+ 74	−480	−406	......
24	4.20 p. m.	U. S.	2881	1211	2775	2466	−1670	−309	+1361	19. 7	+ 86	−473	−387	......
June 26	11.45 a. m.	U. S.	2841	1384	2830	2763	−1457	− 67	+1390	19. 6	+ 10	−466	−456	......
28	10.25 a. m.	{ U. S.	2833	1359	2824	2697	−1474	−127	+1347	19. 4	− 29	−360	−389	}
28	10.45 a. m.	{ Pr.	2835	1349	2824	2688	−1486	−136	+1350	19. 5	− 12	−389	−401	} −398
28	4.30 p. m.	....	2828	1392	2827	2719	−1436	−108	+1328	19. 1	− 54	−349	−403	}
														−400
PRUSSIAN THERMOMETER SCALE DOWN, U. S. UP.														
June 30	10.00 a. m.	{ Pr.	2608	1180	1829	1689	−1428	−140	+1288	20. 6	− 181	+498	−317	}
30	11.30 a. m.	{ U. S.	2603	1188	1805	1694	−1415	−111	+1304	21. 3	− 330	+638	−308	}
30	0.20 p. m.	{ Pr.	2601	1179	1810	1679	−1422	−131	+1291	21. 2	− 315	+632	−317	} −310
30	0.35 p. m.	{ U. S.	2599	1172	1811	1688	−1427	−123	+1304	21. 2	− 324	+629	−305	}
30	3.25 p. m.	{ Pr.	2587	1197	1817	1708	−1390	−109	+1281	20. 5	− 215	+526	−311	}
30	3.40 p. m.	{ U. S.	2615	1175	1824	1686	−1440	−138	+1302	20. 5	− 227	+532	−305	......
July 1	9.40 a. m.	Pr.	2608	1193	1821	1678	−1415	−143	+1272	20. 7	− 239	+554	−315	......
1	11.55 a. m.	{ U. S.	2539	1098	1736	1589	−1441	−147	+1294	21. 1	− 327	+618	−291	}
1	0.10 p. m.	{ Pr.	2542	1084	1744	1572	−1458	−172	+1286	21. 2	− 333	+614	−281	} −290
1	3.15 p. m.	{ U. S.	2547	1099	1741	1595	−1448	−146	+1302	21. 0	− 293	+585	−292	}
1	3.30 p. m.	{ Pr.	2546	1086	1748	1583	−1460	−165	+1295	21. 0	− 306	+594	−288	}
														−300

UNITED STATES COAST SURVEY.—COMPARISONS OF PRUSSIAN AND U. S. PENDULUM STANDARDS, 1877.

BOTH THERMOMETER SCALES DOWN.

Date.	Time.	Which first.	Prussian.		U. S.		Prussian *minus* comparator.	U. S. *minus* comparator.	U. S. *minus* Prussian.	Mean mercurial ther.	Prussian met. ther.	U. S. met. ther.	U. S. – Pr. ther.
			Below.	Above.	Below.	Above.							
										°			
Oct. 16	....	U. S.	2386	1603	1936	2478	− 783	+ 542	+1325	16.2	−547	+291	+838
16	10.12 a. m.	Pr.	2379	1600	1938	2470	− 779	+ 532	+1311	16.4	−571	+239	+810
16	10.35 a. m.	U. S.	2372	1591	1937	2466	− 781	+ 529	+1310	16.7	−632	+216	+848
16	10.51 a. m.	Pr.	2363	1584	1931	2470	− 779	+ 539	+1318	16.9	−645	+177	+822
16	11.23 a. m.	U. S.	2353	479	1886	1367	−1874	− 519	+1355	17.2	−766	+ 30	+796
16	11.32 a. m.	Pr.	2374	486	1952	1386	−1888	− 566	+1322	17.3	−769	+ 43	+812
16	11.58 a. m.	U. S.	2371	505	1951	1398	−1866	− 553	+1313	17.2	−805	+ 26	+831
16	1.09 p. m.	U. S.	2482	1694	1879	2356	− 788	+ 477	+1265	17.2	−805	+ 31	+836
16	1.16 p. m.	Pr.	2487	1686	1873	2364	− 801	+ 491	+1292	17.1	−806	+ 44	+850
17	11.10 a. m.	Pr.	2434	1623	1887	2397	− 811	+ 510	+1321	16.3	−596	+212	+808
17	11.20 a. m.	U. S.	2434	1608	1895	2379	− 826	+ 484	+1310	16.3	−614	+199	+813
17	11.42 a. m.	Pr.	2421	1608	1888	2384	− 813	+ 496	+1309	16.4	−671	+151	+822
17	11.53 a. m.	U. S.	2408	1610	1879	2395	− 798	+ 516	+1314	16.3	−686	+137	+823

Results of comparisons between the pendulum-meter and No. 49, also between the pendulum itself and No. 49, have already been given, under the head of *Coefficient of Expansion*.

The result before given of comparison between the pendulum and No. 49 was obtained when the screws holding the knife-edge in place were tightly screwed up. With these screws loosened, the difference was at 10° C.

$$\text{Pendulum No. 49} = -\ 190^{\mu}.6,$$

the result before obtained being $-199^{\mu}.5$ for 10° C. Neither result is of any value in determining the length of the pendulum.

The measures of length of the pendulum, as compared with the pendulum-meter, corrected for the value of micrometer-screw revolutions, reduced to heavy end down for the pendulum, and to metallic thermometer down and mean of lines 99.99, 100.00, and 100.01 of the meter, give for Pendulum *minus* U. S. pendulum-meter—

	μ
At Geneva	−198.7
At Paris	−181.2
At Berlin	−177.1
At Kew	−174.0
At Hoboken	−165.1

The injury the pendulum received between the Geneva observations and those at Paris, and the work necessary to remedy this injury, account for the great discrepancy between the length at Geneva and that found elsewhere. Taking the remaining measures, however, their discordance will be seen to be far greater than can be accounted for by errors of observation, while the progressive character of this disagreement renders it unlikely that it is due to the greater tightness of the screws holding the knife-edges at one place than at another. It is no greater than may be probably ascribed to accumulations of oxide, &c., under the bearing surfaces of the knives. The comparisons in detail are given in the following tables:

UNITED STATES COAST SURVEY.—PENDULUM AT GENEVA.—LENGTH.

[Heavy end up is corrected by +1μ.0 to bring it to heavy end down.]

Date. Heavy end (d.) down, or (u.) up.	ABOVE.		P − St.	BELOW.		P − St.	Pend. longer than stand. uncorrected.	CORRECTION.		Corrected length.
	Stand.	Pend.		Stand.	Pend.			Above.	Below.	
										μ
Sept. 5 d.	2266	1661	−605	2101	3473	+1372	−1977	−2	+7	−198.6
d.	2219	1667	−542	2120	3599	+1479	−2021	−1	+7	−202.9
u.	2195	1675	−520	2117	3591½	+1474½	−1994½	−1	+7½	−199.3
u.	2185	1687	−498	2130	3628	+1498	−1996	−1	+7	−199.4
Sept. 6 u.	2250	1694	−556	2122	3595	+1473	−2029	−1	+7	−202.7
u.	2190	1690½	−499½	2131	3621	+1490	−1989½	−1	+7½	−198.8
d.	2195	1657	−538	2121	3552	+1431	−1969	−1	+7	−197.7
d.	2184	1687½	−496½	2126	3602½	+1476½	−1973	−1	+7	−198.1
Sept. 7 u.	2253	1677	−576	2134	3595½	+1461½	−2038	−1	+7	−203.6
8 u.	2239	1657½	−581½	2152	3588½	+1436½	−2018	−1	+7	−201.6
Sept. 9 d.	2257	1730	−527	2146	3581	+1435	−1962	−1	+7	−197.0
d.	2223	1723	−500	2170	3673	+1503	−2003	−1	+8	−201.2
u.	2210	1691	−519	2298	3748	+1450	−1969	−1	+7	−196.7
u.	2199	1707	−492	2317	3816½	+1499½	−1992	−1	+7	−199.0
Sept. 13 u.	2084	1234½	−849½	1708	2865½	+1157½	−2007	−2	+6	−200.5
u.	2036	1224	−812	1703	2877	+1174	−1986	−2	+6	−198.4
d.	2040	1258½	−781½	1725	2894½	+1169½	−1951	−2	+6	−195.9
d.	2072	1283	−789	1762	2915	+1153	−1942	−2	+6	−195.0
Sept. 14 u.	2149	1248	−901	1752	2812	+1060	−1961	−2	+5	−195.8
u.	2113	1263	−850	1733	2900	+1167	−2017	−2	+6	−201.5
d.	2101	1253	−848	1732	2812	+1080	−1928	−2	+5	−193.5
d.	2089	1257	−832	1747	2891	+1144	−1976	−2	+6	−198.4
Sept. 15 d.	2137	1275	−862	1739	2830	+1091	−1953	−2	+5	−196.0
d.	2119	1278	−841	1742	2887	+1145	−1986	−2	+6	−199.4
u.	2114	1276	−838	1738	2830	+1092	−1930	−2	+5	−192.7
u.	2113	1265	−848	1750	2898	+1148	−1996	−2	+6	−199.4
Sept. 16 d.	2140	1273½	−866½	1733	2853½	+1120½	−1987	−2	+6	−199.5
d.	2121	1265½	−855½	1742	2882½	+1140½	−1996	−2	+6	−200.4
u.	2122	1260½	−861½	1742	2853½	+1111½	−1973	−2	+6	−197.1
u.	2094	1263½	−830½	1737	2913½	+1176½	−2007	−2	+6	−200.5
Mean										−198.7

UNITED STATES COAST SURVEY.—PENDULUM AT PARIS.—LENGTH.

[Heavy end up is corrected by +1μ.0 to bring it to heavy end down.]

Date. Heavy end (d.) down, and (u.) up.	ABOVE.		P − St.	BELOW.		P − St.	Pend. longer than stand. uncorrected.	CORRECTION.		Corrected length.
	Stand.	Pend.		Stand.	Pend.			Above.	Below.	
1876.										μ
Jan. 25 u.	2373	1467	−906	1541	2444	+ 903	−1809	−3	+4	−180.6
u.	2378	1479	−899	1552	2413	+ 861	−1760	−3	+3	−175.6
d.	2356	1426	−930	1550	2365	+ 815	−1745	−3	+3	−175.1
d.	2367	1461	−906	1536	2400	+ 864	−1760	−3	+3	−176.6
Jan. 26 u.	2417	1451	−966	1558	2392	+ 834	−1800	−3	+3	−179.6
u.	2370	1445	−925	1547	2402	+ 855	−1780	−3	+3	−177.6
d.	2376	1437	−939	1554	2393	+ 839	−1778	−3	+3	−178.4
d.	2372	1459	−913	1547	2414	+ 867	−1780	−3	+3	−178.6
Jan. 28 d.	2400	1473	−927	1564	2406	+ 842	−1769	−3	+3	−177.5
d.	2363	1456	−907	1546	2391	+ 845	−1752	−3	+3	−175.8
u.	2374	1428	−946	1547	2387	+ 840	−1786	−3	+3	−178.2
Jan. 29 u.	2382	1433	−949	1554	2390	+ 836	−1785	−3	+3	−178.1
u.	2362	1401	−961	1539	2405	+ 866	−1827	−3	+3	−182.3
Change of knife-edges.										
Feb. 2 u.	2093	1792	−301	1283	2815	+1532	−1833	−1	+6	−183.0
u.	2112	1832	−280	1278	2797	+1519	−1799	−1	+6	−179.6
d.	2132	1836	−296	1291	2779	+1488	−1784	−1	+6	−179.1
Feb. 3 d.	2148	1837	−311	1279	2787	+1508	−1819	−1	+6	−182.6
Feb. 4 d.	2144	1855	−289	1280	2812	+1532	−1821	−1	+6	−182.8
d.	2141	1854	−287	1274	2796	+1522	−1809	−1	+6	−181.6
u.	2140	1839	−301	1281	2812	+1531	−1832	−1	+6	−182.9
u.	2136	1847	−289	1280	2792	+1512	−1801	−1	+6	−179.8
Change of knife-edges.										
Feb. 9 d.	2143	1844	−299	1268	2817	+1549	−1848	−1	+6	−185.5
d.	2153	1832	−321	1275	2801	+1526	−1847	−1	+6	−185.4
u.	2139	1844	−295	1272	2813	+1541	−1836	−1	+6	−183.3
u.	2138	1859	−279	1273	2826	+1553	−1832	−1	+6	−182.9
Feb. 14 u.	2130	1883	−247	1272	2883	+1611	−1858	−1	+6	−185.5
u.	2135	1868	−257	1274	2826	+1552	−1809	−1	+6	−180.6
d.	2336	1570	−766	1523	2523	+1000	−1766	−2	+4	−177.2
d.	2377	1592	−785	1537	2536	+ 999	−1784	−2	+4	−179.0
Change of knife-edges.										
Feb. 21 d.	2405	1604	−801	1542	2550	+1008	−1809	−2	+4	−181.5
d.	2399	1603	−796	1531	2558	+1027	−1823	−2	+4	−182.9
u.	2388	1586	−802	1541	2544	+1003	−1805	−2	+4	−180.1
u.	2403	1606	−797	1542	2559	+1017	−1814	−2	+4	−181.0
Feb. 22 u.	2397	1593	−804	1545	2605	+1060	−1864	−2	+4	−186.0
u.	2383	1587	−796	1545	2547	+1002	−1798	−2	+4	−179.4
d.	2361	1591	−770	1544	2517	+ 973	−1743	−2	+4	−174.9
d.	2394	1597	−797	1544	2527	+ 983	−1780	−2	+4	−178.6
Mean										−180.3
Corrected for compression of standard										−181.2

UNITED STATES COAST SURVEY.—PENDULUM AT BERLIN.—LENGTH.

[Heavy end up corrected by $+1^{\mu}.0$ to bring it to heavy end down.]

Date.	ABOVE.		P − St.	BELOW.		P − St.	Pend. longer than stand. uncorrected.	CORRECTION.		Corrected length.
	Stand.	Pend.		Stand.	Pend.			Above.	Below.	
1876.										μ
Apr. 19 d.	2293	1862	−431	1175	2487	+1312	−1743	0	+5	−174.8
Apr. 20 d.	2247	1869	−378	1179	2490	+1311	−1689	0	+5	−169.4
d.	2258	1869	−389	1174	2473	+1299	−1688	0	+5	−169.3
u.	2245	1931	−314	1180	2526	+1346	−1660	0	+5	−165.5
u.	2275	1897	−378	1180	2522	+1342	−1720	0	+5	−171.5
Apr. 24 d.	2259	1879	−380	1165	2491	+1326	−1706	0	+5	−171.1
d.	2248	1879	−369	1167	2496	+1329	−1698	0	+5	170.3
u.	2239	1878	−361	1159	2518	+1359	−1720	0	+5	−171.5
Apr. 25 d.	2257	1877	−380	1161	2595	+1434	−1814	rej.		
d.	2257	1895	−362	1159	2558	+1399	−1761	0	+6	−176.7
u.	2224	1853	−371	1157	2540	+1383	−1754	0	+6	−175.0
u.	2245	1878	−367	1162	2543	+1381	−1748	0	+6	−174.4
Apr. 26 d.	2257	1881	−376	1161	2552	+1391	−1767	0	+6	−177.3
d.	2257	1881	−376	1163	2538	+1375	−1751	0	+5	−175.6
u.	2236	1872	−364	1164	2550	+1386	−1749	0	+6	−174.5
u.	2244	1884	−360	1165	2560	+1395	−1755	0	+6	−175.1
Apr. 28 u.	2266	1882	−384	1163	2562	+1399	−1783	0	+6	−177.9
u.	2270	1872	−398	1160	2534	+1374	−1772	0	+5	−176.7
d.	2276	1890	−386	1162	2567	+1405	−1791	0	+6	−179.7
d.	2270	1895	−375	1167	2568	+1401	−1776	0	+6	−178.2
Apr. 29 u.	2242	1870	−372	1157	2593	+1436	−1808	0	+6	−180.4
u.	2226	1872	−354	1156	2577	+1421	−1775	0	+6	−177.1
d.	2227	1882	−345	1156	2545	+1389	−1734	0	+6	−174.0
d.	2247	1889	−358	1153	2558	+1405	−1763	0	+6	−176.9
Apr. 30 u.	2248	1865	−383	1157	2608	+1451	−1834	0	+6	−183.0
u.	2257	1865	−392	1156	2592	+1436	−1828	0	+6	−182.4
d.	2233	1832	−401	1146	2533	+1387	−1788	0	+6	−179.4
d.	2258	1869	−389	1159	2551	+1392	−1781	0	+6	−178.7
May 2 u.	2272	1915	−357	1151	2646	+1495	−1852	0	+6	−184.8
u.	2269	1914	−355	1150	2593	+1443	−1788	0	+6	−178.4
May 3 d.	2262	1872	−390	1152	2603	+1451	−1841	0	+6	−184.7
Mean										−176.1
Corrected for compression of stand										−177.1

UNITED STATES COAST SURVEY.—PENDULUM AT KEW.

[Four observations in parentheses rejected on account of temperature.
Heavy end up corrected by $+1^{\mu}.0$ to reduce to heavy end down.
All observations on mean of 9997th, 9998th, and 9999th lines corrected by $-200^{\mu}.4$ to reduce to mean of 9999th, 10000th, and 10001st.]

Date.	Above. Stand.	Above. Pend.	P − St.	Below. Stand.	Below. Pend.	P − St.	Pend. longer than stand uncorrected.	Correction. Above.	Correction. Below.	Corrected length.
1876.										μ.
June 29 d.	(1936	2299	+363	1003	3124	+2121	−1758	−1	+7	−176.6)
30 d.	1950	....	....	....	....	....	....	....	....	....
d.	1924	2204	+280	1012	3016	+2004	−1724	−1	+7	−173.3
d.	1597	1609	+ 12	2684	2426	− 258	+ 270	0	−1	−173.3
d.	1605	1612	+ 7	2672	2405	− 267	+ 274	0	−1	−172.9
July 6 u.	1868	2697	+829	2733	3299	+ 566	+ 263	−2	+2	−173.1
u.	1781	1635	−145	2666	2246	− 420	+ 274	0	−1	−171.9
July 6 u.	1510	1652	+142	2370	2239	− 131	+ 273	0	0	−172.1
u.	1481	1631	+150	2360	2245	− 115	+ 265	0	0	−172.9
July 10...	(1580	1669	+ 89	2448	2337	− 111	+ 200	0	0	−180.4)
10...	(1536	1602	+ 66	2442	2294	− 148	+ 214	0	− 1	−178.9)
11...	(1515	1616	+101	2434	2307	− 127	+ 228	0	0	−177.6)
....	1476	1601	+125	2434	2291	− 143	+ 268	0	− 1	−173.5
....	1492	1624	+132	2435	2300	− 135	+ 267	0	0	−173.7
Mean										−173.0
Corrected for compression of stand										−174.0

UNITED STATES COAST SURVEY.—PENDULUM AT HOBOKEN.—LENGTH.

Date.	Obs.	ABOVE.		P — St.	BELOW.		P — St.	Pend. longer than stand. uncorrected.	Cor. below.	Corrected length.
		Stand.	Pend.		Stand.	Pend.				
1876.										μ
April 11	P.	2021	2517	+496	2116	2207	+ 91	+405	0	+40.5
11	P.	1985	2152	+167	2072	1884	−187	+354	0	+35.4
11	P.	1997	2142	+145	2062	1888	−174	+319	0	+31.9
12	S.	1975	2173	+198	2075	1831	−244	+442	+1	+44.1
12	S.	2011	2143	+132	2061	1834	−227	+359	+1	+35.8
12	S.	2014	2133	+119	2057	1811	−246	+365	+1	+36.4
13	S.	1980	2104	+124	2062	1818	−224	+368	+1	+36.7
13	S.	2004	2170	+166	2067	1838	−229	+395	+1	+39.4
13	S.	1995	2159	+164	2074	1839	−235	+399	+1	+39.8
14	S.	2004	2129	+125	2052	1815	−237	+362	+1	+36.1
14	S.	1993	2156	+163	2053	1810	−243	+406	+1	+40.5
14	S.	1995	2150	+155	2060	1813	−247	+402	+1	+40.1
16	S.	2028	2218	+190	2077	1882	−195	+385	0	+38.5
16	S.	2018	2217	+199	2071	1873	−198	+397	0	+39.7
16	S.	2020	2207	+187	2070	1877	−193	+380	0	+38.0
16	S.	2021	2213	+192	2067	1866	−201	+393	+1	+39.2
16	S.	2033	2222	+189	2081	1899	−182	+371	0	+37.1
17	S.	2029	2217	+188	2084	1892	−192	+380	0	+38.0
17	S.	2029	2207	+178	2073	1898	−175	+353	0	+35.3
23	S.	2035	2013	− 22	2034	1608	−426	+404	+1	+40.3
23	S.	2042	2017	− 25	2040	1613	−427	+402	+1	+40.1
23	P.	2695	2634	− 61	2750	2317	−433	+372	+1	+37.1
23	P.	2689	2646	− 43	2755	2326	−429	+386	+1	+38.5
23	P.	2023	1970	− 53	2037	1614	−423	+370	+1	+36.9
24	P.	2021	1965	− 56	2010	1604	−406	+350	+1	+34.9
24	P.	1959	1850	−109	1969	1506	−463	+354	+1	+35.3
24	P.	1956	1857	− 99	1115	1540	−435	+336	+1	+33.5
24	P.	1961	1855	−106	1981	1540	−441	+335	+1	+33.4
24	P.	1785	2059	+274	2121	2053	− 68	+342	0	+34.2
24	P.	1829	2091	+262	2133	2071	− 62	+324	0	+32.4
24	P.	1865	2140	+275	2135	2069	− 66	+341	0	+34.1
24	P.	2524	2814	+290	2855	2785	− 70	+369	0	+36.9
24	P.	2523	2811	+288	2829	2802	− 27	+315	0	+31.5
24	P.	1861	2134	+273	2143	2086	− 57	+330	0	+33.0
May 25	P.	2081	2431	+350	1649	1700	+ 51	+299	0	+29.9
25	P.	2083	2452	+369	1651	1709	+ 57	+311	0	+31.1
25	P.	2049	2444	+395	1636	1702	+ 66	+329	0	+32.9
25	P.	2060	2445	+385	1648	1698	+ 50	+335	0	+33.5
25	P.	2065	2457	+392	1639	1700	+ 61	+331	0	+33.1
25	P.	2058	2451	+393	1648	1690	+ 42	+351	0	+35.1
26	S.	1937	2302	+365	1479	1537	+ 58	+307	0	+30.7
26	S.	1907	2288	+381	1483	1541	+ 58	+323	0	+32.3
26	P.	1911	2289	+378	1551	1492	+ 59	+319	0	+31.9
26	P.	1884	2267	+383	1539	1478	+ 61	+322	0	+32.2
27	P.	1883	2147	+259	1964	1908	− 56	+315	0	+31.5
27	P.	1877	2143	+266	1953	1902	− 51	+317	0	+31.7
28	P.	2351	2182	−169	2412	1953	−459	+290	+1	+28.9
June 16	P.	1252	1526	+274	1532	1488	− 44	+318	0	+31.8
16	P.	1968	2501	+533	2238	2467	+229	+304	−1	+30.5
16	S.	1955	2529	+574	2231	2454	+223	+351	−1	+35.2
16	S.	1956	2511	+554	2230	2451	+221	+333	−1	+33.4
Mean										+35.3
Correction reducing to mean of 9999th, 10000th, and 10001st lines of pendulum-meter										−200.4
Length of pendulum										−165.1

CONCLUDED LENGTH OF THE PENDULUM.

The indirect comparisons of the U. S. pendulum-meter with the German meter No. 49 give the following result, when reduced to 15° C.:

	μ
U. S. meter — German pendulum-meter	+131.6
German pendulum-meter — No. 1605	—103.1
No. 1605 — No. 49	— 40.7
U. S. meter — No. 49	— 12.2
Direct comparison, U. S. meter — No. 49	— 20.0

The mean of these values, or $-16^{\mu}.1$, is taken, the likelihood of error of the two methods being estimated as equal. The bad temperature conditions at New York have prevented a more accurate determination of this quantity; but a new determination will be undertaken at the first opportunity.

But we have, for 15° C.:

$$\text{No. 49} - \text{M} = +\overset{\mu}{277.2}, \text{ hence}$$
$$\text{U. S. pendulum-meter} - \text{M} = +261.1$$

The length of the pendulum at the different stations is, therefore, as follows:

	cm.
Geneva	100.00624
Paris	100.00799
Berlin	100.00840
Kew	100.00871
Hoboken	100.00960

CENTER OF MASS.

The quantity $h_d - h_u$, or twice the distance of the center of mass of the pendulum from the center of figure, was observed at the beginning and end of each series of experiments, and also before and after each transposition of knife-edges. The apparatus, method of observing, etc., have been elsewhere described.* Comparisons at the U. S. Office of Weights and Measures show that the 39 centimeters on the staff of the balance, from 17 to 56, are 0.14 mm too long. This correction applied, we obtain for $h_d - h_u$,

	cm.
At Geneva	39.284
At other stations	39.292

These are the values used. The separate observations are shown in the following table:

* The idea of determining the center of mass of a reversible pendulum, instead of moving a weight upon it, belongs exclusively to Bessel.

UNITED STATES COAST SURVEY.—CENTER OF MASS OF REVERSIBLE PENDULUM.

GENEVA.

Date.	Knife at heavy end and direc'n of marked ends.	Firma.	HEAVY END. Reading in middle.	HEAVY END. Reading at end.	LIGHT END. Reading in middle.	LIGHT END. Reading at end.			$h_d - h_u$	Concluded. $h_d - h_u$
1875. Sept. 2	1 forward	Up	17068	01032	56037	00706	16036	55331	39295	39296
		Down...	17069	01028½	56038	00701	16040½	55337	39297	
Sept. 2	2 forward	Up	16086	00049½	56039	00706½	16036½	55332½	39296	39298
		Down...	16086	00053½	56037	00704½	16032½	55332½	39300	
Sept. 3	2 forward	Up	16087	00046	56036	00701½	16031	55334½	39303½	39299
		Down...	160‹8	00045½	56037	00700½	16042½	55336½	39294	
Sept. 9	1 forward	Up	16040	00000½	56035	00695	16039½	55340	39300½	39301
		Down...	16040	00001½	56034	00694½	16038½	55339½	39301	
Sept. 17	2 forward	Up	16046	00001	56028	00689	16045	55339	39294	39295
		Down...	16045	00002½	56027	00688½	16042½	55338½	39296	
Mean at Geneva										39298

Between Geneva and Paris observations, the pendulum was bent and straightened, and the knife-edges were made parallel and ground.

PARIS.

Date.	Knife at heavy end and direc'n of marked ends.	Firma.	HEAVY END. Reading in middle.	HEAVY END. Reading at end.	LIGHT END. Reading in middle.	LIGHT END. Reading at end.			$h_d - h_u$	Concluded. $h_d - h_u$
1876. Jan. 22	1 forward	Up	17024	00977½	56051	00692½	16046½	55358½	39312	39310
		Down...	....	00973½	....	00693	16050½	55358	39307½	
Jan. 29	1 forward	Up	17023	00975	56011	00656	16048	55355	39307	39306
		Down...	17023	00971	....	00654½	16052	55356½	39304½	
Jan. 29	2 forward	Up	17017	00959½	56012	00650	16057½	55362	39304½	39305
		Down...	....	00961	....	00651	16056	55361	39305	
Feb. 8	2 forward	Up	17018	00960	56054	00694	16058	55360	39302	39302
		Down...	17017	00957	......	00692½	16060	55361½	39301½	
Feb. 8	1 back	Up	17014	00960	56052	00696½	16054	55355½	39301½	39302
		Down...	....	00960	....	00696	16054	55356	39302	
Feb. 14	1 back	Up	17016	00961½	56050	00683½	16054½	55366½	39312	39310
		Down...	....	00960	....	00686½	16056	55363½	39307½	
Feb. 14	2 back	Up	17003	00947½	56050	00689	16055½	55361	39305½	39306½
		Down...	....	00946	....	00685½	16057	55364½	39307½	
Feb. 24	2 back	Up	17003	00943	56055	00694	16060	55361	39301	39303
		Down...	....	00947	....	00694½	16056	55360½	39304½	
Mean at Paris										39306

CENTER OF MASS OF REVERSIBLE PENDULUM—Continued.

BERLIN

Date.	Knife at heavy end and direct'n of marked ends.	Firma.	HEAVY END. Reading in middle.	HEAVY END. Reading at end.	LIGHT END. Reading in middle.	LIGHT END. Reading at end.			$h_d - h_u$	Concluded. $h_d - h_u$
1876. Apr. 21	....	Up	17019	00962	56054	00679½	16057	55374½	39317½	39316
		Down	....	00960	....	00681	16059	55373	39314	
Apr. 24	1 forward .	Up	17019	00960	56028	00658½	16059	55369½	39310½	39310
		Down	....	00959	....	00659	16060	55369	39309	
Apr. 27	2 forward .	Up	17009	00947	56030	00657½	16062	55372½	39310½	39312
		Down	....	00945½	....	00653	16063½	55377	39313½	
Apr. 27	2 back	Up	17009	00947½	56040	00668	16061½	55372	39310½	39310
		Down	....	00946½	....	00668½	16062½	55371½	39309	
May 3	....	Up	16996	00931	56038	00669	16065	55369	39304	39300½
		Down	....	00924½	....	00669½	16071½	55368½	39297	
June 3	1 back	Up	16996	00929½	56040	00671	16066½	55369	39302½	39303
		Down	....	00931½	....	00671½	16064½	55368½	39304	
June 3	2 back	Up	17014	00951	56040	00672	16063	55368	39305	39306
		Down	....	00951	....	00670	16063	55370	39307	
Mean at Berlin										39308

KEW.

Date.	Knife at heavy end and direct'n of marked ends.	Firma.	HEAVY END. Reading in middle.	HEAVY END. Reading at end.	LIGHT END. Reading in middle.	LIGHT END. Reading at end.			$h_d - h_u$	Concluded. $h_d - h_u$
1876. June 30	2..........	Up	17014	00938½	56044	00670	16075½	55374	39298½	39293
		Down	....	00940½	....	00683	16073½	55361	39287¼	
July 6	2..........	Up	17004	00944¼	56045	00670½	16059½	55374½	39315	39316
		Down	....	00946	....	00669½	16058	55375½	39317½	
July 6	1..........	Up	17004	00944½	56057	00688¼	16059½	55368½	39309	39307½
		Down	....	00953	....	00700	16051	55357	39306	
July 11	....	Up	16995	00930	56056	00692½	16063	55363½	39300½	39299½
		Down	....	00928½	....	00691	16066½*	55365	39298½	
Mean at Kew										39304

HOBOKEN.

Date.	Knife at heavy end and direct'n of marked ends.	Firma.	HEAVY END. Reading in middle.	HEAVY END. Reading at end.	LIGHT END. Reading in middle.	LIGHT END. Reading at end.			$h_d - h_u$	Concluded. $h_d - h_u$
1877. Apr. 8	2 forward .	Up	16995	00933	56054	00687	16062	55367	39305	39300
		Down	....	00929	....	00693	16066	55361	39295	
June 17	2 forward .	Up	17054	00989	56050	00678	16065	55372	39307	39305
		Down	....	00985	....	00678	16069	55372	39303	
1878. May 6	1 forward .	Up	17046	·01003	56046	00686	16043	55360	39317	39313
		Down	....	00995	....	00685	16051	55361	39310	
Mean at Hoboken										39306

PERIODS OF OSCILLATION AND VALUES OF GRAVITY.

The pendulum was swung in Hoboken in various ways, to wit:

1. The regular set was made on the Geneva support with the bells off. This set cannot be compared with others on the Repsold support, if the reductions be made on the principle of the reversible pendulum, owing to the different ways in which the knives rest on the two supports. The comparison may, however, be made on the principle of the invariable pendulum, so as to eliminate this effect.

2. Sets of experiments were made at various pressures on the Geneva support with the bells on. The knife-edges not having been interchanged, these are strictly only comparable among themselves.

3. Half a regular set was made on the Geneva support with the bells on at about 35° C. There were a few additional experiments at this temperature with heavy end up at different pressures.

4. The pendulum was swung on the Repsold support and also on a very stiff support having the head of the Repsold support as a part of it (so as to have the same bearing on the knives). The object of these experiments was to determine the effect of flexure of the support.

A conspectus of all these experiments is given in the following table:

Periods of oscillation of the pendulum at Hoboken; reduced to one absolute atmosphere and to 15° C., and to values on rigid support without bells or cylinder.

Heavy end down.					Heavy end up.					
Pressure.	Temp. C.	T_d	No. days.	No. thousand oscillations.	Pressure.	Temp. C.	T_u	No. days.	No. thousand oscillations.	
ON GENEVA SUPPORT.										
With bells on.—Regular set to determine gravity.—Knives interchanged.										
in.	°	*s.*			*in.*	°	*s.*			
30	20	1.006344	8	42	30	20	1.006537	8	18	
With bells on.—At high temperatures.										
in.	°	*s.*			*in.*	°	*s.*			
30	35	1.006346	4	22	30	35	1.006544	4	19	Knives interchanged.
					30	34	1.006530	2	6	Knife No. 1.
					$2\frac{1}{4}$	38	1.006533	1	5	Knife No. 1.
					$1\frac{1}{4}$	34	1.006521	1	2	Knife No. 1.
With bells on.—At various pressures.—Knife No. 1 at heavy end.										
in.	°	*s.*			*in.*	°	*s.*			
30	18	1.006349	3	16	30	10	1.006539	3	13	
					29	11	1.006560	2	7	
					27	10	1.006537	1	4	
					$22\frac{1}{2}$	11	1.006549	1	2	
15	18	1.006337	2	20	15	10	1.006545	1	3	
					$7\frac{1}{2}$	10	1.006533	2	10	
5	20	1.006336	1	15						
$1\frac{1}{2}$	20	1.006342	1	21						
					1	10	1.006524	1	11	
					$\frac{3}{4}$	9	1.006530	4	53	
					$\frac{1}{10}$	9	1.006532	4	73	
ON REPSOLD SUPPORT.										
One day, knife No. 1 at heavy end; two days at light end.										
in.	°	*s.*			*in.*	°	*s.*			
30	14	1.006355	3	4	30	14	1.006516	3	4	
ON STIFFEST SUPPORT.										
Knife No. 1 at light end.										
in.	°	*s.*			*in.*	°	*s.*			
30	14	1.006366	1	2	30	15	1.006544	1	2	

The reductions in the above table have been made with the *à priori* constants of atmospheric effect, and with the coefficient of expansion, $18^{\mu}.38$ per degree Centigrade. A correction of $+ 73 \times 10$ for inequality of knives has been applied to the three last results with heavy end up at high temperatures.

The agreement of the several experiments of the regular set is shown by the following table of the observed periods (uncorrected for the effect of the cylinders and of flexure):

Hoboken.—Regular set.

	Obs. T_d	Obs. T_u
	s.	s.
	1.006352	1.006559
	352	560
	361	546
	360	558
	358	544
	350	539
	363	551
	356	534
Mean	1.006357	1.006548

The agreement of the several experiments of the half set at high temperatures is shown in the following table:

Hoboken.—Half set at high temperatures.

Obs. T_d	Obs. T_u
s.	s.
1.006533	1.006709
541	713
536	708
534	708
	706
	716
	708
	709

It will be seen that the mean results of the experiments of the regular set agree as well as could be expected with those made at high temperatures; which shows both that the coefficient of expansion is correct, and also that the correction for bell-glasses is happily not in error. Further to compare these two sets of experiments we may calculate the mean T^2 which is to be used when the reduction is made on the principle of the reversible pendulum, and also the mean T^2 which is to be used when the reduction is made on the principle of the invariable pendulum. Denoting the former by [T^2 Rev.] and the latter by [T^2 Inv.] we have algebraically

$$[T^2 \text{ Rev.}] = \frac{T_d^2\, h_d - T_u^2\, h_u}{h_d - h_u} \qquad [T^2 \text{ Inv.}] = \frac{T_d^2\, h_d + T_u^2\, h_u}{h_d + h_u}$$

The values will be

	[T^2 Inv.]	[T^2 Rev.]
	s^2	s^2
From regular set	1.012846	1.012410
From ½ set at high temperatures	1.012850	1.012399
Difference, in seconds, per day	$0^s.3$	$0^s.5$

We may next examine the experiments at various pressures. Their concordance with one another is very good; but the reader will hardly desire to see the single experiments set forth here; particularly as all the means are given in the tables at the end of this paper. We may exhibit [T^2 Inv.] and [T^2 Rev.] for pairs of experiments under nearly the same pressure but in reversed positions. But these results can have no value in the determination of gravity; nor can they be expected to accord with those just obtained, for, not to speak of the non-interchange of knife-edges, the observations in the two positions were taken at intervals of months, under conditions very different in many respects, and were never intended to be used for obtaining gravity, but only to show the variation of the period and decrement of arc with the pressure.

	[T^2 Inv.]	[T^2 Rev.]
From experiments at 30 inches down and 30 inches up.....	$1.012855(s)^2$	$1.012425(s)^2$
From experiments at 15 inches down and 15 inches up.....	1.012842	1.012376
From experiments at 5 inches down and 7½ inches up.....	1.012832	1.012389
From experiments at 1½ inches down and 1 inch up.......	1.012836	1.012424
From mean of experiments at ½ and ¼ inch down and experiments at $\frac{4}{10}$ inch up..................................	1.012841	1.012415

The agreement is sufficient to show that the coefficient of atmospheric pressure is well determined.

We pass now to the experiments on the Repsold support and on the stiffest support. These were not very carefully made, being only intended to show that the effect of flexure was really what calculation had predicted. Upon these supports the knife-edge rested on steel instead of glass, and consequently the reductions on the principle of the reversible pendulum are not comparable with results of experiments on the Geneva support until the slip shall have been measured on both stands. The reductions on the principle of the invariable pendulum are, however, comparable.

	[T^2 Inv.]	[T^2 Rev.]
From experiments on stiffest support	$1.012879(s)^2$	$1.012512(s)^2$
From experiments on Repsold support	1.012855	1.012514
From regular set	1.012846	
Δ Repsold and stiffest support, in seconds, per day.	1ˢ.0	0ˢ.1
Δ Repsold support and regular set	0ˢ.6	

We may now proceed to compare the results at the European stations. First, the results of the single experiments at each station will be compared, with only such corrections as vary from day to day. Next, the values of [T^2 Inv.] and [T^2 Rev.] will be given for each station after correcting them for the wear of the edges so as to reduce them to what they would have been for Paris, just after the knives had been ground. Lastly, we shall use the determinations by the principle of the reversible pendulum of the absolute length of the seconds' pendulum (still uncorrected for slip) at each station, in combination with the determinations of relative gravity on the principle of the invariable pendulum, in order to find four independent values of the length of the seconds' pendulum at each station. These, being corrected for elevation, will be comparable with the results of other experiments. The Hoboken experiments on the Geneva support cannot be used to determine [T^2 Rev.] until the slip has been ascertained; those made at Hoboken, on the Repsold support, must, therefore, be used in place of them for the present.

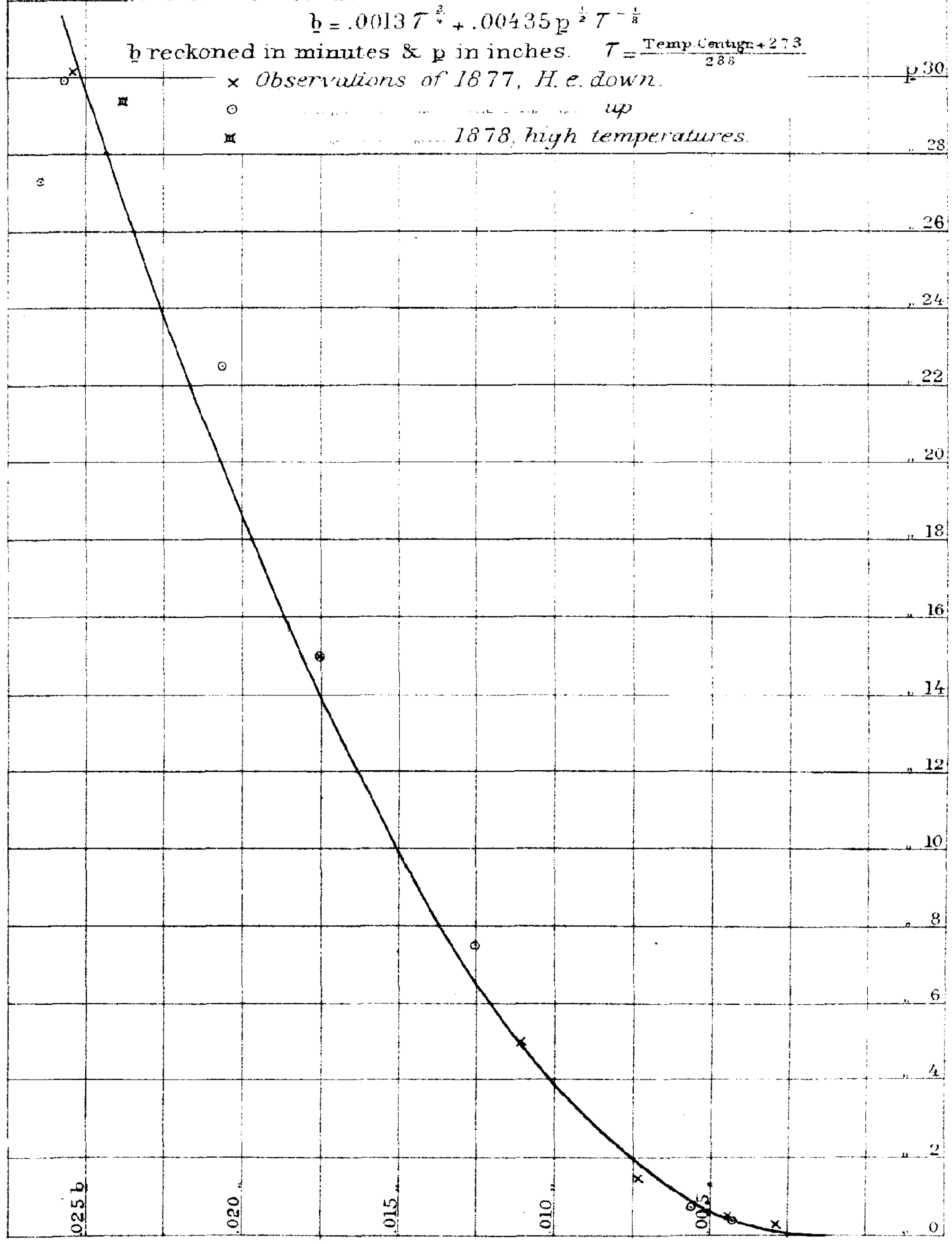
Paris.—Periods of oscillation.

	Heavy end down.	Heavy end up.
	s.	*s.*
	1.006051	1.006192
	048	210
	047	190
	048	195
	048	185
	052	185
	062	208
	053	213
Mean..........	1.006051	1.006197

The results on the last two days at Paris were affected by excessive damp.

U. S. Geodetic Survey Pendulum at Hoboken.

Curve showing Logarithmic part of the decrement, of the amplitude of oscillation at different pressures, compared with the formula.



Berlin.—Periods of oscillation.

	Heavy end down.	Heavy end up.
	s.	*s.*
	1.005899	1.006052
	901	037
	896	026
	890	036
	901	037
	896	034
	899	046
	895	034
Mean..........	1.005898	1.006038

Kew.—Periods of oscillation.

	Heavy end down.	Heavy end up.
	s.	*s.*
	1.005935	1.006077
	30	68
	29	73
	27	64
	25	70
	31	71
	28	64
	31	66
	29	70
	30	66
		59
		63
		72
		73
		66
		74
		77
		67
Mean..........	1.005930	1.006069

The following table shows the results of Paris, Berlin, and Kew in comparison:

	[T^2 Inv.]	[T^2 Rev.]	Diff.
Paris	1.0121042(s)^2	1.0116956(s)^2	4086
Berlin	1.0117925	1.0113934	3991
Kew	1.0118560	1.0114548	4012

These differences involve double the square roots of the sums of the squares of the errors of the periods of oscillation with heavy end down and with heavy end up. Therefore, the Berlin and Kew differences are remarkably close together, while that at Paris is rather divergent. The experiments at Hoboken show a wide discrepancy in this difference, owing to the use of the Geneva support. Consequently the [T^2 Rev.] cannot be used.

Now, reducing the values of [T^2 Rev.] to mean solar time and dividing into the length of our pendulum at Paris we obtain the length of the seconds' pendulum at the several stations. The ratios of the [T^2 Rev.] at the different stations, being inversely as the length of the seconds' pendulum, may be used to obtain the length of the seconds' pendulum at any station from the value deduced from the [T^2 Rev.] at any other station. So that at each station we shall have not only a value of the seconds' pendulum deduced from the [T^2 Rev.] of that station but also two other values deduced from the [T^2 Rev.] of the two other stations. These three values will have nearly equal weight. They are as follows:

For Paris.	*m.*
From Paris observations	.9939390
From Berlin observations	02
From Kew observations	20
Mean	.9939337
Reduction to sea-level	+ 163
Seconds' pendulum at Paris reduced to sea-level	.9939500

For Berlin.	*m.*
From Paris observations	.9942362
From Berlin observations	.9942452
From Kew observations	.9942382
Mean	.9942399
Reduction to sea-level	+ 83
Seconds' pendulum at Berlin reduced to sea-level	.9942482

For Kew.	*m.*
From Paris observations	.9941757
From Berlin observations	.9941830
From Kew observations	.9941740
Mean	.9941776
Reduction to sea-level	+ 14
Seconds' pendulum at Kew reduced to sea-level	.9941790

No comparison can be attempted between these results and those of previous experimenters until the former have been corrected for the slip of the knives and the latter have been reduced anew according to modern methods. These matters will be treated in the second part of this report with results which will be found satisfactory.

The pendulum at Geneva was virtually a different pendulum from that used at the other stations, because of the accident that befel it after the Geneva experiments. These experiments can, therefore, only be reduced on the principle of the reversible pendulum. The concordance of the single experiments is shown in the following table:

GENEVA.

T_d^2	T_u^2
1.012599	1.012814
581	775
589	797
607	793
593	789
580	767
582	763
598	783

The resulting value of the length of the seconds' pendulum after correcting for flexure in the manner explained under that heading is

Length of seconds' pendulum at Geneva.

	m.
Experiments of Coast Survey	0.993556
Professor Plantamour's result	0.993550

The appended tables show the details of the experiments at the different stations.

Respectfully submitted.

C. S. PEIRCE,
Assistant.

UNITED STATES GEODETIC SURVEY.—PENDULUM AT PARIS, 1876.

HEAVY END DOWN.

Date.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
	h. m. s.	*s.*								
Jan. 26	26 42.545									
	39 55.317	792.772	.045	792.727	788	1.0059987	+23	−79	+800	1.0060731
	3 41.846	1426.529	.042	1426.487	1418	59922	+23	−79	+796	662
	54 30.966	3049.120	.032	3049.088	3031	59644	+23	−79	+792	380
					(5237)					(1.0060509)
Jan. 28	36 04.848									
	49 17.572	792.724	.045	792.679	788	1.0059378	+23	−62	+788	1.0060127
	13 00.006	1422.514	.042	1422.472	1414	59915	+23	−62	+781	657
	3 57.273	3057.187	.031	3057.156	3039	59743	+23	−62	+776	480
					(5241)					(1.0060475)
				Change of knife-edges, from No. 2 to No. 1.						
Feb. 2	49 57.380									
	3 6.109	788.729	.047	788.682	784	1.0059720	+23	−59	+766	1.0060450
	26 56.663	1430.554	.042	1430.512	1422	59859	+23	−59	+769	592
	17 44.781	3048.118	.032	3048.086	3030	59690	+23	−59	+769	423
					(5236)					(1.0060473)
Feb. 4	21 35.132									
	35 26.122	830.990	.047	830.943	826	1.0059843	+ 8	−25	+808	1.0060634
	58 24.349	1378.227	.040	1378.187	1370	59759	+ 8	−25	+806	548
	43 50.526	2726.177	.027	2726.150	2710	59594	+ 8	−25	+815	392
					(4906)					(1.0060477)
				Change of knife-edges, from No. 1 to No. 2.						
Feb. 9	42 04.281									
	56 09.340	845.059	.049	845.010	840	1.0059642	+ 8	−15	+852	1.0060487
	19 09.561	1380.221	.042	1380.179	1372	59613	+ 8	−15	+845	451
	11 36.262	3146.701	.035	3146.666	3128	59674	+ 8	−15	+830	497
					(5340)					(1.0060484)
Feb. 14	22 51.895									
	36 55.946	844.051	.052	843.999	839	1.0059583	+ 8	−09	+765	1.0060347
	59 57.201	1381.255	.043	1381.212	1373	59811	+ 8	−09	+760	570
	55 45.135	3347.934	.036	3347.898	3328	59790	+ 8	−09	+756	545
					(5540)					(1.0060521)
				Change of knife-edges, from No. 2 to No. 1.						
Feb. 21	18 49.010									
	35 29.053	1000.043	.056	999.987	994	1.0060231	+ 8	−41	+475	1.0060673
	55 51.888	1222.835	.035	1222.800	1215½	60054	+ 8	−41	+475	496
	48 50.446	3178.558	.035	3178.523	3159½	60209	+ 8	−41	+469	645
					(5369)					(1.0060617)
Feb. 22	48 01.742									
	03 15.250	913.508	.054	913.454	908	1.0060066	+ 8	−36	+400	1.0060438
	28 4.196	1488.946	.043	1488.903	1480	60155	+ 8	−36	+402	529
	18 38.379	3034.183	.032	3034.151	3016	60182	+ 8	−36	+400	554
					(5404)					(1.0060528)

PENDULUM AT PARIS, 1876—Continued.

HEAVY END UP.

Date.	Time of transit.		Intervals.	Cor. for arc.	Corr. time.	Number oscilla-tions.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
	h. m.	*s.*	*s.*								
Jan. 26	33	30.148									
	38	20.301	350.153	.017	350.136	348	1.0061379	+23	−183	+692	1.0061911
	49	42.111	621.810	.018	621.792	618	61359	+23	−183	+690	1889
	11	52.245	1330.134	.015	1330.119	1322	61415	+23	−183	+685	1940
						(2288)					(1.0061922)
Jan. 29	27	10.797									
	32	56.946	346.149	.020	346.129	344	1.0061899	+23	−152	+640	2410
	43	22.790	625.844.	.019	625.825	622	61500	+23	−152	+639	2010
	06	45.096									
	28	57.261	1332.166	.014	1332.151	1324	1.0061563	+23	−152	+631	2065
						(2290)					(1.0062102)
Change of knife-edges, from No. 1 to No. 2.											
Feb. 2	42	06.507									
	47	52.620	346.113	.021	346.092	344	1.0060814	+23	−127	+616	1326
	58	20.449	627.829	.019	627.810	624	61058	+23	−127	+614	1568
	20	28.601	1327.152	.015	1327.137	1319	61692	+23	−127	+614	2202
						(2287)					(1.0061897)
Feb. 4	37	42.073									
	43	50.238	368.265	.021	368.244	366	1.0061311	+08	−050	+641	1910
	53	51.932	601.694	.019	601.675	598	61455	+08	−050	+648	2061
	14	5.338	1213.406	.014	1213.392	1206	61294	+08	−050	+649	1901
						(2170)					(1.0061946)
Change of knife-edges, from No. 2 to No. 1.											
Feb. 9	15	38.392									
	21	46.660	368.268	.023	368.245	366	1.0061339	+08	−031	+690	2006
	31	50.357	603.697	.018	603.679	600	61317	+08	−031	+681	1975
	54	54.778	1384.421	.015	1384.406	1376	61090	+08	−031	+682	1749
						(2342)					(1.0061847)
Feb. 14	57	20.800									
	3	27.026	366.226	.024	366.202	364	1.0060494	+08	−017	+685	1170
	13	40.787	613.761	.017	613.744	610	61377	+08	−017	+685	2053
	37	49.623	1448.836	.015	1448.821	1440	61257	+08	−017	+683	1931
						(2414)					(1.0061847)
Change of knife-edges, from No. 1 to No. 2.											
Feb. 21	9	17.620									
	16	01.129	403.509	.024	403.485	401	1.0061930	+08	−090	+389	2237
	25	41.719	580.590	.017	580.573	577	61924	+08	−090	+389	2231
	47	33.776	1312.057	.017	1312.040	1304	61656	+08	−090	+389	1963
						(2282)					(1.0062078)
Feb. 22	42	43.316									
	49	23.814	400.498	.024	400.474	398	1.0062161	+08	−087	+376	2458
	59	17.467	593.653	.018	593.635	590	61610	+08	−087	+373	1904
	21	13.570	1316.103	.014	1316.089	1308	61842	+08	−087	+368	2131
						(2296)					(1.0062129)

UNITED STATES COAST SURVEY.—PENDULUM AT BERLIN, 1876.

HEAVY END DOWN.

Date.	Time of transit.			Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
	h.	*m.*	*s.*	*s.*								
Apr. 20	7	46	6.276									
	8	00	27.356	861.080	.051	861.029	856	1.0058750	− 20	−05	+235	1.0058960
		25	22.154	1494.798	.044	1494.754	1486	58910	− 20	−05	+229	9114
	9	15	45.845	3023.691	.032	3023.659	3006	58744	− 20	−05	+219	8938
							(5348)					(1.0058990)
Apr. 24	7	58	29.726									
	8	12	56.855	867.429	.050	867.079	862	1.0058853	− 05	−50	+163	1.0058961
	8	37	11.415	1454.560	.043	1454.517	1446	58900	− 05	−50	+154	8999
	9	26	50.885	2979.470	.032	2979.438	2962	58872	− 05	−50	+141	8958
							(5270)					(1.0059011)
Change of knife-edges, from No. 2 to No. 1.												
Apr. 25	7	52	42.427									
	8	5	34.992	772.565	.046	772.519	768	1.0058841	+ 20	−47	+133	1.0058947
	8	31	10.033	1535.041	.046	1534.995	1526	58945	+ 20	−47	+127	9045
	9	22	11.466	3061.433	.032	3061.401	3043½	58817	+ 20	−47	+120	8910
							(5337½)					(1.0058954)
Apr. 26	7	24	10.687									
	7	39	9.489	898.802	.053	898.749	893½	1.0058747	+ 20	−44	+160	1.0058883
	8	03	22.545	1453.056	.043	1453.013	1444½	58934	+ 20	−44	+153	9063
	8	53	58.330	3035.785	.032	3035.753	3018	58824	+ 20	−44	+135	8935
							(5356)					(1.0658959)
Apr. 28	9	55	51.699									
	10	10	35.938	884.239	.052	884.187	879	1.0059011	− 20	−18	+117	1.0059090
	10	35	1.069	1465.131	.043	1465.088	1456½	58963	− 20	−18	+110	9035
	11	26	23.675	3082.606	.033	3082.573	3064½	58949	− 20	−18	+103	9014
							(5400)					(1.0059031)
Apr. 29	10	4	4.873									
	10	19	11.260	906.387	.053	906.334	901	1.0059201	− 24	00	+077	1.0059254
	10	44	35.226	1523.966	.044	1523.922	1515	58891	− 24	00	+072	8939
	11	33	55.599	2960.373	.032	2960.341	2943	58923	− 24	00	+069	8968
							(5359)					(1.0059007)
Change of knife-edges, from No. 1 to No. 2.												
Apr. 30	9	48	32.618									
	10	3	18.235	885.617	.052	885.565	880½	1.0057524	+1266	−09	+078	1.0058857
	10	27	33.128	1454.893	.043	1454.850	1446½	57726	+1266	−09	+074	9057
	11	17	49.445	3016.317	.032	3016.285	2999	57636	+1266	−09	+072	8965
							(5326)					(1.0058970)
May 2	10	9	4.306									
	10	23	33.445	869.139	.051	869.088	864	1.0058889	− 20	−24	+078	1.0058923
	10	47	48.008	1454.563	.043	1454.520	1446	58921	− 20	−24	+076	8953
	10	37	29.499	2981.491	.032	2981.459	2964	58904	− 20	−24	+070	8930
							(5274)					(1.0058934)

PENDULUM AT BERLIN, 1876—Continued.

HEAVY END UP.

Date.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
	h. m. s.	s.								
Apr. 20	10 39 53.808									
	45 21.794	327.986	.018	327.968	326	1.0060368	−20	− 13	+158	1.0060493
	55 55.608	633.814	.019	633.795	630	60238	−20	− 13	+158	60363
	11 17 55.051	1319.443	.014	1319.429	1311½	60458	−20	− 13	+157	60582
					(2267½)					
Apr. 24	10 39 34.322									
	44 44.200	309.878	.017	309.861	308	1.0060422	−05	−114	+102	1.0060405
	55 42.192	657.992	.020	657.972	654	60734	−05	−114	+101	60716
	11 18 22.348	1360.156	.014	1360.142	1352	60240	−05	−114	+098	60219
					(2314)					
Change of knife-edges, from No. 1 to No. 2.										
Apr. 25	10 40 17.215									
	46 13.008	295.793	.020	295.773	294	1.0060306	+20	−106	+081	1.0060301
	57 10.481	656.473	.020	657.453	653½	60490	+20	−106	+080	60484
	11 19 31.024	1340.543	.014	1340.529	1332½	60255	+20	−106	+079	60248
					(2280)					
Apr. 26	10 24 38.854									
	29 50.246	311.392	.017	311.375	309½	1.0060582	+20	− 89	+089	1.0060602
	40 46.193	655.947	.020	655.927	652	60230	+20	− 89	+088	60249
	11 3 16.807	1350.614	.014	1350.600	1342½	60335	+20	− 89	+086	60352
					(2304)					
Apr. 28	7 26 20.727									
	31 12.494	291.767	.017	291.750	290	1.0060345	−20	− 52	+144	1.0060417
	42 45.149	692.655	.021	692.634	688½	60044	−20	− 52	+139	60111
	8 5 17.247	1352.098	.014	1352.084	1344	60141	−20	− 52	+131	60200
					(2322½)					
Apr. 29	7 49 31.597									
	55 27.757	356.160	.020	356.140	354	1.0060452	−24	− 03	+098	1.0060523
	8 5 47.463	619.706	.018	619.688	616	59870	−24	− 03	+095	59938
	28 19.558	1352.095	.014	1352.081	1344	60126	−24	− 03	+091	60190
					(2314)					
Change of knife-edges, from No. 2 to No. 1.										
Apr. 30	7 58 1.799									
	8 3 17.204	315.405	.018	315.387	313½	1.0060191	−15	− 06	+097	1.0060267
	13 51.007	633.803	.018	633.785	630	60079	−15	− 06	+095	60153
	35 46.924	1315.917	.014	1315.903	1308	60420	−15	− 06	+088	60487
					(2251½)					
May 2	8 9 14.734									
	14 24.588	309.854	.017	309.837	308	1.0059643	−20	− 48	+081	1.0059656
	25 6.451	641.863	.019	641.844	638	60251	−20	− 48	+080	60263
	46 46.262	1299.811	.013	1299.798	1292	60356	−20	− 48	+077	60365
					(2238)					

UNITED STATES GEODETIC SURVEY.—PENDULUM AT BERLIN, 1876.—SECOND READING OF FILLETS.

HEAVY END DOWN.

Date.	Time of transit.			Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
	h.	*m.*	*s.*	*s.*								
Apr. 20	7	46	6.276									
	8	0	25.343	859.067	.051	859.016	854	1.0058735	− 20	−05	+235	1.0058945
	8	25	22.154	1496.811	.044	1496.767	1488	58918	− 20	−05	+229	9122
	9	15	45.846	3023.692	.032	3023.660	3006	58749	− 20	−05	+219	8943
							(5348)					(1.0058993)
Apr. 24	7	58	29.727									
	8	12	56.855	867.128	.050	867.078	862	1.0058910	− 05	−50	+163	1.0059018
	8	37	11.417	1454.562	.043	1454.519	1446	58914	− 05	−50	+154	9013
	9	26	50.882	2979.465	.032	2979.433	2962	58855	− 05	−50	+141	8941
							(5270)					(1.0059017)
Apr. 25	7	52	42.430									
	8	5	34.999	772.569	.046	772.523	768	1.0058893	+ 20	−47	+133	1.0058999
	8	31	.030	1535.031	.046	1534.985	1526	58879	+ 20	−47	+127	8979
	9	22	11.470	3061.440	.032	3061.408	3043½	58840	+ 20	−47	+120	8933
							(5337½)					(1.0058956)
Apr. 26	…	…	…									
	…	…	…	…	…	…	…	…	…	…	…	…
	…	…	…	…	…	…	…	…	…	…	…	…
	…	…	…	…	…	…	…	…	…	…	…	…
Apr. 28	9	55	52.194									
	10	10	35.934	883.740	.052	883.688	878½	1.0059055	− 20	−18	+117	1.0059134
	10	35	1.062	1465.128	.043	1465.085	1456½	58942	− 20	−18	+110	9014
	11	26	23.673	3082.611	.033	3082.578	3064½	58992	− 20	−18	+103	9057
							(5399½)					(1.0059058)
Apr. 29	10	4	4.892									
	10	19	11.260	906.368	.053	906.315	901	1.0058990	− 24	00	+077	1.0059043
	10	44	35.225	1523.965	.044	1523.921	1515	58884	− 24	00	+072	8932
	11	33	55.598	2960.373	.032	2960.341	2943	58923	− 24	00	+069	8968
							(5359)					(1.0058969)
Apr. 30	9	48	32.615									
	10	3	18.248	885.633	.052	885.581	880½	1.0057706	+1266	−09	+078	1.0059041
	10	27	33.126	1454.878	.043	1454.835	1446½	57691	+1266	−09	+074	9022
	11	17	49.446	3016.330	.032	3016.288	2999	57646	+1266	−09	+072	8975
							(5326)					(1.0058997)
May 2	10	9	4.303									
	10	23	33.441	869.138	.051	869.087	864	1.0058877	− 20	−24	+078	1.0058911
	10	47	48.007	1454.566	.043	1454.523	1446	58942	− 20	−24	+076	8974
	11	37	29.497	2981.490	.032	2981.458	2964	58900	− 20	−24	+070	8926
							(5274)					(1.0058936)

PENDULUM AT BERLIN, 1876.—SECOND READING OF FILLETS—Continued.

HEAVY END UP.

Date.	Time of transit.	Intervals.	Cor. for arc	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
	h. m. s.	*s.*								
Apr. 20	10 39 53.808									
	10 45 21.795	327.987	.018	327.969	326	1.0060399	−20	− 13	+158	1.0060524
	10 55 55.606	633.811	.019	633.796	630	60254	−20	− 13	+158	60379
	11 17 55.056	1319.450	.014	1319.432	1311½	60480	−20	− 13	+157	60604
					(2267½)					
Apr. 24	10 39 34.319									
	10 44 44.199	309.880	.017	309.863	308	1.0060427	− 5	−114	+102	1.0060410
	10 55 42.186	657.987	.020	657.967	654	60657	− 5	−114	+101	60639
	11 18 22.345	1360.159	.014	1360.145	1352	60244	− 5	−114	+ 98	60223
					(2314)					
Apr. 25	10 40 17.219									
	10 46 13.007	295.788	.020	295.768	294	1.0060136	+20	−106	+ 81	1.0060131
	10 57 10.480	657.473	.020	657.453	653½	60490	+20	−106	+ 80	60484
	11 19 31.001	1340.521	.014	1340.507	1332½	60090	+20	−106	+ 79	60083
					(2280)					
Apr. 26	....									
	....	....	....	....	....	....	....	....	....	....
	....	....	....	....	....	....	....	....	....	....
	....	....	....	....	....	....	....	....	....	....
Apr. 28	7 76 20.728									
	7 31 12.491	291.763	.017	291.746	290	1.0060207	−20	− 52	+144	1.0060279
	7 42 45.140	692.649	.021	692.628	688½	59956	−20	− 52	+139	60023
	8 5 17.248	1352.108	.014	1352.094	1344	60223	−20	− 52	+131	60282
					(2322½)					
Apr. 29	7 49 31.594									
	7 55 27.735	356.141	.020	356.121	354	1.0059915	−24	− 3	+ 98	1.0059986
	8 5 47.463	619.728	.018	619.710	616	60227	−24	− 3	+ 95	60295
	8 28 19.558	1352.095	.014	1352.081	1344	60126	−24	− 3	+ 91	60190
					(2314)					
Apr. 30	7 58 1.801									
	8 3 17.204	315.403	.018	315.385	313½	1.0060128	−15	− 6	+ 97	1.0060204
	8 13 51.007	633.803	.018	633.785	630	60079	−15	− 6	+ 95	60153
	8 35 46.925	1315.918	.014	1315.904	1308	60428	−15	− 6	+ 88	60495
					(2251½)					
May 20	8 9 14.732									
	8 14 24.593	309.861	.017	309.844	308	1.0059870	−20	− 48	+ 81	1.0059883
	8 25 6.441	641.848	.019	641.829	638	60016	−20	− 48	+ 80	60028
	8 46 46.260	1299.819	.013	1299.806	1292	60418	−20	− 48	+ 77	60427
					(2238)					

UNITED STATES COAST SURVEY.—PENDULUM AT KEW, JULY, 1876.

HEAVY END DOWN.

Date.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
		h. m. s.	*s.*								
July 1	A	10 21 7.884									
	D	10 36 32.320	924.436	.055	924.381	919	1.0058552	+1269	−41	−293	1.0059487
	D	11 2 46.510	1574.190	.045	1574.145	1565	58435	+1269	−41	−296	367
	D	11 51 56.675	2950.165	.029	2950.136	2933	58425	+1269	−41	−298	355
						(5417)					(1.0059381)
July 2	D	6 42 58.365									
	C	6 58 35.845	937.480	.054	937.426	932	1.0058219	+1281	−50	−291	1.0059159
	C	7 21 52.006	1396.161	.041	1396.120	1388	58501	+1281	−50	−300	432
	B	8 12 44.764	3052.758	.032	3052.726	3035	58405	+1281	−50	−305	331
						(5355)					(1.0059328)
July 3	A	6 52 30.199									
	D	7 10 46.638	1096.439	.059	1096.380	1090	1.0058532	+1275	−60	−329	1.0059418
	A	7 31 19.845	1233.207	.035	1233.172	1226	58499	+1275	−60	−333	381
	A	8 21 53.837	3093.992	.033	3093.959	3076	58384	+1275	−60	−334	265
						(5392)					(1.0059322)
July 4	B	8 47 30.365									
	D	6 2 52.780	922.415	.053	922.362	917	1.0058473	+1271	−53	−311	1.0059380
	A	6 26 30.049	1417.269	.042	1417.227	1409	58389	+1271	−53	−322	285
	A	7 17 51.973	3081.924	.032	3081.892	3064	58394	+1271	−53	−328	284
						(5390)					(1.0059301)
July 4	B	10 9 10.507									
	A	10 24 41.982	931.475	.053	931.422	926	1.0058553	+1271	−51	−334	1.0059439
	B	10 48 10.172	1408.190	.042	1408.148	1400	58200	+1271	−51	−333	087
	B	11 40 42.517	3152.345	.033	3152.312	3134	58430	+1271	−51	−329	321
						(5460)					(1.0059281)
Change of knife-edges, from No. 1 to No. 2.											
July 7	D	6 25 10.062									
	B	6 40 30.456	920.394	.052	920.342	915	1.0058383	+1297	−29	−330	1.0059321
	C	7 4 35.883	1445.427	.042	1445.385	1437	58351	+1297	−29	−342	277
	C	7 56 21.945	3106.062	.033	3106.529	3088	58384	+1297	−29	−357	295
						(5440)					(1.0059295)
July 7	B	10 31 28.644									
	B	10 47 16.214	947.570	.054	947.516	942	1.0058556	+1297	−26	−388	1.0059439
	B	11 11 18.607	1442.393	.042	1442.351	1434	58236	+1297	−26	−388	119
	D	12 4 26.142	3186.535	.033	3186.502	3168	58403	+1297	−26	−386	288
						(5544)					(1.0059270)
July 8	B	6 28 6.506									
	A	6 43 50.039	943.533	.055	943.478	938	1.0058400	+1310	−22	−342	1.0059346
	B	7 7 46.412	1436.373	.042	1436.331	1428	58340	+1310	−22	−345	283
	B	7 59 32.460	3106.048	.033	3106.015	3088	58339	+1310	−22	−346	281
						(5454)					(1.0059293)
July 9	B	7 16 52.359									
	A	7 32 33.873	941.514	.054	941.460	936	1.0058333	+1287	−43	−318	1.0059259
	B	7 56 32.249	1438.376	.042	1438.334	1430	58280	+1287	−43	−325	199
	B	8 48 46.480	3134.231	.032	3134.199	3116	58405	+1287	−43	−330	319
						(5482)					(1.0059277)
July 10	D	7 22 39.981									
	D	7 38 20.486	940.505	.054	940.451	935	1.0058299	+1300	−50	−255	1.0059294
	B	8 3 34.305	1513.819	.043	1513.776	1505	58312	+1300	−50	−261	301
	A	8 53 51.827	3017.522	.032	3017.490	3000	58300	+1300	−50	−269	281
						(5440)					(1.0059289)

PENDULUM AT KEW, JULY, 1876—Continued.

HEAVY END UP.

Date.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
		h. m. s.	*s.*								
July 1	....	12 4 38.635									
	....	12 10 35.776	357.141	.020	357.121	355	1.0059746	+1269	− 92	−248	1.0060675
	....	12 22 3.866	688.090	.020	688.070	684	59503	+1269	− 92	−251	429
	....	12 43 40.584	1296.718	.013	1296.705	1289	59775	+1269	− 92	−253	699
						(2328)					(1.0060612)
July 2	....	5 12 52.612									
	....	5 19 54.123	421.511	.019	421.492	419	1.0059475	+1281	−114	−213	1.0060429
	....	5 30 29.920	635.797	.017	635.780	632	59810	+1281	−114	−221	756
	....	5 53 52.226	1402.306	.013	1402.293	1394	59491	+1281	−114	−226	432
						(2445)					(1.0060515)
July 2	....	8 26 28.214									
	....	8 32 50.506	382.292	.021	382.271	380	1.0059763	+1281	−117	−263	1.0060664
	....	8 42 56.119	605.613	.018	605.595	602	59718	+1281	−117	−269	613
	....	9 5 38.207	1362.088	.014	1362.074	1354	59631	+1281	−117	−272	523
						(2336)					(1.0060569)
July 3	....	5 36 32.199									
	....	5 42 50.446	378.247	.022	378.225	376	1.0059176	+1275	−140	−243	1.0060068
	....	5 53 32.262	641.816	.019	641.797	638	59514	+1275	−140	−248	401
	....	6 16 20.400	1368.138	.014	1368.124	1360	59755	+1275	−140	−257	633
						(2374)					(1.0060481)
July 3	....	9 39 10.524									
	....	9 45 43.885	393.361	.023	393.338	391	1.0059795	+1275	−133	−284	1.0060653
	....	9 56 00.545	616.660	.018	616.642	613	59413	+1275	−133	−287	268
	....	10 19 1.766	1381.221	.014	1381.207	1373	59774	+1275	−133	−291	625
						(2377)					(1.0060538)
July 4	....	4 45 55.609									
	....	4 52 31.981	396.372	.023	396.349	394	1.0059619	+1271	−126	−232	1.0060532
	....	5 3 15.808	643.827	.019	643.808	640	59500	+1271	−126	−237	408
	....	5 26 34.123	1398.315	.014	1398.301	1390	59717	+1271	−126	−247	615
						(2424)					(1.0060547)
July 4	....	7 35 40.095									
	....	7 42 16.473	396.378	.022	396.356	394	1.0059797	+1271	−119	−274	1.0060675
	....	7 52 12.002	595.529	.017	595.512	592	59324	+1271	−119	−276	200
	....	8 15 6.168	1374.166	.015	1374.151	1366	59671	+1271	−119	−278	545
						(2352)					(1.0060480)
July 4	....	9 5 28.179									
	....	9 12 10.591	402.412	.023	402.389	400	1.0059725	+1271	−118	−269	1.0060609
	....	9 22 30.259	619.668	.018	619.650	616	59253	+1271	−118	−271	135
	....	9 45 32.484	1382.225	.014	1382.211	1374	59752	+1271	−118	−274	631
						(2390)					(1.0060499)
July 4	....	12 2 46.318									
	....	12 9 18.675	392.357	.023	392.334	390	1.0059846	+1271	−114	−273	1.0060730
	....	12 19 48.451	629.776	.019	629.757	626	60016	+1271	−114	−275	898
	....	12 42 28.501	1360.050	.014	1360.036	1352	59438	+1271	−114	−276	319
						(2368)					(1.0060540)

PENDULUM AT KEW, JULY, 1876—Continued.

HEAVY END UP—Continued.

Date.	Method.	Time of transit.			Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
					Change of knife-edges, from No. 2 to No. 1.								
		h.	m.	s.	s.								
July 7	A	5	22	10.009									
	B	5	28	34.302	383.293	.022	383.271	381	1.0059606	+1297	− 68	−247	1.0060588
	A	5	39	4.045	629.743	.019	629.724	626	59488	+1297	− 68	−253	464
	A	6	1	21.989	1337.944	.014	1337.930	1330	59624	+1297	− 68	−264	589
								(2337)					(1.0060555)
July 7	D	8	40	40.053									
	D	8	47	14.392	394.339	.023	394.316	392	1.0059082	+1297	− 64	−304	1.0060011
	D	8	57	56.216	641.824	.019	641.805	638	59639	+1297	− 64	−308	564
	D	9	20	52.395	1376.179	.015	1376.164	1368	59673	+1297	− 64	−314	592
								(2398)					(1.0060490)
July 7	B	9	30	20.421									
	B	9	37	0.814	400.393	.023	400.370	398	1.0059548	+1297	− 63	−320	1.0060462
	B	9	47	32.548	631.734	.019	631.715	628	59156	+1297	− 63	−321	069
	B	10	10	10.642	1358.094	.014	1358.080	1350	59852	+1297	− 63	−322	764
								(2376)					(1.0060530)
July 7	C	12	22	54.115									
	D	12	29	22.447	388.332	.022	388.310	386	1.0059845	+1297	− 56	−323	1.0060763
	D	12	40	16.361	653.914	.020	653.894	650	59908	+1297	− 56	−324	825
	D	13	2	52.405	1356.044	.014	1356.030	1348	59570	+1297	− 56	−325	486
								(2384)					(1.0060623)
July 8	C	5	21	20.981									
	D	5	28	2.341	392.360	.023	392.337	390	1.0059923	+1310	− 47	−265	1.0060921
	D	5	38	48.184	645.843	.020	645.823	642	59548	+1310	− 47	−270	541
	D	6	1	48.372	1380.188	.014	1380.174	1372	59577	+1310	− 47	−278	562
								(2404)					(1.0060615)
July 8	B	9	12	54.420									
	B	9	19	14.691	380.271	.022	380.249	378	1.0059497	+1310	− 57	−270	1.0060480
[Rejected.]	B	9	30	30.691	676.000	.020	675.980	672	59226	+1310	− 57	−273	206
	B	9	52	20.428	1309.737	.013	1309.724	1302	59324	+1310	− 57	−276	301
								(2352)					(1.0060303)
July 9	D	6	14	46.203									
	A	6	22	2.823	436.620	.022	436.598	434	1.0059862	+1287	−106	−232	1.0060811
	D	6	31	48.308	525.485	.018	525.467	582	59570	+1287	−106	−238	513
	D	6	54	24.353	1356.045	.015	1356.030	1348	59570	+1287	−106	−249	502
								(2364)					(1.0060562)
July 9	D	9	16	14.154									
	D	9	22	40.466	386.312	.023	386.289	384	1.0059609	+1287	− 98	−292	1.0060506
	D	9	32	56.136	615.670	.018	615.652	612	59673	+1287	− 98	−293	569
	D	9	55	24.160	1348.024	.014	1348.010	1340	59776	+1287	− 98	−294	671
								(2336)					(1.0060639)
July 10	B	5	28	2.285									
	B	5	34	48.716	406.431	.023	406.408	404	1.0059604	+1300	−114	−216	1.0060574
	B	5	44	58.357	609.641	.018	609.623	606	59785	+1300	−114	−213	758
	B	6	8	22.704	1404.347	.014	1404.333	1336	59692	+1300	−114	−217	661
								(2346)					(1.0060571)
July 10	B	9	35	56.718									
	D	9	43	26.400	449.682	.023	449.659	447	1.0059485	+1300	−115	−226	1.0060444
	C	9	53	7.879	581.479	.019	581.460	578	59862	+1300	−115	−231	816
	C	10	15	41.910	1354.031	.014	1354.017	1346	59562	+1300	−115	−237	510
								(2371)					(1.0060572)

UNITED STATES COAST SURVEY.—PENDULUM AT HOBOKEN.

HEAVY END DOWN.

Date.	Chr.	Method.	Time of transit.			Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
			h.	*m.*	*s.*	*s.*								
June 11	380	D	15	33	13.345									
		A		49	3.937	950.592	.056	950.536	944½	1.0063913	− 57	−60	−363	1.0063433
		B	16	16	18.874	1634.937	.048	1634.889	1624½	3952	− 57	−60	−363	472
		B	17	8	24.798	3125.924	.031	3125.893	3106	4047	− 57	−60	−363	567
									(5675)					(1.0063518)
June 14	387	A	13	59	52.591									
		D	14	14	44.815	891.224	.051	891.178	885½	1.0064069	−288	−63	−377	1.0063341
		B		38	14.865	1410.050	.043	1410.007	1401	4289	−288	−63	−377	561
		D	15	30	14.824	3119.959	.035	3119.924	3100	4273	−288	−63	−377	545
									(5386½)					(1.0063516)
June 15	380	D	15	22	38.937									
		B		35	12.769	753.832	.044	753.788	749	1.0063923	− 68	−51	−436	1.0063368
		D		59	17.029	1444.260	.046	1444.215	1435	4214	− 68	−51	−436	659
		D	16	45	50.878	2793.849	.027	2793.822	2776	4200	− 68	−51	−436	645
									(4960)					(1.0063607)
June 16	380	B	14	28	36.999									
		B		43	4.571	867.572	.050	867.522	862	1.0064056	− 68	−32	−472	1.0063484
		B	15	10	53.272	1668.701	.049	1668.651	1658	4240	− 68	−32	−472	668
		D		59	55.059	2941.787	.029	2941.758	2923	4174	− 68	−32	−472	602
									(5443)					(1.0063604)
								Change of knife-edges.						
June 17	202	B	15	54	32.711									
		D	16	10	51.967	979.256	.055	979.200	973	1.0063720	+142	−54	−432	1.0063376
		B	16	36	40.868	1548.901	.044	1548.857	1539	4048	+142	−54	−432	704
		D	17	26	28.880	2988.012	.029	2987.982	2969	3934	+142	−54	−432	590
									(5481)					(1.0063584)
June 19	202	B	14	15	25.293									
		B		29	40.743	855.450	.050	855.399	850	1.0063525	+240	−39	−456	1.0063270
		B		55	18.561	1537.818	.047	1537.771	1528	3946	+240	−39	−456	691
		D	15	45	54.822	3036.261	.038	3036.223	3017	3717	+240	−39	−456	462
									(5395)					(1.0063497)
June 20	202	D	15	4	14.856									
		D		20	0.891	946.035	.053	945.981	940	1.0063631	+240	−59	−416	1.0063396
June 22	202	C	14	31	41.930									
		A		44	53.972	792.042	.047	791.995	787	1.0063471	+336	−46	−386	1.0063375
		B	15	12	33.042	1659.070	.050	1659.020	1648½	3814	+336	−46	−386	718
		C	16	1	24.137	2931.095	.029	2931.066	2912½	3745	+336	−46	−386	649
									(5348)					(1.0063630)
June 29	202	D	10	7	24.685									
		B		21	10.950	826.265	.048	826.217	821	1.0063547	+426	−53	−404	1.0063516
		A		46	1.924	1490.974	.046	1490.927	1481½	3636	+426	−53	−404	605
		B	11	30	27.294	2665.370	.030	2665.340	2648½	3582	+426	−53	−404	551
									(4951)					(1 0063561)

UNITED STATES GEODETIC SURVEY.—PENDULUM AT HOBOKEN, 1877.

HEAVY END UP.

Date.	Chr.	Method.	Time of transit.			Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
			h.	*m.*	*s.*	*s.*								
June 11	202	C	14	11	38. 249									
		D		18	15. 367	397. 118	. 020	397. 097	394½	1. 0065830	+ 94	−139	−306	1. 0065479
		A		29	24. 260	668. 893	. 016	668. 877	664½	5872	+ 94	−139	−306	5521
		A		48	31. 797	1147. 537	. 010	1147. 526	1140	6019	+ 94	−139	−306	5668
									(2199)					(1. 0065590)
June 14	387	B	16	13	38. 972									
		C		19	53. 944	374. 972	. 021	374. 951	372½	1. 0065820	−288	−141	−323	1. 0065068
		C		30	5. 987	612. 033	. 018	612. 015	608	6038	−288	−141	−323	5286
		D		56	20. 856	1574. 869	. 016	1574. 853	1564½	6594	−288	−141	−323	5842
									(2545)					(1. 0065596)
June 15	380	A	14	15	1. 901									
		A		20	22. 026	320. 125	. 017	320. 108	318	1. 0066299	− 68	−114	−378	1. 0065739
		B		30	52. 683	630. 657	. 018	630. 639	626½	6069	− 68	−114	−371	5516
		C		53	24. 044	1351. 361	. 013	1351. 348	1342½	5909	− 68	−114	−364	5363
									(2287)					(1. 0065457)
June 16	380	B	13	22	44. 800									
		B		28	45. 183	360. 383	. 019	360. 374	358	1. 0066304	− 68	− 73	−399	1. 0065764
		C		39	27. 931	642. 748	. 018	642. 729	638½	6233	− 68	− 73	−394	5698
		D	14	1	19. 039	1351. 108	. 012	1311. 096	1302½	5998	− 68	− 73	−391	5466
									(2299)					(1. 0065577)
							Change of knife-edges.							
June 17	202	B	14	47	8. 665									
		B		53	0. 975	352. 310	. 018	352. 292	350	1. 0065497	+142	−119	−368	1. 0065152
		D	15	5	0. 698	719. 723	. 019	719. 703	715	5780	+142	−119	−368	5435
		D		25	36. 797	1236. 099	. 011	1236. 088	1228	5861	+142	−119	−368	5516
									(2293)					(1. 0065435)
June 19	202	"XOXO"	16	11	16. 343									
		D		17	40. 855	384. 512	. 021	384. 491	382	1. 0065222	+240	− 97	−386	1. 0064979
		A		29	19. 952	699. 097	. 019	699. 078	694½	5927	+240	− 97	−386	5684
		D		48	43. 042	1163. 090	. 010	1163. 079	1155½	5591	+240	− 97	−386	5348
									(2232)					(1. 0065389)
June 20	202	A	14	7	40. 020									
		A		17	48. 012	607. 992	. 017	607. 975	604	1. 0065815	+240	−136	−345	1. 0065574
		B		38	36. 656	1248. 644	. 011	1248. 633	1240½	5564	+240	−136	−345	5323
									(1844½)					
June 22	202	D	16	22	58. 721									
		C		27	32. 028	273. 307	. 012	273. 295	271½	1. 0066110	+336	−112	−321	1. 0066013
		C		37	27. 926	595. 898	. 015	595. 883	592	5586	+336	−112	−321	5489
		B		57	35. 297	1207. 371	. 011	1207. 360	1199½	5531	+336	−112	−321	5434
									(2063)					(1. 0065512)
June 29	202	A	9	6	0. 137									
		B		10	58. 584	298. 447	. 016	298. 431	296½	1. 0065133	+426	−121	−335	1. 0065103
		B		21	44. 824	646. 240	. 019	646. 220	642	5734	+426	−121	−335	5704
		D		40	39. 187	1134. 363	. 012	1134. 351	1127	5231	+426	−121	−335	5201
									(2065½)					(1. 0065343)

UNITED STATES GEODETIC SURVEY.—PENDULUM AT HOBOKEN, 1877.—SWINGS AT LOW PRESSURES.

HEAVY END DOWN.

[All temperatures in low-pressure and high-temperature experiments are to be corrected by $+0^{\circ}.3$ C.]

Date.	Pressure.	Temperature.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Period, corrected.
	In.	°		*h. m. s.*	*s.*						
Sept. 25	30.25	67.6	A	10 48 26.194							
			A	11 7 45.625	1159.431	.044	1159.387	1152	1.0064123	+ 13	1.0064136
			A	11 27 25.156	1179.531	.022	1179.509	1172	4258	+ 13	271
			A	11 57 54.835	1829.679	.014	1829.665	1818	4164	+ 13	177
								(4142)			(1.0064173)
Sept. 26	15.19	66.1	A	10 48 22.909							
			A	11 44 26.903	963.994	.042	963.952	958	1.0062129	+ 13	1.0062142
			A	11 41 32.780	2225.877	.053	2225.824	2212	2495	+ 13	508
			A	12 23 24.403	2511.623	.019	2511.604	2496	2516	+ 13	529
								(5666)			(1.0062457)
Sept. 27	30.25	68.5	A	8 50 47.365							
			A	9 1 15.420	628.055	.032	628.023	624	1.0064471	+ 13	1.0064484
			A	9 19 34.448	1099.028	.030	1098.998	1092	4084	+ 13	097
			A	9 58 14.881	2320.433	.026	2260.407	2246	4145	+ 13	158
								(3962)			(1.0064194)
Sept. 29	0.50	68.3	D	10 7 12.887							
			B	10 38 21.285	1868.398	.146	1868.252	1857	1.0060592	+128	1.0060720
			A	11 8 15.753	1794.468	.127	1794.341	1783½	0785	+128	0913
			B	11 37 59.106	1783.353	.112	1783.241	1772½	0598	+128	0726
			B	12 5 17.109	1638.003	.098	1637.905	1628	0842	+128	0970
			D	12 35 59.323	1842.214	.091	1842.123	1831	0748	+128	0876
			D	13 7 48.930	1909.607	.085	1909.522	1898	0705	+128	0833
			B	13 42 28.560	2079.630	.082	2079.548	2067	0706	+128	0834
			D	14 12 50.654	1822.094	.061	1822.033	1811	0922	+128	1050
			D	14 57 16.812	2666.158	.076	2666.082	2650	0687	+128	0815
			D	15 24 6.555	1609.745	.040	1609.705	1600	0656	+128	0784
			B	15 55 2.794	1856.237	.042	1856.195	1845	0677	+128	0805
			D	16 25 57.012	1854.218	.037	1854.181	1843	0677	+128	0805
			D	17 9 32.831	2615.819	.042	2615.777	2600	0681	+128	0809
			B	17 29 4.907	1172.076	.017	1172.059	1165	0592	+128	0720
			D	18 26 24.687	3439.780	.044	3439.736	3419	0649	+128	0777
			B	18 57 57.142	1892.455	.020	1892.435	1881	0792	+128	0920
			A	19 34 33.931	2196.789	.020	2196.769	2183½	0767	+128	0895
								(33834½)			(1.0060879)
Oct. 1	4.99	68.3	A	8 18 49.961							
			D	8 49 38.866	1848.905	.121	1848.784	1837½	1.0061410	+ 9	1.0061419
			B	10 19 42.950	5404.084	.166	5403.918	5371	1288	+ 9	297
			B	10 51 26.588	1903.638	.028	1903.610	1892	1364	+ 9	373
			D	11 23 1.196	1894.611	.021	1894.590	1883	1535	+ 9	544
			D	12 11 51.086	2929.890	.012	2929.878	2912	1394	+ 9	403
			B	12 31 21.201	1170.115	.009	1170.106	1163	1101	+ 9	110 rej.
								(13895½)			(1.0061379)
Oct. 1	1.50	68.0	B	12 48 29.189							
			B	13 24 56.944	2247.755	.161	2247.594	2234	1.0060850	+ 9	1.0060859
			B	13 53 7.292	1680.348	.100	1680.248	1670	1365	+ 9	1374
			D	14 28 10.769	2103.477	.096	2043.381	2050	0760	+ 9	0769
			D	15 1 39.367	2068.598	.074	2068.524	2056	0914	+ 9	0923
			D	15 33 22.956	1903.589	.055	1903.534	1892	0962	+ 9	0971
			D	16 1 31.528	1748.572	.040	1748.532	1738	0596	+ 9	0605
			B	16 41 46.912	2355.384	.045	2355.339	2341	1252	+ 9	1261
			D	17 6 24.907	1477.995	.020	1477.975	1469	1095	+ 9	1104
			B	18 16 19.357	4194.450	.046	4194.404	4169	0935	+ 9	0944
			A	18 41 44.122	1524.765	.012	1524.753	1515½	0990	+ 9	0999
								(21115½)			(1.0060999)

PENDULUM AT HOBOKEN, 1877.—SWINGS AT LOW PRESSURES—Continued.

HEAVY END DOWN—Continued.

Date.	Pressure.	Temperature.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Period, corrected.
	In.	°		h. m. s.	s.						
Oct. 3	0.25	68.5	D	9 7 46.910							
			D	9 37 20.754	1773.844	.148	1773.696	1763	1.0060669	+ 9	1.0060678
			C	10 14 43.954	2243.200	.168	2243.032	2229½	0695	+ 9	704
			D	10 39 46.646	1502.692	.104	1502.588	1493½	0850	+ 9	859
			B	11 7 16.707	1650.061	.107	1649.954	1640	0695	+ 9	704
			D	11 37 36.784	1820.077	.118	1819.959	1809	0580	+ 9	589
			D	12 6 5.210	1708.426	.098	1708.328	1698	0825	+ 9	834
			B	12 41 6.999	2101.789	.105	2101.684	2089	0718	+ 9	727
			B	13 13 34.839	1947.840	.089	1947.751	1936	0697	+ 9	706
			D	13 38 10.828	1475.989	.066	1475.923	1467	0825	+ 9	834
			B	14 8 4.704	1793.878	.071	1793.808	1783	0617	+ 9	626
			B	14 36 45.175	1720.471	.065	1720.406	1710	0854	+ 9	863
			D	15 11 42.930	2097.755	.069	2097.686	2085	0844	+ 9	853
			D	15 39 47.164	1084.234	.052	1684.182	1674	0824	+ 9	833
			B	16 3 44.881	1437.717	.041	1437.676	1429	0714	+ 9	723
			D	16 33 16.614	1771.733	.048	1771.685	1761	0676	+ 9	685
			D	17 6 8.567	1971.953	.047	1971.906	1960	0745	+ 9	754
			D	17 34 10.790	1682.223	.037	1682.190	1672	0913	+ 9	922
			B	18 10 26.929	2176.139	.044	2176.095	2163	0542	+ 9	551
			B	18 49 43.208	2356.279	.042	2356.237	2342	0790	+ 9	799
								(34704)			(1.0060746)
Oct. 5	14.98	64.4	D	9 0 1.140							
			B	9 17 52.833	1071.693	.063	1071.630	1065	1.0062253	+129	1.0062382
			B	9 54 48.623	2215.790	.068	2215.722	2202	2316	+129	445
			D	11 2 54.925	4086.302	.038	4086.264	4061	2211	+129	340
								(7328)			(1.0062378)
Oct. 5	14.98	64.3	A	11 13 7.823							
			D	11 32 58.754	1190.931	.067	1190.864	1133½	1.0062223	+129	1.0062352
			D	12 7 46.697	2087.943	.061	2087.882	2075	2082	+129	211
			B	13 18 12.864	4226.167	.038	4226.129	4200	2212	+129	341
								(7458½)			(1.0062306)
Oct. 5	30.16	64.7	A	13 30 39.866							
			B	13 42 22.876	703.010	.037	702.973	698½	1.0064037	+129	1.0064166
			B	14 2 26.846	1203.670	.034	1203.636	1196	3846	+129	3975
			A	14 49? 17.896	2811.350	.027	2811.324	2793½	3805	+129	3934
								(4688)			(1.0063979)
Oct. 5	30.16	64.5	D	14 58 45.016							
			B	15 10 56.685	731.669	.039	731.630	727	1.0063686	+129	1.0063815
			D	15 30 7.029	1150.344	.032	1150.312	1143	3972	+129	4101
			B	15 15 15.206	2708.177	.027	2708.150	2691	3731	+129	3860
								(4561)	....		(1.0063913)

PENDULUM AT HOBOKEN, 1877.—SWINGS AT LOW PRESSURES—Continued.

HEAVY END UP.

Date.	Pressure.	Temperature.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Period, corrected.
1877.	*In.*	°		*h. m. s.*	*s.*						
Nov. 30	0. 39	....	A	13 50 25. 592							
		51. 6	A	14 45 4. 562	3278. 970	. 240	3278. 730	3260	1. 0057454	+367	1. 0057821
		51. 3	A	15 25 16. 464	2411. 902	. 111	2411. 791	2398	57510	+367	877
		51. 0	A	16 17 6. 318	3109. 854	. 095	3109. 759	3092	57435	+367	802
		50. 6	A	17 15 16. 274	3189. 956	. 064	3489. 892	3470	57326	+367	693
		50. 5	A	17 38 42. 309	1406. 035	. 017	1406. 018	1398	57353	+367	720
								(13618)			(1. 0057784)
Dec. 4	7. 47	48. 0	D	11 10 8. 716							
			B	11 18 38. 728	510. 012	. 032	509. 980	507	1. 0058777	+369	1. 0059146
			D	12 29 8. 732	4230. 004	. 081	4229. 923	4205	59270	+369	9639
			B	12 31 40. 604	151. 872	. 000	151. 872	151	57748	+369	8117
								(4863)			(1. 0059540)
Dec. 8	0. 35	48. 7	B	10 46 34. 876							
			B	11 30 42. 130	2647. 254	. 195	2647. 069	2632	1. 0057253	+353	1. 0057606
			B	12 30 42. 790	3600. 660	. 156	3600. 504	3580	57274	+353	627
			B	13 34 50. 774	3847. 984	. 091	3847. 893	3826	57221	+353	574
			B	14 31 44. 256	3413. 482	. 044	3413. 438	3394	57272	+353	625
			B	15 27 37. 353	3353. 097	. 026	3353. 071	3334	57202	+353	555
								(16766)			(1. 0057597)
Dec. 8	0. 42	48. 8	D	15 33 29. 256							
			B	16 33 2. 802	3573. 546	. 194	3573. 352	3553	1. 0057281	+353	1. 0057634
			B	17 43 40. 999	4238. 197	. 122	4238. 075	4214	57131	+353	484
			B	19 10 34. 758	5213. 759	. 070	5213. 689	5184	57270	+353	623
			B	19 45 48. 809	2114. 051	. 015	2114. 036	2102	57260	+353	613
			D	20 34 54. 576	2945. 767	. 014	2945. 753	2929	57197	+353	550
								(17982)			(1. 0057579)
Dec. 10	0. 67	49. 4	B	12 5 30. 825							
			D	12 44 21. 293	2330. 468	. 170	2330. 298	2317	1. 0057389	+389	1. 0057778
			D	13 27 16. 098	2574. 805	. 117	2574. 688	2560	57375	+389	764
			D	14 51 33. 072	5056. 974	. 112	5056. 862	5028	57403	+389	792
			B	15 28 30. 747	2217. 675	. 023	2217. 652	2205	57379	+389	768
								(12110)			(1. 0057778)
Dec. 10	0. 72	48. 8	B	15 31 29. 314							
			B	16 34 47. 093	3797. 779	. 166	3797. 613	3776	1. 0057238	+389	1. 0057627
			B	17 43 24. 675	4117. 582	. 082	4117. 500	4094	57401	+389	790
			B	19 1 41. 431	4696. 756	. 040	4696. 716	4670	57208	+389	597
								(12540)			(1. 0057669)
Dec. 12	29. 87	50. 8	D	13 3 14. 877							
			D	13 38 42. 675	2127. 798	. 024	2127. 774	2114	1. 0065156	+283	1. 0065439
Dec. 12	29. 95	52. 1	B	15 39 35. 317							
			B	17 00 12. 625	4837. 308	. 039	4837. 269	4806	1. 0065062	+283	1. 0065349
Dec. 12	29. 03	51. 1	B	17 24 27. 023							
			B	18 15 46. 956	3079. 933	. 031	3079. 902	3060	1. 0065039	+283	1. 0065322
Dec. 14	30. 24	48. 6	B	18 15 36. 941							
			B	18 52 1. 055	2184. 114	. 026	2184. 088	2170	1. 0064922	+329	1. 0065247
Dec. 16	27. 20	50. 2	D	7 24 47. 072							
			D	8 1 47. 321	2220. 249	. 038	2220. 211	2206	1. 0064419	+356	1. 0064775
Dec. 16	27. 20	50. 2	D	8 10 58. 858							
			B	8 44 6. 577	1987. 719	. 036	1987. 683	1975	1. 0064218	+356	1. 0064574

PENDULUM AT HOBOKEN, 1877.—SWINGS AT LOW PRESSURES—Continued.

HEAVY END UP—Continued.

Date.	Pressure.	Temperature.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Period, corrected.
1877.	*In.*	°		*h. m. s.*	*s.*						
Dec. 16	29.15	50.2	B	9 06 56.915							
			D	9 40 39.025	2022.110	.033	2022.077	2009	1.0065092	+356	1.0065448
Dec. 16	29.05	50.5	D	9 51 20.849							
			D	10 22 28.955	1868.106	.030	1868.076	1856	1.0065065	+356	1.0065421
Dec. 16	22.45	50.9	B	11 21 36.680							
			B	11 59 43.101	2286.421	.036	2286.385	2272	1.0063402	+356	1.0063670
Dec. 17	15.02	50.4	B	12 22 32.587							
			D	13 20 16.792	3464.205	.063	3464.142	3443	1.0061406	+376	1.0061782
Dec. 17	7.45	50.5	D	16 1 52.821							
			D	17 26 51.046	5098.225	.093	5098.132	5068	1.0059455	+376	1.0059831
Dec. 19	0.64	50.1	D	12 38 2.610							
			B	13 39 30.870	3688.260	.216	3688.044	3667	1.0057388	+335	1.0057723
			B	14 42 54.668	3803.898	.104	3803.694	3782	57361	+335	696
			D	15 44 36.884	3702.216	.048	3702.168	3681	57506	+335	841
								(11130)			(1.0057752)
Dec. 19	1.00	50.0	B	15 51 9.164							
			B	16 53 14.595	3725.431	.162	3725.269	3704	1.0057421	+335	1.0057756
			D	17 55 43.088	3748.493	.072	3748.421	3727	57475	+335	810
			B	18 56 0.814	3617.726	.032	3617.694	3597	57531	+335	866
								(11028)			(1.0057813)
Dec. 22	0.38	49.7	B	11 44 24.745							
			B	12 7 34.722	1389.977	.088	1389.889	1382	1.0057084	+406	1.0057490
			B	14 3 40.572	6965.847	.248	6965.599	6926	57174	+406	580
			B	15 2 54.889	3554.320	.053	3554.267	3534	57349	+406	755
			D	17 7 56.626	7501.737	.051	7501.686	7459	57227	+406	633
			B	17 34 40.764	1604.136	.005	1604.131	1595	57249	+406	654
								(20896)			(1.0057628)
Dec. 22	0.81	49.5	D D	18 30 28.843							
			D D	19 2 12.749	1903.906	.041	1903.865	1893	1.0057390	+406	1.0057796
			D B	19 46 50.573	2677.823	.036	2677.787	2662½	57416	+406	822
								(4555½)			(1.0057813)
Dec. 23	0.46	49.0	D B	4 22 53.989							
			D	4 48 42.354	1548.365	.047	1548.318	1539½	1.0057278	+406	1.0057684
			D D	5 19 8.774	1826.420	.043	1826.377	1816	57142	+406	548
								(3355½)			(1.0057611)
Dec. 23	0.82	49.0	D	6 40 36.960							
			B	7 13 43.405	1986.445	.135	1986.310	1975	1.0057266	+406	1.0057672
			D	8 19 39.117	3955.712	.145	3955.567	3933	57379	+406	785
			D	9 47 1.082	5241.965	.076	5241.889	5212	57347	+406	753
			D	14 16 35.199	1774.116	.012	1774.104	1764	57279	+406	685
								(12884)			(1.0057741)
Dec. 23	30.55	49.6	D	10 37 55.017							
			B	11 10 22.653	1947.636	.037	1947.599	1935	1.0065111	+406	1.0065517
Dec. 23	30.55	48.9	D	11 14 36.834							
			B	11 48 2.889	2006.055	.043	2006.012	1993	1.0065289	+406	1.0065695

UNITED STATES GEODETIC SURVEY.—PENDULUM AT HOBOKEN.—SWINGS AT HIGH TEMPERATURE.

HEAVY END UP.

Date.	Pressure.	Temp. F.	Method.	Time of transit.			Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp. to 35° C.	Period, corrected.
1878.	*In.*	°		*h.*	*m.*	*s.*	*s.*								
Apr. 24	1. 24	....	D	11	52	17. 358									
		92. 5	B	12	5	25. 088	787. 730	. 042	787. 688	783	1. 0059872	+ 8	......	+100	1. 0059980
		92. 6	D	12	18	20. 755	775. 667	. 035	775. 632	771	60078	+ 8	......	+ 91	60177
		92. 0	D	12	30	31. 137	730. 382	. 028	730. 354	726	59972	+ 8	......	+128	60108
										(2280)					(1. 0060088)
Apr. 26	2. 25	....	B	11	3	13. 343									
		99. 3	B	11	24	0. 896	1247. 553	. 038	1247. 515	1240	1. 0060605	+ 8	......	−244	1. 0060369
		100. 8	D	11	53	54. 768	1793. 872	. 034	1793. 838	1783	60785	+ 8	......	−316	60477
		101. 5	B	12	24	6. 767	1811. 999	. 022	1811. 977	1801	60949	+ 8	......	−352	60605
										(4824)					(1. 0060497)
Apr. 24	29. 888	....	B	10	8	11. 240									
		89. 9	D	10	19	46. 835	695. 595	. 030	695. 565	691	1. 0066064	+ 8	−59	+178	1. 0066191
		90. 5	B	12	32	16. 830	749. 995	. 012	749. 983	745	66886	+ 8	−59	+157	66992
		91. 0	D	10	50	8. 937	1072. 107	. 006	1072. 101	1065	66696	+ 8	−59	+135	66780
										(2501)					(1. 0066670)
Apr. 26	29. 886	....	D	8	49	46. 929									
		96. 5	B	8	57	4. 888	437. 959	. 020	437. 939	435	1. 0067563	+ 8	−60	− 78	1. 0067459
		97. 0	D	9	5	31. 263	506. 375	. 011	506. 314	503	66885	+ 8	−60	−100	66733
		97. 8	D	9	25	41. 330	1210. 067	. 010	1210. 057	1202	67030	+ 8	−60	−135	66843
										(2140)					(1. 0066936)
Apr. 26	29. 890	....	D	9	28	27. 344									
		97. 8	D	9	33	9. 253	281. 909	. 018	281. 891	280	1. 0067536	+ 8	−60	−135	1. 0067349
		97. 9	D	9	42	12. 885	543. 632	. 016	543. 616	540	66963	+ 8	−60	−135	66776
		98. 6	B	10	5	51. 382	1418. 497	. 015	1418. 482	1409	67296	+ 8	−60	−164	67080
										(2229)					(1. 0067039)
Apr. 30	30. 000	....	D	9	4	59. 956									
		102. 8	B	9	12	1. 115	421. 859	. 019	421. 840	419	1. 0067780	− 9	−85	−328	1. 0067358
		102. 8	B	9	21	49. 060	587. 945	. 014	587. 931	584	67312	− 9	−85	−328	66890
		102. 7	D	9	44	53. 336	1384. 276	. 010	1384. 266	1375	67389	− 9	−85	−328	66967
										(2378)					(1. 0067017)
Apr. 30	30. 007	....	B	9	47	45. 072									
		102. 7	D	9	55	33. 220	468. 448	. 027	468. 121	465	1. 0067263	− 9	−87	−328	1. 0066839
		102. 5	D	10	5	27. 224	594. 004	. 016	593. 988	590	67593	− 9	−87	−320	67177
		102. 3	B	10	26	40. 780	1273. 556	. 011	1273. 545	1265	67549	− 9	−87	−313	67140
										(2320)					(1. 0067060)
May 2	29. 897	....	D	11	05	33. 347									
		95. 2	D	11	13	58. 729	505. 382	. 025	505. 357	502	1. 0066873	−45	−61	− 28	1. 0066739
		95. 4	D	11	28	44. 682	885. 953	. 016	885. 937	880	67466	−45	−61	− 36	67324
		95. 6	B	11	46	26. 759	1062. 077	. 006	1062. 071	1055	67024	−45	−61	− 43	66875
										(2437)					(1. 0067007)
May 2	29. 892	....	D	11	49	35. 047									
		98. 0	D	11	55	31. 417	356. 370	. 019	356. 351	354	1. 0066412	−45	−60	− 64	1. 0066243
		96. 3	D	12	03	26. 619	475. 202	. 013	475. 189	472	67563	−45	−60	− 71	67387
		96. 7	D	12	22	53. 429	1166. 810	. 012	1166. 798	1159	67282	−45	−60	− 85	67092
										(1985)					(1. 0067011)
May 10	29. 899	....	D	10	11	18. 725									
		91. 1	D	10	17	37. 254	378. 529	. 017	378. 512	376	1. 0066809	+28	−62	+135	1. 0066910
		91. 5	D	10	27	11. 098	573. 844	. 013	573. 831	570	67211	+28	−62	+114	67291
		92. 3	D	10	51	59. 023	1487. 925	. 011	1487. 914	1478	67077	+28	−62	+ 85	67128
										(2424)					(1. 0067132)
May 10	29. 892	....	B	10	55	38. 701									
		92. 5	D	11	3	12. 735	454. 034	. 022	454. 012	451	1. 0066785	+28	−60	+ 78	1. 0066831
		92. 5	D	11	13	22. 820	610. 085	. 014	610. 071	606	67178	+28	−60	+ 78	67224
		92. 8	D	11	33	30. 913	1208. 093	. 010	1208. 083	1200	67358	+28	−60	+ 64	67390
										(2257)					(1. 0067734)
May 11	29. 968	....	B	9	42	29. 411									
		93. 7	D	9	48	56. 995	387. 584	. 016	387. 568	385	1. 0066701	+28	−78	+ 28	1. 0066679
		94. 0	D	9	58	28. 845	631. 850	. 012	571. 838	568	67570	+28	−78	+ 21	67541
		94. 4	B	10	21	35. 106	1386. 260	. 010	1386. 250	1377	67175	+28	−78	0	67125
										(2330)					(1. 0067153)
	29. 980	....	D	10	25	16. 895									
		94. 4	B	10	32	48. 935	452. 040	. 024	452. 016	449	1. 0067171	+28	−81	0	1. 0067118
		94. 3	B	10	42	40. 894	591. 959	. 012	591. 947	588	67126	+28	−81	+ 7	67080
		94. 0	D	11	10	49. 181	1688. 287	. 012	1688. 575	1677	67233	+28	−81	+ 21	67201
										(2714)					(1. 0067161)

PENDULUM AT HOBOKEN.—SWINGS AT HIGH TEMPERATURE—Continued.

HEAVY END DOWN.

Date.	Pressure.	Temp. F.	Method.	Time of transit.			Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp. to 35° C.	Period, corrected.
1878.	*In.*	°		*h.*	*m.*	*s.*	*s.*								
May 4	29.794	....	D	9	19	15.340									
		96.0	B	9	31	37.201	741.861	.041	741.820	737	1.0065400	−45	−16	− 75	1.0065264
		96.8	D	9	54	29.156	1371.955	.039	1371.916	1363	65414	−45	−16	−108	65245
		97.3	D	10	59	34.625	3905.469	.031	3905.438	3880	65563	−45	−16	−133	65369
										(5980)					(1.0065327)
May 5	29.868	....	B	9	13	26.830									
		94.0	D	9	26	12.843	766.013	.038	765.975	761	1.0065375	−45	−24	+ 25	1.0065329
		94.2	B	9	49	22.897	1390.054	.036	1390.018	1381	65301	−45	−24	+ 8	65240
		94.8	D	10	54	11.255	3888.358	.028	3888.330	3863	65571	−45	−24	− 17	65485
										(6005)					(1.0065409)
May 6	30.027	....	D	11	05	53.067									
		95.1	D	11	15	40.900	587.833	.026	587.807	584	1.0065188	+54	−40	− 33	1.0065169
		95.1	B	11	33	32.896	1071.996	.028	1071.968	1065	65427	+54	−40	− 33	65408
		95.1	B	12	30	31.132	3418.236	.030	3418.206	3396	65389	+54	−40	− 33	65370
										(5045)					(1.0065356)
May 8	29.843	....	B	9	57	19.379									
		96.5	D	10	9	35.213	735.834	.042	735.792	731	1.0065554	−59	−21	− 91	1.0065383
		96.1	D	10	30	23.375	1248.162	.036	1248.126	1240	65533	−59	−21	− 75	65378
		94.0	B	11	23	35.137	3191.762	.034	3191.728	3171	65367	−59	−21	+ 25	65312
										(5142)					(1.0065338)

NOTE.—For addendum to this appendix see end of volume.