A Quincuncial Projection of the Sphere.

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For meteorological, magnetological and other purposes, it is convenient to have a projection of the sphere which shall show the connection of all parts of the surface. This is done by the one shown in the plate. It is an orthomorphic or conform projection formed by transforming the stereographic projection, with a pole at infinity, by means of an elliptic function. For that purpose, l being the latitude, and θ the longitude, we put

$$\cos^2 \phi = \frac{\sqrt{1 - \cos^2 l \cos^2 \theta} - \sin l}{1 + \sqrt{1 - \cos^2 l \cos^2 \theta}},$$

and then $\frac{1}{2} F\phi$ is the value of one of the rectangular coördinates of the point on the new projection. This is the same as taking

$$\cos am\,(x + y\sqrt{-1})\,(\text{angle of mod.} = 45^\circ) = \tan \frac{p}{2}\,(\cos\theta + \sin\theta\sqrt{-1}),$$

where x and y are the coördinates on the new projection, p is the north polar distance. A table of these coördinates is subjoined.

Upon an orthomorphic potential the parallels represent equipotential or level lines for the logarithmic projection, while the meridians are the lines of force. Consequently we may draw these lines by the method used by Maxwell in his Electricity and Magnetism for drawing the corresponding lines for the Newtonian potential. That is to say, let two such projections be drawn upon the same sheet, so that upon both are shown the same meridians at equal angular distances, and the same parallels at such distances that the ratio of successive values of $\tan \frac{p}{2}$ is constant. Then, number the meridians and also the parallels. Then draw curves through the intersections of meridians with meridians, the sums of numbers of the intersecting meridians being constant on any one curve. Also, do the same thing for the parallels. Then these curves will represent the meridians and parallels of a new projection having north poles and south poles wherever the component projections had such poles.

Functions may, of course, be classified according to the pattern of the projection produced by such a transformation of the stereographic projection with a pole at the tangent points. Thus we shall have—

1. Functions with a finite number of zeros and infinites (algebraic functions).
2. Striped functions (trigonometric functions). In these the stripes may be equal, or may vary progressively, or periodically. The stripes may be simple, or themselves compounded of stripes. Thus, sin (a sin z) will be composed of stripes each consisting of a bundle of parallel stripes (infinite in number) folded over onto itself.
3. Chequered functions (elliptic functions).
4. Functions whose patterns are central or spiral.

I. *Table of Rectangular Coördinates for Construction of the "Quincuncial Projection."*

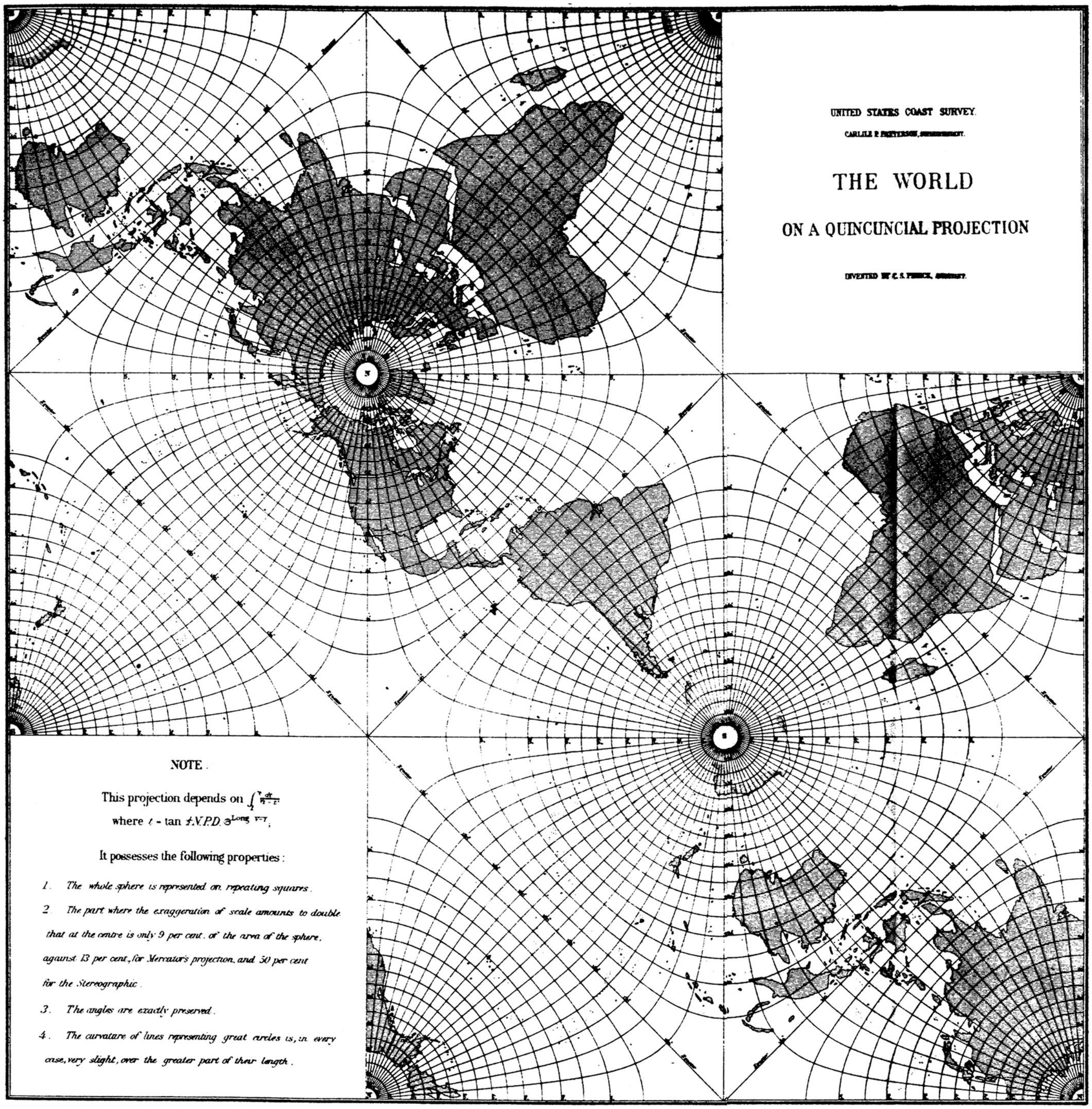
x (for longitudes in upper line). y (for longitudes in lower line.)

LAT.	0° 90	5° 85	10° 80	15° 75	20° 70	25° 65	30° 60	35° 55	40° 50	45° 45	50° 40	55° 35	60° 30	65° 25	70° 20	75° 15	80° 10	85° 5	LAT.
85°	.033	.033	.033	.032	.031	.030	.029	.027	.025	.024	.021	.019	.017	.014	.011	.009	.006	.003	85°
80	.067	.066	.066	.064	.063	.061	.058	.055	.051	.047	.043	.038	.033	.028	.023	.017	.012	.006	80
75	.100	.100	.099	.097	.094	.091	.087	.082	.077	.071	.065	.058	.050	.042	.034	.026	.017	.009	75
70	.135	.134	.133	.130	.127	.122	.117	.110	.103	.095	.087	.077	.067	.057	.046	.035	.023	.012	70
65	.169	.169	.167	.163	.159	.154	.147	.139	.130	.120	.109	.097	.085	.072	.058	.044	.029	.015	65
60	.205	.204	.201	.198	.192	.185	.177	.168	.157	.145	.131	.117	.102	.086	.070	.053	.036	.018	60
55	.241	.240	.237	.232	.226	.218	.208	.197	.184	.170	.154	.138	.120	.102	.082	.062	.042	.021	55
50	.278	.277	.274	.269	.261	.251	.240	.227	.212	.196	.178	.159	.139	.117	.095	.072	.048	.024	50
45	.317	.316	.312	.306	.297	.286	.273	.258	.241	.223	.202	.181	.158	.134	.109	.083	.055	.028	45
40	.357	.356	.351	.344	.334	.321	.307	.290	.270	.250	.228	.204	.179	.151	.123	.094	.063	.032	40
35	.400	.398	.393	.384	.373	.358	.341	.322	.301	.279	.254	.228	.200	.170	.139	.106	.071	.036	35
30	.446	.443	.437	.427	.413	.396	.377	.356	.332	.308	.281	.253	.222	.190	.155	.119	.081	.041	30
25	.495	.492	.484	.471	.455	.435	.414	.391	.365	.338	.309	.279	.246	.211	.174	.134	.091	.046	25
20	.548	.545	.534	.518	.498	.476	.452	.426	.398	.369	.339	.307	.272	.235	.195	.151	.104	.053	20
15	.609	.604	.589	.568	.544	.517	.490	.461	.432	.401	.369	.336	.300	.262	.219	.173	.121	.062	15
10	.681	.672	.649	.620	.590	.559	.528	.497	.466	.434	.401	.367	.330	.291	.248	.200	.143	.076	10
5	.775	.752	.713	.673	.635	.600	.566	.532	.500	.467	.433	.399	.363	.324	.282	.234	.177	.102	5
0	1.000	.841	.774	.723	.679	.639	.602	.567	.533	.500	.467	.433	.398	.361	.321	.277	.226	.159	0

II. *Preceding Table Enlarged for the Spaces Surrounding Infinite Points.*

x (for longitudes in upper line). y (for longitudes in lower line).

LAT.	0°	1°	2°	3°	4°	5°	6°	8°	10°	12½°	15°		75°	77½°	80°	82°	84°	85°	86°	87°	88°	89°	LAT.
	90	89	88	87	86	85	84	82	80	77½	75		15	12½	10	8	6	5	4	3	2	1	
15°	.609	.609	.608	.607	.606	.604	.602	.596	.589	.579	.568		.173	.147	.121	.098	.074	.062	.050	.038	.025	.013	15°
12½	.643	.643	.642	.641	.639	.636	.634	.627	.618	.606	.594		.185	.159	.131	.107	.082	.069	.055	.042	.028	.014	12½
10	.681	.681	.680	.678	.675	.672	.668	.659	.649	.635	.620		.200	.173	.143	.118	.091	.076	.062	.047	.031	.016	10
8	.715	.714	.713	.710	.706	.702	.697	.686	.674	.658	.641		.213	.185	.155	.129	.100	.085	.069	.052	.035	.018	8
6	.753	.752	.750	.746	.741	.735	.728	.714	.700	.681	.662		.227	.199	.169	.142	.112	.095	.078	.060	.040	.020	6
5	.775	.774	.770	.765	.759	.752	.745	.729	.713	.692	.673		.234	.207	.177	.150	.119	.102	.084	.065	.044	.022	5
4	.798	.797	.793	.786	.779	.770	.761	.743	.725	.704	.683		.242	.215	.185	.158	.128	.110	.092	.071	.049	.025	4
3	.825	.823	.817	.808	.798	.788	.778	.757	.738	.715	.693		.250	.224	.194	.168	.137	.120	.101	.079	.055	.029	3
2	.857	.853	.843	.831	.819	.806	.794	.772	.750	.726	.703		.259	.233	.204	.178	.148	.131	.112	.090	.065	.035	2
1	.899	.889	.872	.854	.839	.824	.810	.785	.763	.737	.713		.268	.243	.215	.190	.161	.144	.126	.105	.079	.046	1
0	1.000	.929	.899	.877	.857	.841	.825	.798	.774	.747	.723		.277	.253	.226	.202	.175	.159	.143	.123	.101	.071	0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THE WORLD
ON A QUINCUNCIAL PROJECTION
NOTE.
This projection depends on $\int \frac{dt}{\sqrt{1-t^4}}$
where $t = \tan \tfrac{1}{2} N.P.D.\, e^{\text{Long} \sqrt{-1}}$;
It possesses the following properties:
1. The whole sphere is represented on repeating squares.
2. The part where the exaggeration of scale amounts to double that at the centre is only 9 per cent. of the area of the sphere, against 13 per cent., for Mercator's projection, and 50 per cent for the Stereographic.
3. The angles are exactly preserved.
4. The curvature of lines representing great circles is, in every case, very slight, over the greater part of their length.