civilians divide it into *movable* and *immovable*, the latter embracing land and its incidents. See French *Code Civil*, Liv. II. tit. 1.

The air and the high seas are not the subject of ownership, although an act obstructing the right of another to their common use may be an actionable wrong.   (S.E.B.)

Prophecy (in theology) [Gr. προφητεία, the gift of interpreting the will of the gods]: Ger. *Prophezeiung*; Fr. *prophétie*; Ital. *profezia*. Literally, prediction of future events; but in a larger sense, the organ through which a progressive revelation of divine truth is effected by means of inspired human agents called prophets.

In this larger sense prophecy is the human complement of inspiration. Through inspiration the prophet is informed with his revelatory message. The prediction of future events may or may not be a feature of prophecy in the more fundamental sense. The term prognosis is sometimes used as in general equivalent to prophecy.

Literature: KUENEN, The Prophets and Prophecy in Israel (London, 1877); EDERSHEIM, Prophecy and Hist. in relation to the Messiah (London, 1885); WITSIUS, De Prophetis et Prophetia, Miscellan. Sacer, tom. i; THOLUCK, Die Propheten u. ihre Weissagungen (2nd ed., 1860); W. H. GREEN, Moses and the Prophets (N.Y., 1883); BRIGGS, Messianic Prophecy (N.Y. and Edinb., 1886).   (A.T.O.)

Prophet: see PROPHECY.

Propitiation [Lat. *propitiatio*, from *propitiare*, to appease]: Ger. *Versöhnung*; Fr. *propitiation*; Ital. *propiziazione*. The act of placating a divinity who is supposed to be in a state of displeasure with the one who performs the propitiatory action. In Christian theology, that aspect of the Atonement by virtue of which it appeases the judicial wrath of God against the sinner.

Propitiation is to be distinguished from expiation, which is the suffering through which the propitiation is effected. Propitiation has direct reference to the divine wrath through which it is appeased. Expiation is more directly related to guilt. By expiating the guilt of sin the divine wrath is propitiated.

Literature: see ATONEMENT, JUSTIFICATION, and IMPUTATION.   (A.T.O.)

Proportion [Lat. *pro* + *portio*, a part, a share]: Ger. *Proportion*; Fr. *proportion*; Ital. *proporzione*. (1) The aesthetically agreeable or harmonious relation to the whole of the unequal parts of any object.

Distinguished in this regard from symmetry, which refers to the relation of like and equal parts, with emphasis primarily upon their relations to one another rather than upon their relation to the whole. Like symmetry, proportion is distinguished from harmony by reference to quantitative relations, whereas harmony refers rather to qualitative relations.

(2) Applied also to the relations of objects in their entirety to some norm, as illustrated by the expression 'well proportioned.'

The term proportion is applied, aside from objects of nature, most appropriately to sculpture, architecture, and painting as involving drawing; secondarily and more metaphorically to music, poetry, the drama, and the novel. It is sometimes used loosely and untechnically as implying mere adaptation, and occasionally even as synonymous with symmetry.

The history of proportion as an aesthetic category has been essentially identical with that of harmony and symmetry. Zeising and Köstlin afford illustrations of recent modes of analytical treatment of proportion. Cf. HARMONY, SYMMETRY, GOLDEN SECTION, and BALANCE.

Literature: KÖSTLIN, Aesthetik (1869); ZEISING, Aesthetische Forschungen (1855); DAY, The Sci. of Aesthetics (1893).   (J.R.A.)

Proposition [Lat. *propositum*, from *proponere*, to place before]: Ger. *Satz*; Fr. *proposition*; Ital. *proposizione*. A JUDGMENT (q. v.) expressed in words.

The term judgment denotes an 'axiomatic concept' (Zindler, *Sitzber. Akad. Wiss. Wien*, 118, ix. 32) which it is difficult, if not impossible, to define; but for the purpose of logic a judgment may be said to be an association of experiences or elements of experiences which has been made an object of reflection by a conscious mind, and whose validity has been accepted by it. By an association is meant not merely a coexistence or a sequence, but an association of any complicated kind whatever, or any more or less explicit analysis of a mental whole into parts (cf. JUDGMENT). Thus *A conquers B* means that *A* is associated with *B* in the relation of conqueror, or that *B* is apprehended as *B* as conquered by *A*. Further, by an experience is meant any object of consciousness whatever, or any association of objects of consciousness; by the validity of the connection is meant merely its reality, or its occurrence in whatever universe, real or fictional or imagined, happens to be the background of the subject under discussion.   (C.L.F.–J.M.B.)

Cf. the view of proposition given under

SUBJECT (in logic, 2), which is in certain respects different from what follows.

(C.S.P.—J.M.B.)

I. *The Import of Propositions.* It follows from the definition of the proposition that it must consist of at least three different members, two terms (between which the relation is said to hold) and another word whose function is to express at once the nature of the connection between them and the asseveration of that connection. (This double force of the copula is adverted to by Bradley, *Princ. of Logic*, 22.) In *Armies conquer countries*, we may think of *armies* and *countries* as the objects of consciousness, and of *conquer* as specifying the nature of the relation and at the same time asserting that it holds. But such a proposition as *A conquers B* can, if there is any occasion for it, be broken up differently, viz. into *A is one-of-the-conquerors-of-B*. Whether *B* or one-of-the-conquerors-of-*B* be regarded as the second of the related objects of consciousness is merely a matter of convenience, and will be determined in any actual case by considering whether other propositions, which it is desired to combine with this as data towards conclusions, have *B* or one-of-the-conquerors-of-*B* among their terms. Or, again, we can always decide which is for the moment the way in which we are regarding the proposition by considering whether in its inverted form it is the statement *B is conquered by A* or *One of those who conquer B is A* which interests us. To discover the three elements involved in *A runs*, we have, again, simply to invert it, *One who runs is A*. And the fact that there is no proposition which cannot be expressed in an exactly equivalent inverted form proves that this analysis of the proposition into two terms and a copulative connecting link is justified.

But there is one particular relation that we have by far the most frequently to deal with in reasoning—the relation of *b* invariably following upon *a*, or of *a* as the sufficient antecedent of *b*. This relation is variously expressed in words—*a* is-followed-by-*b*, *a* implies *b*, *a* is-indicative-of *b*, *a* is-a-sufficient-condition-of *b*, If *a* then *b*, The objects *a* are-included-among the objects *b*, or All *a* is *b* (where *a* and *b* may themselves be propositions, instead of simple terms, without altering the essential character, for logic, of the relation). In order to hold this relation present in consciousness in its purely abstract form, freed from all those variations of language which, rich in meaning though they may be, are entirely inessential to the purposes of logic, it is absolutely necessary to represent it by some symbol. Formal logic, as ordinarily treated in the books, is only semi-formal. It has been agreed, since the time of the earliest writers upon the subject, to allow terms to enter into propositions shorn of the special implications which follow upon their different meanings, and to represent them by colourless letters of the alphabet; it is only carrying this admirable device for abstracting from the inessential a little further if we represent the simple copula of *All a is b* by some symbol. We shall make use of the form $<$, a modification of that suggested by Peirce, for this purpose, and we shall write $a < b$ for any one of the copulative relations which have just been variously put into words. De Morgan regards this relation as sufficiently characterized by the fact that it is transitive, but that is a statement that needs modification. We shall then have for the formal representation of *Not all a is b* (corresponding to the plan of indicating *what is not a* by $\bar{a}$), the same sign with a horizontal mark indicating negation over it, as $a \nleq b$. It will also add greatly to facility of expression if we write ∞ and o for the SPECIAL TERMS (q. v.) of logic, *everything* and *nothing* (or what exists and what does not exist). Innovations are difficult to make, and there was long and strenuous opposition to the introduction of the special quantity o into arithmetic and algebra; but it seems that the time has come when these simple aids towards extracting the essential from the accidental in logic should be used. Cf. SYMBOLIC LOGIC, ad fin., and TERM (negative).

This view, that the import of the proposition is to affirm some sort of connection between two objects of consciousness, dates from Aristotle. A favourite view of recent years is to maintain that in the simple judgment, *A is B*, there is both an analysis and a synthesis—that *A as being B* is given first, as an integral element of consciousness, and that the work of forming the judgment consists in first separating the concepts and then reuniting them by means of the connecting copula (cf. JUDGMENT). This is doubtless a correct account of the manner of forming immediate judgments, but it is not correct as a description of propositions. The examples of the proposition which are usually studied by the logicians are so cut and dried that it is difficult to detect its real essence; it is necessary to consider it in the process of being

formed, to see what it is really like. The act of the mind in the presence of such a situation as occurs in the following incident may be taken as the typical instance of the judgment : 'A prelate said, " My first penitent was a murderer"; a distinguished nobleman entered the room at that moment, and, after greetings, remarked to the assembled company, "You may not know that I was His Eminence's first penitent."' In the presence of two premises like this, we do not first get the nobleman and the murderer fused together in a mental content and then separate them in order to reunite them ; but the two premises being held before the mind as a mental whole (that is, it being recognized that they hold good at one and the same time and place), an instinctive excision of the common element takes place, and the hearer finds himself forced to accept the hitherto unsuspected relation 'This nobleman is, then, a murderer !' (that is, the nobleman and murderer are, in this instance, one and the same object). It is the two premises—and, after they have been in part restated in the conclusion, it is this proposition—which brings about for the mind of the reasoner the conviction that the two descriptions appertain to one and the same object ; that is, there is produced in his mind the complex conception 'this nobleman as being a murderer.' Expressed logically, it is through the compelling force upon the human mind of the fundamental principle of the syllogism (Sigwart), or it is, in psychological terms, through the irresistible impulse of the mind for *putting this and that together* when a common element (in this case 'first penitent') leads to their inclusion in a larger whole, that this proposition comes into consciousness. It is only after the two premises and the conclusion which they involve have effected the junction, for the hearer, of nobleman and murderer that the 'immanence' for consciousness of the one in the other exists, in a way which may then be explicitly declared in words (proposition). (C.L.F.)

The difference of view on this subject is probably to be accounted for by the distinction between judgment and proposition, especially when that difference is accounted for genetically by the 'communicative' or 'declarative' function of language. The normal psychological process seems to be the formation of judgment by the acceptance of an enlarged (synthetic) whole of mental content, and then the rise of proposition, in words, by analysis and for communication ; this in the mind of the speaker. But in the mind of the *hearer*, to whom the relation expressed in the proposition is new, this procedure is apparently reversed : two more or less familiar terms are given to him in language, joined in a certain relation ; and he, by accepting this relation, forms a new judgment. The logician, if he restrict himself to the point of view of the hearer, should deal strictly with propositions and their communication from one mind to another, holding that they are always synthetic ; the psychologist, approaching the proposition from the point of view of mental process and meaning, finds that the proposition is always the analytic issue of an earlier judgment. Even when the hearer *hears* the proposition 'a is b,' it is not *his* proposition until he has gained the judgment ab and recognized the relation of the parts a and b to each other or to the whole. (J.M.B., C.L.F., G.F.S.)

The nature of the relation between terms which is expressed by the simple copula of *All a is b*, $a < b$, has been variously taken to be (1) the inclusion of the group of objects represented by a among the group of objects represented by b ; (2) the implication of the sensations or ideas b by the sensations or ideas a (Mill); or (3) the attachment to the group of objects a of the qualities involved in the meaning of b. (The purpose of any given asseveration is either to make an addition to the groups of objects already known to have the quality b, or else to add to the qualities already known to be possessed by the objects a ; we can indicate by the emphasis whether we mean, for instance, '*Man* also is a primate,' or 'Man is a *primate* also.' So Venn and Baldwin.)

The reason that so many different views on this matter are possible would seem to be very simple : every term is a double-edged machine—it effects the separating out of a certain group of objects, and it epitomizes a certain complex of marks. It follows that no one of the above accounts of the nature of the proposition is complete ; any statement involves in full a fourfold implication. Whoever says, for instance, that 'All politicians are statesmen' must be prepared to maintain that the objects politicians are the same as some of the objects statesmen, and are in the possession of all of the qualities of statesmen ; and also that the quality-complex politician entails the quality-complex statesman, and is indicative of the presence of some of the objects statesmen. It is open to the psychologist to show that in any given instance one

or the other of these several meanings is what the speaker has most prominently in his mind, and we are able to put the abstract copula of $a < b$ into such a form of words as shall accentuate now one and now another of them (as, The a's *are included among* the b's, Being a *entails* being b); but from any one all the others can immediately be inferred, and hence, for the logician, the full content of the proposition involves them all. In dealing with this subject, logicians have been in the habit (as Keynes remarks) of committing the fallacy of exclusiveness, that is, of denying that one side of the shield is silver, on the ground that one side of the shield is gold.

The same doctrine is applicable to the compound proposition. *If a is b, c is d*, means that the instances of *a* being *b* are included *among* the instances of *c* being *d*, and that the truth of *a is b entails* the truth of *c is d*. The two different meanings are severally made prominent in *Whenever a is b, c is d* and *If a is b, c is d*; the former is more apt to be of empirical origin and the latter to be derived from other propositions, but either follows from the other, and hence, for logic, each is equivalent to the other. Writers on symbolic logic sometimes (Schröder, Venn) develop the subject in terms of class-inclusion only (McColl in terms of implication only), and seem to think that some necessary connection is herein involved. But in doing this they have alienated the logicians of the regular school, not unnaturally, and they have introduced perfectly needless restrictions; $ab < o$ means not simply that the class ab does not exist, but also that a's which are b, b's which are a, and *things which are* both a and b are all non-existent. It is true that to use the *language* of one or the other of the several meanings of the proposition is almost unavoidable, but the class-meaning and the attribute-meaning carries each the other with it; they are, like the gold and the silver sides of the shield, inextricably welded together. Herein appears the great advantage to be gained by the free use of the generalized copula $a < b$, which is *defined* as indicating explicitly all four of the implications; there is no form of *words* which does not seem to commit us more or less to one or the other of them, to the exclusion of the rest.

The metaphysician is able to think away all reference to an objective world, and to regard his experiences as merely ordered sequences among the elements and combinations of elements of his own consciousness. But the hypothesis that there is an objective reality standing in a one-to-one correspondence to this play of elements of consciousness is deeply ingrained in the human mind, and it is inextricably involved in our forms of utterance. Every term used to mark out an element of consciousness and to aid in conveying an intelligible statement to the listener posits the application to certain portions of reality (even the subjective world of the utterer is part of the objective world of the receiver), as well as giving him marks by which to recognize them. This reference to reality is implicitly present in every proposition; in some propositions it is present explicitly. In *Everything is material, All is vanity, Whatever is, is right, Everything is toil and trouble, Alles ist Gefühl*, the subject of the proposition is the whole of the universe, whatever it may be which is the subject of discourse. But any proposition can be immediately thrown into the form in which this reality reference is explicit. *All a is b* is the same thing as *Everything is $\bar{a}$ or b*, or $(a < b) = (\infty < \bar{a} + b)$ (where the symbol + is used for the word *or*). This fact has led some writers to define the proposition as a 'description of reality,' and to say that reality is the 'real subject' in every proposition. This leads us to the consideration of the distinction between subject and predicate (for in the equivalence just stated, a, which was subject, has become an element of the predicate). When, e. g., A and B are alike individuals, what is the difference between A *is* B and B *is* A? A proposition, in its living form, is something which is set up—proposed—by the utterer for the acceptance of the receiver; the former throws into his subject all that he knows the latter is already willing to grant him, and to this he adds in the predicate what constitutes the new information to be conveyed by the sentence. The difference becomes patent in such sentences as these: *The large round red-covered table at which he is writing is dusty; The large round dusty table at which he is writing is covered with red*. The utterer chooses the first of these sentences when he knows that the receiver has noticed all the other determinants of the table—that to tell him of them would be throwing time away—but that he has not yet noticed that the table is dusty. All the part of the sentence before the *is* is thrown in to enable the receiver to pick out with certainty the object which is referred to; what

comes after the *is* is the matter freshly added. In the second sentence it is the knowledge that the object marked out by all the words of the subject is covered with red which is about to be introduced into the receiver's mind. In *A is B* we say, 'I know that you know who *A* is, perhaps you don't also know that he is the same person as *B*.' Now, the sentence $\infty \lessdot \bar{a} + b$, *Everything is non-a or b*, is correctly called a description of reality; but in the form *Everything which is a is b*, that is (to put in explicitly the implicit reference to reality), $\infty\, a \lessdot b$, it is not ∞, or the actual, which is being described, but only a limited portion of it, namely, so much of it as is *a*. It is, therefore, not correct to say that the proposition is a description of reality; it is such only when it is in a certain form. A proposition is a 'description' of whatever may chance to be its subject. It would be just as correct, and no more so, to say that every universal proposition *is* an affirmation of non-existence, because it can be thrown into the form $a\,\bar{b} \lessdot 0$. It is that, but it is something more. Bosanquet says that the real subject of discourse in any proposition is 'outside' the proposition; it would be more correct to say that only part of that outside term is the subject, except after the proposition has been thrown explicitly into the form $\infty \lessdot \bar{a} + b$. There is no one form of a series of equivalent proposition which gives their 'real' meaning; their real meaning can only be got by considering all their forms. (See below, pp. 368–9.)

II. *Kinds of Propositions.* Propositions are divided as regards the source of their validity into (1) empirical, (2) immediate, (3) derived.

Empirical propositions are the results of valid inductions. Immediate propositions are of several different kinds: (*a*) axioms, (*b*) postulates, (*c*) definitions, (*d*) verbal propositions, (*e*) propositions of immediate perception. (*a*) Axiomatic propositions are probably also empirical in the last analysis, but they are the result of such early experience, and they have become so deeply embedded in all our knowledge, that they are practically indistinguishable from what they would be if they were innate. (*b*) Only general propositions are properly called axioms; particular propositions, affirming existence, are postulates, as—what Poincaré calls a fundamental postulate of mechanics—something is constant, or the fundamental postulate of logic, something exists, or the postulate of geometry, figures can be superimposed. (*c*) With de-

finitions or definitive propositions belong also any general assumptions temporarily taken for granted within a given argument. (*d*) Verbal propositions merely set forth in the form of explicit copula and predicate what has been already taken for granted in the meaning of the subject. 'What is taken for granted' is, of course, a term relative to the intelligence and knowledge of the person addressed, but it must always include at least the indispensable signification of the subject, that is, so much as is involved in its definition. (These propositions are also called —by Kant—analytic, but a less dignified name is preferable for propositions whose nature it is to be trivial.) Both 'real' and 'verbal' statements are *non-formal*; that is, the distinction is one that cannot be made until it is known what *a* and *b* stand for in $a \lessdot b$. Besides these there is a different sort still, which is valid (or not), quite independently of the meaning of the terms, as *a is always a*; *That a is b is the same thing as that non-b is non-a*. (*e*) These are usually singular propositions, as 'I feel cold at this moment,' and are not of frequent use in arguments. The essential characteristic of a proposition (1) is that it can be proved, but only by induction, which is more or less uncertain, and liable to overthrow as knowledge advances; a proposition (3) can be derived deductively from (1) or (2) or both together, and its degree of validity depends upon that of its premises; a proposition (2) cannot be proved at all. The axioms are frequently regarded as being necessary; they are certainly essential to our continuing to think without being overcome by mental dizziness and nausea. Sigwart regards all derived propositions as necessary (*Logik*, 2nd ed., 210), but those derived purely or in part from empirical propositions are surely not so—nothing can have greater validity than its source.

Propositions are simple, complex, or compound. A simple proposition is one in which the grammatical subject and the grammatical predicate are regarded each as a single logical term. In a complex proposition the subject or predicate, or both, are broken up, in the course of the argument, into separate elements, as when 'The undevout astronomer is mad' is transformed into 'Any astronomer is either devout or mad.' A compound proposition is a statement in which one, at least, of the terms is itself a proposition: as, *Never do mortals sin that angels do not weep*; *That some a is b and not any a is b should both be*

true is impossible. The latter is a doubly compound proposition.

III. *The Existence of Terms.* Do universal propositions imply the existence of their subjects? From *All a is b,* are we safe in concluding that *a's* exist? The answer to this question is that in the statements of real life there is no general rule. For the most part we should regard it as waste of time to speak much about things which do not exist, yet we can say *All disobedience is punished* without in the least asserting that disobedience ever occurs. But in formal logic, where terms have become *a* and *b* and we know nothing about the meanings of our concepts, it is necessary to adopt some fixed convention in this matter; if any implications of this sort are made by propositions in general, we must know exactly what they are and be able to state them explicitly. The convention which many logicians accept is this: *Some a is b,* since it affirms the existence of *a* which is *b,* must be taken as implying the separate existence of both *a* and *b.* But in the case of *No a is b,* there is no difficulty whatever in admitting that one way in which it may become a valid statement is by our knowing that *a* or *b* (one or the other) does not exist at all. Moreover, it is indispensable that we should have in logic propositions that are the exact denials of each other; and hence if *Some a is b* is taken as meaning *Some a is b, and a and b both exist,* we must mean, in full, by *No a is b* that *No a is b or else a or b is non-existent.* It follows that *No a is b* cannot be taken as asserting the existence of either *a* or *b.* It is, however, an error to say that it makes no implication of existence; if there is no *a* which is *b,* then everything must be either non-*a* or else non-*b* (unless we are taking account of that imaginary universe in which nothing exists), and hence certainly either non-*a* or else non-*b* must exist. So in the proposition *All that is non-a is b,* we do not assert the existence of non-*a,* it is true; but the proposition is exactly equivalent to *All but a is b,* and this certainly affirms that *a* or *b,* one or the other, exists. On the other hand, the immediate denial of this last, *Not all but a is b,* though a particular proposition, makes no affirmation in regard to the terms that explicitly enter it; but it is equivalent to *Some non-a is non-b,* and hence it does affirm, by a necessary implication, the existence of both non-*a* and non-*b.* Hence the rule that is sometimes stated—particular

propositions imply the existence of their terms, universal propositions make no implication in regard to the existence of their terms—is not correct. If it is said, for instance, that all is vanity, things that are vanity are certainly affirmed to exist (if anything exists). If the rule is limited to subjects—i. e. that particulars imply, while universals do not imply, the existence of their subjects—then it is true except in the case of two of the particular propositions of the complete scheme, *Not all but a is b* and *Some besides a is b;* but it does not give us all the information in regard to existence that we have the right to demand of the logician. The complete rule is this: *Express every universal proposition in the equivalent form, All but x is y, and every particular proposition in the equivalent form, Some x is y; then the particular proposition affirms the existence of both x and y, and the universal proposition affirms the existence of either x or y.* With this convention it is to be noted that *Some a is b* does not follow from *All a is b,* except with the aid of the explicitly stated minor premise, *There is some a.* [In most cases, however, the existence is not 'asserted' explicitly, but rather taken for granted. It should be added, also, that on another view, there is absolutely no difference between universal and particular propositions in the matter of their reference to existence; and much may be said for such a view.—J.M.B.]

A large amount of bad reasoning has been expended upon the question of the existence of terms, mostly due, of course, to the non-comprehension of what those who uphold the above doctrine mean by existence. The word is unfortunately chosen, for it has unavoidable metaphysical and psychological implications which invite confusion; existence in the sense of being something that we are capable of thinking about must of course belong to every term that is an element of a comprehensible statement, but that is not the sort of existence that the logician has in mind. It would be better to substitute for it the word *occurrence,* meaning occurrence within that field of thought which the speaker is talking about (see UNIVERSE OF DISCOURSE); when we say *Nothing has happened,* we do not mean to assert that nothing has happened within the planet Mars, nor that nothing has happened among the microbes. The word occurrence has the additional reason for its use that nothing else is possible in speaking of compound propositions (to which the above

doctrine is wholly applicable). When we deny *If a is b, c is d*, we affirm the (actual or possible) occurrence of *a is b* in conjunction with *c is not d*, and hence, all the more, the (actual or possible) occurrence of each in itself; but when we affirm it, we do not affirm that *a is b* ever occurs, but only that what is sure to occur is the *alternation* of either *a is not b* or *c is d*. There is an admirable and exhaustive discussion of this topic in Keynes' *Formal Logic*, chap. vii, Part i, 3rd ed., which ought to render further misconception impossible.

The correct doctrine in regard to subjects of propositions, though often overlooked, is of early date. Petrus Hispanus says (see Prantl, *Gesch. d. Logik*, iii. 61) that *omnis* in the major premise of a syllogism does not imply the actual existence of the objects which it sums up. It should be added that when we know the meaning of our terms and the sources of our propositions, we often have more knowledge than this about existence; if the proposition is empirical, there is usually implication of existence, as All roads lead to Rome; if it is derived from the axioms and fundamental principles of any science or of real life, there is not, as Whatever gods there are, are just.

The psychologists (if not the logicians) are right in saying that in the predicate the reference to objectivity, though implied, is not explicitly in the mind of the speaker; but that is accounted for by the application of what is frequently referred to as the *principle of parsimony*. It is sufficient if a single term carries the objective reference for the whole sentence, and to repeat this reference explicitly in the predicate would be tautologous. It is not predicates only which are without it, but also all elements of the subject, except one; thus we cannot think of *ab*, in *ab* ⋖ *c*, *architect-bankers are clever*, except as architects who are bankers, or as bankers who are architects, or as *things which* are at once architects and bankers. In citizen-student and student-citizen, only one word is a noun and the other is practically an adjective. (Wundt writes them, to indicate this distinction, *cS* and *sC* respectively.) The predicate-word, then (just like the determinant-word), though fully capable of bearing the reference to reality, has it, in general, only in abeyance. In contraposition, where subject and predicate change places, the reality-reference remains in the subject, ∞ *s* ⋖ *c* becomes ∞ $\bar{c}$ ⋖ $\bar{s}$ (*Whatever is a student is*

a citizen* becomes *Whatever is a non-citizen is a non-student*). Adjectives arise, in general, as predicate-words, but they may at any moment become, by themselves, subject-words, by throwing in the proper special term to carry them, as *Things which are blue are cold*. An adjective and a noun each represents a mental experience of more or less complexity held for the moment to be integral; the source of their difference is to be found in the doctrine of 'natural kinds' (see KIND). All the subjects of *x* constitute the application of *x*, and all the predicates of *x* constitute the signification or the implication of *x*; now adjectives are, in general, words of wide application but of very little implication, but nouns, on the other hand, are the names of natural kinds of great depth. We can say, e.g., *Things which are white are the brightest things in the field of view, Things which are white do not fade, Things which are white are easily soiled, Things which are white are suggestive of purity*, and that is almost all. The word represents a term of extremely little depth; *being white* has almost no further implication. It is for this reason that in the constructing of language it has remained solely an adjective. It is not the case that a word is a predicate-word because it is an adjective; it is an adjective because it is (owing to its little depth) almost exclusively a predicate-word.

The attempt to introduce propositions with quantifical predicates into formal logic has been over and over again shown to be a mistaken one; such a proposition can always be expressed in terms of two propositions of the regular scheme.

IV. *The Scheme of Propositions.* The proposition in *is* will usually contain, besides its essential elements, two modifying words, one of quantity and one of quality, as

Some | *a* | is | not | *b*,

and will thus consist in all of five constituents of different sorts. *Some* and *all* are indicators of particular and universal quality respectively; *not* and *the absence of any word* are what stand respectively for negative and affirmative quality. These several indicators may be treated by the logician as separate proposition-elements (and usually are so treated), or their implications may all be thrown into the copula, and we shall have thus what may be called the *figured* copula, as *a is-wholly b, a is-not-wholly b*. In the compound proposition this is the regular mode of expression; we say *p is-indicative-of*

q, That a is b is-not-incompatible-with c being d.
Different symbolic copulas (modifications of
<) may be devised for all the different
relations of this sort, and the transformation
from one to another may be made by
mechanical rules. How many of these es-
sentially different relations are there? The
ordinary logic recognizes only four, and of
these one is to a certain extent recalcitrant
to rule, for the reason that it is in fact
a member of a different scheme. Instead of
Some a is-not b, we ought to express this
member of the group of four as *Not all a is b.*
This is the form in which it appears in
Aristotle, and it frequently retains this form
in the works of the schoolmen, as appears in
the fact that the symbolic letters which stand
for the several propositions, *A, E, I, O*, are
said to be (but upon perhaps insufficient
authority) the characteristic vowels of πᾶς,
οὐδείς (οὐδέν), τίς, οὐ πᾶς (Prantl, *Gesch. d.
Logik*, xv. 277, and iv. 153–4). The pro-
positions admitted into any scheme should be
propositions which are the immediate denials
one of another, as *Some a is b, Not any a is b.*
Pairs of immediate denials are

(*a*) *All a is b,*
(*ā*) *Not all a is b,*

and

(*m̄*) *Some a is-not b,*
(*m̄̄*) *No a is-not b.*

Either (*a*) and (*ā*) should be regarded as the
canonical forms, or else (*m̄*) and (*m̄̄*); to mix
them up, as is done, is a pity, for the rules
for TRANSFORMATION (q. v., in logic) apply
very differently to the incongruous pair (*a*)
and (*m̄̄*), and hence much confusion arises.
The right pair to choose is, of course, (*a*) and
(*ā*); *All that glitters is gold* is properly
denied by *Not all that glitters is gold.*

The actual number of different statements
that are possible in terms of *x* and *y* and
their contradictory terms $\bar{x}$ and $\bar{y}$ (excluding
double negatives) is eight. This is at once
evident if we express everything that can be
said in the form of propositions of existence
and of non-existence; thus the combinations
of *a* and *b* and their negatives are ab, $\bar{a}b$, $a\bar{b}$,
$\bar{a}\bar{b}$, and since each one of these combinations
can be said to exist (a particular proposition,
There is some a which is b, or *Some a is b*) or
to be non-existent (a universal proposition,
There is no a which is b, or *No a is b*), it is
evident that eight different statements of fact
are possible. These, of course, remain diffe-
rent, no matter what the form in which they
may be expressed. One reason why logic

commonly recognizes only four out of this set
of eight is that it has fought shy of negative
terms, and especially of negative terms as
subjects. This is strange, because Aristotle
gives, in his most fundamental group of
propositions (those in one term only), four
with negative subjects, as *Not-a exists, Not-a
exists not*, &c. It is, however, De Morgan to
whom we owe not only the generalization of
the copula (which, he says, he has 'made as
abstract as the terms'), but also the full intro-
duction into logic of negative terms as subjects
as well as predicates, and the setting out of
the eight propositions of a complete scheme.
De Morgan did not, however, devise appro-
priate copulas for the several statements to be
made; but one does not have to search far, in
the language of real life, to find such, and
when they are found, the eight things to be
said can all be said by means of them, very
simply, without the use of any negative terms
whatever. The letters *A, E, I, O* being no
longer adequate, we may take *i* and *o* and
their negatives to stand for the symmetrical
copulas—those in which subject and pre-
dicate are simply commutable—and the
unsymmetrical letters, *a* and *u* (*u* is perhaps
sufficiently unsymmetrical), to stand for those
copulas with which subject and predicate
cannot be interchanged without change of
sign. We shall then have

(*a*) All *x* is *y*,   (*ā*) Not all *x* is *y*,
(*ū*) None but *x* is *y*,   (*u*) Something besides
                                      x is *y*,
(*ī*) No *x* is *y*,   (*i*) Some *x* is *y*,
(*o*) All but *x* is *y*,   (*ō*) Not all but *x* is *y*.

The first two copulas in each column are non-
symmetrical: *None but x is y* can only be
inverted into *None but non-y is non-x*, and
Not all x is y only into *Not all non-y is non-x*;
in the last four propositions all terms are
simply commutable.

Language furnishes us with perfectly ade-
quate forms of expression for these eight
modes of connection in the compound proposi-
tion as well as in the simple proposition.
Thus we have

(*a*) If it rains it pours,   (*ā*) Though it rains it
                                      does not always pour,
(*ū*) Not unless it rains   (*u*) Besides when it
    does it pour,   rains it sometimes
                                      pours,
(*ī*) Never when it rains   (*i*) Sometimes when
    does it pour,   it rains it pours,
(*o*) Unless it rains it   (*ō*) Not always except
    pours,   when it rains does
                                      it pour.

With the aid of negative terms, and the special terms 'nothing' and 'everything,' any given statement can be made in four different ways with any one copula:

(a_1) All a is b,   (a_3) Nothing is a and $\bar{b}$,

(a_2) All $\bar{b}$ is $\bar{a}$,   (a_4) Everything is either $\bar{a}$ or b;

and similarly their denials, the particulars. Or, for the compound proposition,

(a_1) If a is b, c is d,   (a_3) Never is a b, that c is not d,

(a_2) If c is not d, a is not b,   (a_4) Always either a is not b or c is d.

Now logic should either examine *all* these forms of speech, which are perfectly easy and natural in real life, or it should require everything to be reduced to *one* model form of expression before consenting to study it. It is one of the most remarkable of the curiosities of science that, of the four possible modes of universal statement, a certain two,

(a)   All a is b,
(i)   No a is b,

should have been regarded as canonical in dealing with terms, but a different two,

(a_1)   If a is b, c is d,
(a_4)   Either a is b or c is d,

should alone have been considered to exist in dealing with propositions. Symmetry and harmony and beauty of treatment were attainable only by admitting the complete scheme of possible statements which language has made for us. To admit for the simple and the compound proposition incongruous pairs was strangely ill advised; in this matter unconscious language-construction has shown itself far more intelligent than conscious logic-making. There is no arguing in terms of the so-called hypotheticals, disjunctives, &c., that has not its exact parallel in arguing in simple propositions; and the compound proposition as an element of an argument would not have required any different treatment from that given to terms, had it not been for this forced disjunction between their admitted modes of expression. It is for this reason, among others—to show the parallelism between simple and compound propositions— that the consideration of the full scheme of propositions is essential. (It is proper to regard *if* as a copula, for the actual meaning of If a is b, c is d, is Given that a is b, *it will always follow* that c is d. By an ellipsis of language we are allowed to say all this with the one little word *if*; but logic deals with the real connections of terms and of proposi-

tions, and its first task is to abstract from accidents of language.)

Wundt has proposed a set of symbols to stand for certain of these relations; but the relations which he symbolizes are neither complete nor symmetrically chosen, and the symbols which he makes use of have no connection with each other, nor do they lend themselves to negation. A better set can be constructed by means of variations of the fundamental $<$, so devised as to exhibit plainly the different relations which the copulas bear to each other and to the propositions which they represent:

$a < b$ All a is b. $a \leqq b$ Not all a is b.	$a \overline{<} b$ None but a is b. $a < b$ Some besides a is b.
$a \overline{\vee} b$ No a is b. $a \vee b$ Some a is b.	$a \vee b$ All but a is b. $a \overline{\vee} b$ Not all but a is b.

The negation of any relation is indicated by a line drawn over the corresponding affirmative copula. (This line is then to be regarded as an integral part of the copula to which it is attached.) As thus constructed, these symbols have certain marked coincidences with the relations that they symbolize; they are to this extent of the nature of diagrams, and fitted to hold in mind in a simple form the things that are reasoned about. These coincidences are: (1) Commutative relations are represented by a symmetrical copula; non-commutative relations by a non-symmetrical copula. (2) A copula involving an odd number of lines (i. e. three) is universal; a copula involving an even number of lines (i. e. two or four) is particular. (3) There is a simple rule for the transformation from any one copula to another, which need not be stated here.

The names of these several signs of relation are for universals: the copula of sufficiency, the copula of indispensableness, the copula of incompatibility (or non-concurrence, or exclusion), the copula of exhaustion; and these, together with their negatives (which express particular connections), constitute an exact enumeration of all the simple logical relations in which any two concepts or any two events can stand to each other.

For logic considered as the art and science of drawing conclusions, and of testing the validity of purported conclusions drawn by others, there is no difference between the simple proposition 'Whatever is a is b' (1) and the compound proposition (i. e. the proposition about propositions) 'Whenever a is b, it-follows-that c is d,' or 'That a is b entails that c is d' (2). But for that ultimate analysis of the meaning of the proposition which properly comes within the domain of psychology or of epistemology, but which has usually been given over to the logician, there is a distinct difference between them. In the case of (1), the full import is that the universe is such that the assumed substratum for those affections of consciousness which we have summed up under a is coincident in time and space (that is, occupies the same point of space at the same instant of time) with the corresponding substratum of the sensation-congeries b. This is what is at the bottom of our affirmation that the objects a are identical with some of the objects b. But when we affirm in (2) identity of occurrence of a is b with some cases of occurrence of c is d, the mind makes (one knows not why) a *far less complete fusion* of a is b with c is d than it makes of a and b in a is b; thus a is b may be an occurrence in South Africa, and c is d, however invariably attendant upon it, may be an occurrence in China; all that is affirmed is sufficiency in the antecedence of the protasis for the occurrence of the apodosis. Thus in 'Whenever the bell rings, the curtain falls,' our whole system of interpretation of conscious experience imposes upon us a far closer fusion in the *bell* which *rings*, and in the *curtain* which *falls*, than in the compound event which is constituted by their conjoint occurrence. There is more frequently perhaps an interval of time permissible in the compound event than in the compound object, but that makes no difference in the distinction here insisted upon. There is a certain fixed, continuing substratum in the case of a which later comes to be b; and there fails to be any such bearer of the event a is b, which is followed by the event c is d. (C.L.F.)

The forms of words which embody existential judgments ('sea-serpents exist') and impersonal judgments ('it rains') have occasioned much discussion. The former is discussed under JUDGMENT, where the alternative theories are also stated. As to the impersonal, the view indicated under PREDICATION is very current and goes very well with the theory of the existential. It makes the subject of the impersonal the entire sphere of reality (cf. UNIVERSE OF DISCOURSE) within which the observed or indicated phenomena present themselves. Genetically (and linguistically), the impersonal is a very early form. The child says merely 'rain,' assuming the world of fact to which his and others' experience are common. Cf. INDEX (in exact logic).

Literature: see the authors cited; the literature under the topics referred to; the general treatises on LOGIC, and BIBLIOG. C, 1, b, and 2, l.　　(J.M.B., C.L.F.)

Prosody [Gr. προσῳδία, what accompanies the song]: Ger. *Prosodie*; Fr. *prosodie*; Ital. *prosodia*. The term prosody is used by the Greek grammarians in relation to all the marks, including accents, breathings, quantities, &c., which might be added to the letters in the writing of lines of verse. In modern use it applies solely to the recognition of rhythmical form in verse. This rhythm is dependent on the orderly recurrence of long syllables in quantitative verse, of stressed syllables in accentual verse.　　(B.I.W.)

Prospection (-ive) and **Prospective Reference**: see ORIGIN *versus* NATURE.

Prostration [Lat. *prostratio*, an overthrowing or subverting]: Ger. *Prostration*, *Erschöpfung* (exhaustion); Fr. *prostration*; Ital. *prostrazione*. A serious and often sudden loss of strength; likewise a great depression; e.g. the mental and physical prostration sequent to the loss of a friend, to the shock of an accident, &c. Cf. SHOCK.

The term 'nervous prostration' indicates the feebleness and depression characteristic of NEURASTHENIA (q.v.).　　(J.J.)

Prosyllogism [for deriv. see SYLLOGISM]: Ger. *Prosyllogismus*, *Vorschluss*; Fr. *prosyllogisme*; Ital. *prosillogismo*. A syllogism whose conclusion is a premise of another. See CHAIN SYLLOGISM, and EPISYLLOGISM. (C.S.P.)

Protagoras. (480 cir.–411 B.C.) Studied under Democritus. Lived and taught in Athens. He was the first who taught philosophy and rhetoric for money, and the first to call himself a Sophist.

Protanopia [Gr. πρῶτος, first, + αν + ὄπτομαι, fut. ὄψομαι, I see]: Ger. *Protanopie*; Fr. *protanopie* (suggested); Ital. *protanopia* (suggested). The name proposed by v. Kries for what was formerly called red-blindness.

It is now known (especially by means of the monocular cases) that partial COLOUR-BLINDNESS (q.v.) or dichromasy, with a few exceptional