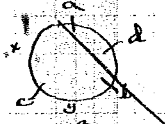
Notes for my Treatise on Arithmetic

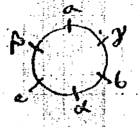
Elementary Principles

Any two numbers are equal or else one is greater than the other or else one is 0^{∞} ~~But by is~~ Using ∞ to mean positive logarithmic infinity.

Addition

~~Q~~ The numbers form a cycle ◯ Let a, b, c, be three different numbers. Let $B\binom{ab}{c}$ mean ~~from~~ between a ~~to~~ and b (inclusive of both) in that way in which c is not between a and b. Then every number is either $B\binom{ab}{c}$ or $B\binom{bc}{a}$ or $B\binom{ca}{b}$ and a alone is both $B\binom{ca}{b}$ and $B\binom{ab}{c}$ etc. If a=b only a is $B\binom{ab}{c}$ and ~~it is some number~~ only c is both $B\binom{bc}{a}$ and $B\binom{ca}{b}$ But what numbers are one and what the other is indeterminate.





If d is $B\binom{ab}{c}$ then ~~c is $B\binom{ab}{d}$ while a is $B\binom{cd}{b}$~~

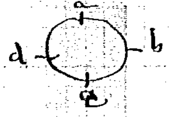
~~and b is $B\binom{cd}{a}$~~ and whatever is $B\binom{ca}{b}$ or $B\binom{bc}{a}$ is $B\binom{ab}{d}$

or hence and whatever is $B\binom{ad}{b}$ or $B\binom{db}{a}$ is $B\binom{ac}{d}$

Therefore is α is $B\binom{bc}{a}$ and β is $B\binom{ca}{b}$ and γ is $B\binom{ab}{c}$

α, β, γ one ~~a~~ is $B\binom{bc}{\alpha}$ ~~b is~~ Hence b is $B\binom{\alpha\beta}{c}$ and c is $B\binom{\alpha\beta}{b}$

while a is $B\binom{\alpha\beta}{c}$



If d is $B\binom{ac}{b}$ then b is $B\binom{ac}{d}$

Hence if d is $B\binom{ac}{b}$ a is $B\binom{bd}{c}$

For any this is ~~For either and~~ $B\binom{bd}{c}$ or $B\binom{bc}{d}$ or $B\binom{cd}{b}$

~~But if a were d alone is both $B\binom{bd}{a}$ and $B\binom{ad}{c}$~~

But if a were $B\binom{bc}{d}$ d would be $B\binom{bc}{a}$ and not $B\binom{ac}{b}$ as supposed unless d=c

and if a were $B\binom{cd}{b}$ b would be $B\binom{cd}{a}$ and not $B\binom{ac}{d}$ as we find unless b=c

Hence the best way to write d is $B\binom{ac}{b}$ is ~~best~~ b,d ⊕ a,c

Restate, next leaf.

The general idea of separation is highly important.

Let ι_{xyz} mean "the x is on the same side as y of z where z is a barrie

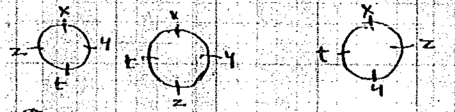
So that in all cases ~~[illegible]~~ $\Pi_x \Pi_z \iota_{xxz}$ and $\iota_{xyz} = \iota_{yxz}$

~~A cycle~~ To say that objects form a cycle is to say that any two of them form a barrier. Let β_{xyz} mean x is the barrier formed by x and y

So that $\beta_{xyz} = \beta_{xzy}$

Let σ_{xyzt} ~~σ_{xyz}~~ mean "t is on the same side ~~of~~ as z of the barrier formed b

Then $\sigma_{xyzt} = \sigma_{xytz} = \sigma_{yxzt}$ and $\Pi_z \sigma_{yxzz}$



σ_{xyzt} σ_{xyzt}

σ_{ytxz} σ_{xtyz}

$\sigma_{xyzt} = \sigma_{ztxy}$

But since we have

~~xyzt~~ xytz
xzyt xzty
xtyz xtzy

tx

xyzt xyzt xyzt
~~xz~~ yz ytzx zxyt
tx y x

~~xy~~ abc—acb
x cab—cab

abcd
bacd
bcad
bcda

abc
~~x~~bac
bca

xyzt xytz
xtzy xtyz
xzty xzyt

zxty—zxyt
ztxy—ztyx
zyxt—zytx

yzxt—yztx
ytxz—ytzx
yxtz—yxzt

xyzt—xytz
xzyt xzty
xtyz xtzy

~~xyzt—xytz~~
~~xzty xzyt~~

xyzt—xytz
~~xtyz~~ xtzy
xyzt xytz

ytza ytaz
yatz yazt
ytza ytaz

tazy tayz
tyaz—tyza
t

xyzt → xytz

xytz xytz
xzty xtyz

1234—1243
1423—1432
1342—1324

x 123—132
231—213
312—321

1234—1243
2143—2134

1234 1243
1423 1432
1342 1324

4321 4312
4132 4123
4213 4231

3124 3142
3412 3421
3241 3214

2314 2341
2431 2413
2143 2134

1234
3124

~~1234~~

1234 1234 1234
4123 2413 2341

1234 1234 1234
3142 4312 3421
4321
~~2413~~
~~1234~~
~~2134~~

1234—1243
1423—4123
3124 3142
3412 4312
2314 2341
2431 4231
~~1234—1243~~

1234 1243
1423 4123

3241 3214
3421 4321

1234 | 1234 | 1234
4231 | 3214
4132 | 3124
4123 | 3142

1234—1324
~~3241—3421~~
~~1243—4123~~
~~2143—2134~~
2143 2413
4321 4231
3412 3142

1234 1243
1423 1432
1342 1324

3124 3142
3412 3421
3241 3214

2314 2341
2431 2413
2143 2134

3124
3241 3214
3412 3421

4321 4312
4132 4123
4213 4231

2431 2413
[illegible] [illegible]

1234—1324
3421 3241
~~1342~~
~~4213~~
2134

The proper relation is evidently $\boxed{\begin{smallmatrix} a & b \\ d & c \end{smallmatrix}}$ or better $\left(\begin{smallmatrix} & a & \\ d & & b \\ & c & \end{smallmatrix}\right)$ or for sake of printing $\cancel{\left(\begin{smallmatrix} a & b \\ & c \end{smallmatrix}\right)}$

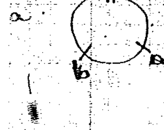
$\cancel{\left(\begin{smallmatrix} a \\ c \\ d \end{smallmatrix}\right)}$ $\left(\begin{array}{c|c} a & b \\ d & c \end{array}\right)$ $\left(\begin{array}{c|c} a & d \\ b & c \end{array}\right)$

Addition — Has but one value except $\infty + (-\infty)$ and $(-\infty) + \infty$ which is indet.

$~~a + 0 = a~~$ $0 + 0 = 0$ $\infty + \infty = \infty$

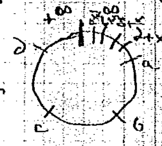
? If $\left(\begin{smallmatrix} a, & b \\ d, & c \end{smallmatrix}\right)$ then $\left(\begin{smallmatrix} a+x, & b+x \\ d+x, & c+x \end{smallmatrix}\right)$? $(-\infty) + (-\infty) = -\infty$

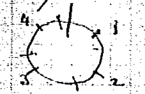
a



If $\left(\begin{smallmatrix} -\infty, & \infty \\ a, & b \end{smallmatrix}\right)$, then $\left(\begin{smallmatrix} -\infty & +\infty \\ a+x & b+x \end{smallmatrix}\right)$

If $\left(\begin{smallmatrix} a, & b \\ d, & c \end{smallmatrix}\right)$ then $\left(\begin{smallmatrix} a+x, & b+x \\ d+x, & c+x \end{smallmatrix}\right)$ provided $\left(\begin{smallmatrix} \infty & \infty \\ b & a \end{smallmatrix}\right)$





~~0 + 1~~

sum of whole numbers is a whole number

~~(a+b)+c~~ $0 + 0 = 0$

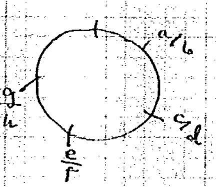
$Ex + y = E(x+y)$

$x + Ey = E(x+y)$

$\frac{1}{0}$ $\frac{1}{1}$ $\frac{0}{1}$

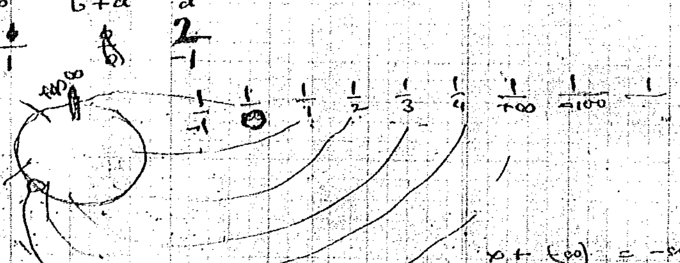
$\frac{1}{-2}$ $\frac{1}{-1}$ $\frac{a}{c}$ $\frac{0}{\infty}$ $\frac{1}{0}$

$\frac{a+b}{c+d}$ $\frac{b}{d}$



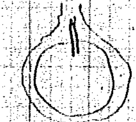
$\frac{a}{b}$ $\frac{a+c}{b+d}$ $\frac{b}{d}$

$\frac{-1}{1}$ $\frac{b}{b}$ $\frac{2}{-1}$



$x + (-\infty) = -\infty$

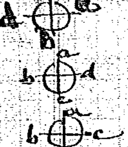
$x (+ 100) =$



We have either $a, b \otimes c, d$

or $a, c \otimes b, d$

or $a, d \otimes b, c$



or $\sigma_{abcd} + \sigma_{acbd} + \sigma_{abdc}$

$\sigma_{abad} \neq 0$ σ

Advance last 3 — Retire last 3

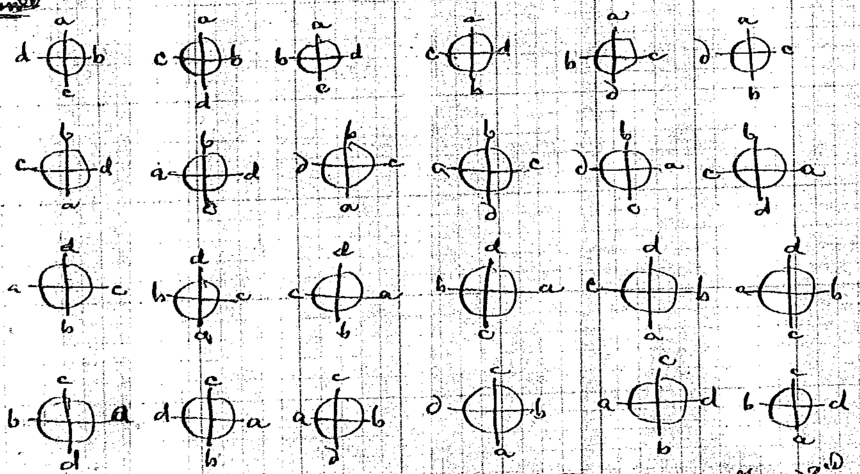
$\sigma_{abcd} = \overline{\sigma}_{abdc} = \overline{\sigma}_{adcb} = \overline{\sigma}_{adbc} = \overline{\sigma}_{acdb} = \overline{\sigma}_{acbd}$

Exchanging two pairs

$= \overline{\sigma}_{bdac} = \overline{\sigma}_{bdca} = \overline{\sigma}_{bcad} = \overline{\sigma}_{bcda} = \overline{\sigma}_{bacd} = \sigma_{badc}$

$= \sigma_{dcba} = \overline{\sigma}_{dcab} = \overline{\sigma}_{dabc} = \overline{\sigma}_{dacb} = \overline{\sigma}_{dbac} = \overline{\sigma}_{dbca}$

$= \overline{\sigma}_{cadb} = \overline{\sigma}_{cabd} = \overline{\sigma}_{cbda} = \overline{\sigma}_{cbad} = \overline{\sigma}_{cdba} = \sigma_{cdab}$



The value of σ remains unchanged by interchange of 1st and 3rd or 2nd and 4th (or both)

or when cyclical order is preserved or reversed

or when the members of each of two pair of letters are interchanged

i.e abcd changed to badc or to cdab or dcba

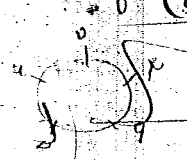
Or when first and third (or second and fourth) either both or neither remain first and third (or second and fourth) in either order

There are three sets of equivalent arrangements

abcd	2 abdc	1 adbc
2 adcb	1 acdb	2 acbd
reverse bcda	1 bdca	2 dbca
badc	2 bacd	1 dacb
similar cdab	1 dcab	1 bcad
2 cbad	1 dbac	bdac
reverse dabc	1 cabd	cadb
dcba	cdba	1 cbda

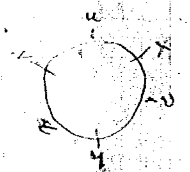
~~If $\begin{pmatrix} x & y \\ u & z \end{pmatrix}$ and $\begin{pmatrix} x & u \\ x & v \end{pmatrix}$ then $\begin{pmatrix} u & v \\ y & x \end{pmatrix}$~~ and $\begin{pmatrix} y & x \\ v & u \end{pmatrix}$

~~or (~~



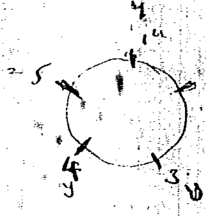
~~If $\begin{pmatrix} x & y \\ u & x \end{pmatrix}$ and either $\begin{pmatrix} x & u \\ x & v \end{pmatrix}$ or $\begin{matrix} x & y \\ v & u \end{matrix}$ $\begin{pmatrix} y & u \\ x & v \end{pmatrix}$~~

then



If $\begin{pmatrix} x & y \\ u & z \end{pmatrix}$ and ~~$\begin{pmatrix} y & z \\ v & x \end{pmatrix}$ $\begin{pmatrix} y & z \\ x & u \end{pmatrix}$~~ $\begin{pmatrix} y & z \\ v & x \end{pmatrix}$

then $\begin{pmatrix} y & u \\ v & x \end{pmatrix}$ and $\begin{pmatrix} x & u \\ v & x \end{pmatrix}$ and $\begin{pmatrix} x & u \\ y & v \end{pmatrix}$



If $\begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix}$ and $\begin{pmatrix} 4 & 5 \\ 3 & 2 \end{pmatrix}$ $\Bigg|$ $\begin{pmatrix} & 5 & \\ 4 & & 1 \\ & 2 & \end{pmatrix}$ $\begin{pmatrix} & 5 & \\ 4 & & 2 \\ & 3 & \end{pmatrix}$

~~i.e. $\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix}$ and $\begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$~~

i.e $\begin{pmatrix} 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $\begin{pmatrix} 4 & 5 \\ 3 & 2 \end{pmatrix}$ $\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$

then $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

and since the hypoth can be written

5 1 3 4

$\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ $\begin{pmatrix} 5 & 4 \\ 2 & 3 \end{pmatrix}$

by the same prin $\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ or $\begin{pmatrix} z & y \\ v & x \end{pmatrix}$

Also since we have $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$

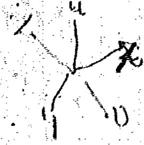
by the same principle $\begin{pmatrix} 3 & 4 \\ 1 & 5 \end{pmatrix}$ or $\begin{pmatrix} x & u \\ y & v \end{pmatrix}$

Thus the sole def. of the relation is

~~If $\begin{pmatrix} y & z \\ x & u \end{pmatrix}$~~ and ~~$y\,x$~~

If $\begin{pmatrix} y & z \\ v & x \end{pmatrix}$ and $\begin{pmatrix} y & x \\ x & u \end{pmatrix}$ If $\begin{pmatrix} y & z \\ v & x \end{pmatrix}$ $\begin{pmatrix} z & y \\ y & x \end{pmatrix}$

then $\begin{pmatrix} v & y \\ x & u \end{pmatrix}$ $\begin{matrix} y & u \\ v & x \end{matrix}$ $\begin{pmatrix} y & u \\ v & x \end{pmatrix}$



or if $\begin{pmatrix} v & y \\ x & z \end{pmatrix}$ $\begin{pmatrix} x & y \\ u & x \end{pmatrix}$

$\begin{matrix} v\,y \\ x\,u \end{matrix}$ If $\begin{pmatrix} z & x \\ y & v \end{pmatrix}$ If $\begin{pmatrix} \alpha & \omega \\ 2 & 1 \end{pmatrix}$ $\begin{pmatrix} \alpha & \omega \\ 3 & 2 \end{pmatrix}$ then $\begin{pmatrix} \alpha & \omega \\ 3 & 1 \end{pmatrix}$

If $\begin{pmatrix} \alpha & \omega \\ 2 & 1 \end{pmatrix}$ $\begin{pmatrix} \alpha & \omega \\ 3 & 2 \end{pmatrix}$ $\begin{pmatrix} \alpha & \omega \\ 4 & 3 \end{pmatrix}$ then $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$

$\begin{pmatrix} \alpha & 1 \\ 4 & \\ 3 & 2 \end{pmatrix}$ $\begin{pmatrix} \alpha & 1 \\ 3 & 2 \end{pmatrix}$ $\begin{pmatrix} \alpha & \omega \\ 4 & 3 \end{pmatrix}$ [illegible] α

$\begin{pmatrix}\alpha & \omega\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha & \omega\\ 2 & 0\end{pmatrix}\begin{pmatrix}\alpha & \omega\\ 2 & 1\end{pmatrix}\begin{pmatrix}\alpha & [illegible]\\ 2 & 0\end{pmatrix}\begin{pmatrix}\omega & 1\\ 2 & 0\end{pmatrix}$

$\begin{matrix}\alpha & \omega\\ 2 & 0\end{matrix}$

The primitive proposition is that

$$\begin{pmatrix}\alpha & \omega\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha & \omega\\ 2 & 0\end{pmatrix} \sim \begin{pmatrix}\alpha & 1\\ 2 & 0\end{pmatrix}$$

$$\begin{pmatrix}\alpha & \omega\\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha & \omega\\ 2 & 1\end{pmatrix}\begin{pmatrix}\alpha & \omega\\ 3 & 2\end{pmatrix}\begin{pmatrix}\cancel{\alpha} & \omega\\ 4 & \cancel{3}\end{pmatrix} \sim \begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}$$

$$\begin{pmatrix}\alpha & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}\alpha & 1\\ 3 & 2\end{pmatrix}\cancel{\begin{pmatrix}\alpha & 2\\ 4 & 3\end{pmatrix}}$$

$$\begin{pmatrix}[illegible] & 1\\ 3 & 2\end{pmatrix}\cancel{\begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}}$$

$$\begin{pmatrix}\alpha & 0\\ 1 & \omega\end{pmatrix}\begin{pmatrix}\alpha & [illegible]\\ 2 & [illegible]\end{pmatrix}$$

$$\begin{pmatrix}0 & \omega\\ 2 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ \alpha & 3\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & \alpha\end{pmatrix} \sim \begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$$

~~α ω~~ 1 $\begin{pmatrix}\alpha & 1\\ 3 & 2\end{pmatrix}\begin{pmatrix}\alpha & 0\\ 2 & 1\end{pmatrix} \sim \begin{pmatrix}1 & \alpha\ \cancel{[illegible]}\\ 2 & 1\end{pmatrix}$

$$\begin{pmatrix}\alpha & 1\\ 3 & 2\end{pmatrix}\begin{pmatrix}\alpha & 0\\ 3 & 1\end{pmatrix} \sim \begin{pmatrix}0 & 1\\ 3 & 2\end{pmatrix}$$

Secundal Arithmetic.

Required √10

```
10
01
00011
0000101
0000010001101001110
```

```
                          0
                          1
                         10
                        101
                       1100
                      11101
                    1000110
                   10101001
                  110011000
                 1111011001
               100101001010
              1011001101101
             11011000100100
           1000001010110101
          10011101110001110
         101111100111010001
        1110010111100110000
      100010101100000110001
     1010011101111110010010
    11001010001011101010101
   111101000000111000111100
 10010011010011001111001101
101100011100111010111010110
```

Secundal Algrim.
Pascal's Triangles

111100

1

1
1

1
10
1

1
11
11
1

1
100
110
100
1

1
101
1010
1010
101
1

1
110
1111
10100
1111
110
1

1
111
10100
100011
100011
10101
111
1

1
1000
11000
110000
1000110
111000
11100
1000
1 1

1
1001
100100
1010100
111110
111110
1010100
100100
1001
1

1
1010
101101
1111000
11010010
11111100
11010010
1111000
101101
1010
1

1
1011
110111
10100101
101001010
111001110
111001110
101001010
10100101
110111
1011
1

1
1100
1000010
11011100
11110111
1100011000
1110011100
1100011000
11110111
11011100
1000010
1100
1

1
1101
1001110
100011110
1011001011
10100000111
11010110100
11010110100
10100000111
1011001011
100011110
1001110
1101

1
1110
1011011
101101100
111101001
11111010010
101110111011
110101101000

1
1111
10101001
111000111
10101010101
101111011011
1001110011101
1100100100011
1100100100011

1
10000
~~1e~~1011000
1000110000
11100011100
1000100010000
1111101001000
101110010110000
11001001000110

1 2 3
2 7 3½
3 10 3⅓
4 13 3¼
5 19 3⅕
6 26 3⅙
15 12
5 113
6 336

1111
111
11
1101001

1101
1101
1101
1011011

0.5
0.25
.125
0.0625
.03125
0.015625
0.0078125
0.00390625
0.001953125
R 0.0009765625
0.00048828125
0.000244140625
0.0001220703125
0.00006103515625
0.000030517578125
0.0000152587890625
0.00000762939453125
0.000003814697265625
0.0000019073486328125
R 0.00000095367431640625

0.1
0.01
0.001

0.1
1
1
1
1
1
1
1
1
0.0000000001
1
1
1
1
1
1
1
1
1
0.00000000000000000001

```
01
001
.0001
00001
.000001
.0000001
.00000001
.000000001
0000000001
          1
           1
            1
             1
              1
00 0000000 00000001
          1
         01
           1
```

```
.5
.25
.125
.0625
.03125
.015625
.0078125
.00390625
.001953125
.0009765625
.00048828125
.000244140625
.0001220703125
1.00006103515625
.000030517578125
.0000152587890625
.00000762939453125
.000003814697265625
.0000019073486328125
.00000095367431640625
.000000476837158203125
.0000002384185791015625
.00000011920928955078125
.000000059604644775390625
.0000000298023224887695312 5
```

```
03125
 78125
  48828125
   244140625
   1220703125
    6103515625
     152587890625
      3814697265625
       19073486328125
        95367431640625
```

```
Secundal 0.0001 10011
Decimal  .1
           .625
          0375
          03125
          00625
          00390625
          0022437 5
          001953125
```

```
1/25 = 0.000001010001111010111100001
       0.0011001100110011001100110011
            101
            001
              101    101
              000    100
                      101
                     0100
                       101
                      0011
                        101
                        001
                          101
                          010
                           101
                            11
                            101
                             10
                             101
```

Dec

~~0.1 = 0.001100110011~~

~~0.2 = 0~~

Dec

0.1 =	0.00011
0.2 =	0.00110
0.3	0.01001
0.4	0.01100
0.5	0.1
0.6	0.10011
0.7	0.10110
0.8	0.11001
0.9	0.11100

```
0.0000++0++00++0011001100110011
          ++00++00+1001100110011 0011
            ~~+100+1001100+1~~
     +00++00++0++0++00++00++00+100+
          +00++00++00++00++00++00++
           +00++00++00++00++00++00+
               +00++00++00++00++00++
                +00++00++00++00++00+
                   +00++00++00++00++
                    +00++00++00++00+
                       +00++00++00++
                        +00++00++00+
                           +00++00++
                               +00++
                               +00+
   +  +  +++  +++++  +++++  0++  ++00
      +  +    +++ +     +     ++   ++
   ,                           +   0+

0.001010101011110100011110 00011
```

Rule of Direct Division (to obtain the entire circulating secundal of the quotient). Dividend and Divisor are assumed to be odd integers expressed secundally. Then the Rule is as follows:

Operation 1st. Strike off the 1 in the ~~units~~ zero place of the divisor

Operation 2nd. To the truncated remnant, the place of each figure being assumed to be one less than before, add 1, and call the sum the Pontifex.

Operation 3rd. Designate the dividend the zeroth (0th) Pons ~~if it be not equal to a pre~~

Operation 4th. As soon as any Pons, say the nth is found, strike off from the end its last 1 with ~~the~~ whatever 0s may follow it and call what is struck off the nth increment. ~~but~~ And to the truncated remnant, ~~if it be not equal to a previous Pons~~ considered as ending in the zero place, if it be not equal to a previous Pons add the Pontifex, and call the sum the (n+1)th Pons.

But if it be equal to ~~the~~ a Previous Pons, proceed to perform the 5th operation

To express $\frac{1}{101^2}$ in ~~circulating~~ recurring decimal circulant $\frac{10}{11}\Big|^1$

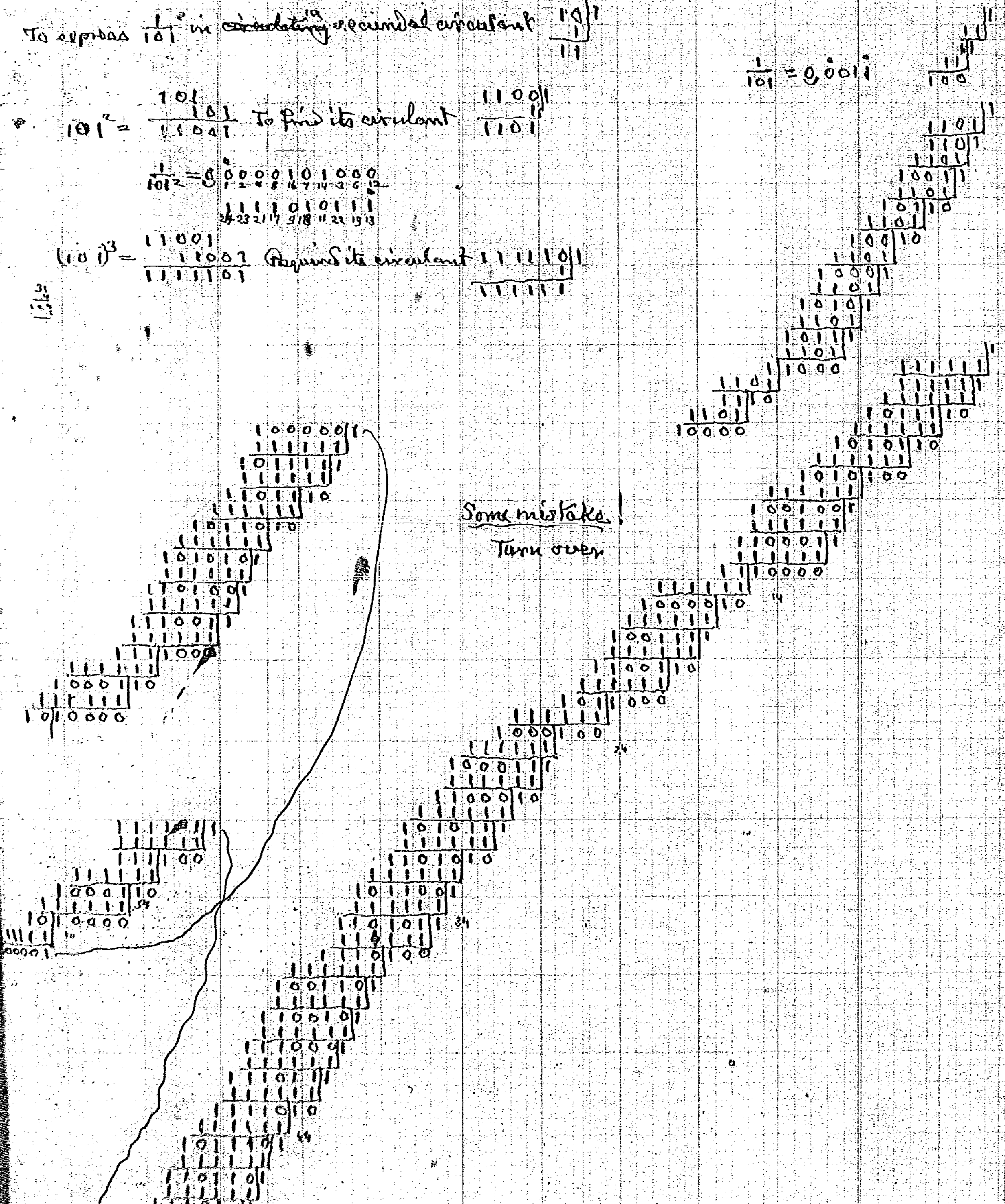
$\frac{1}{101} = 0.\dot{0}01\dot{1}$

$101^2 = \frac{\begin{array}{r}101\\101\end{array}}{11001}$ To find its circulant $\frac{1100}{1101}\Big|^1$

$\frac{1}{101^2} = 0.\underset{1}{0}\,\underset{2}{0}\,\underset{4}{0}\,\underset{8}{0}\,\underset{16}{1}\,\underset{7}{0}\,\underset{14}{1}\,\underset{3}{0}\,\underset{6}{0}\,\underset{12}{0}$

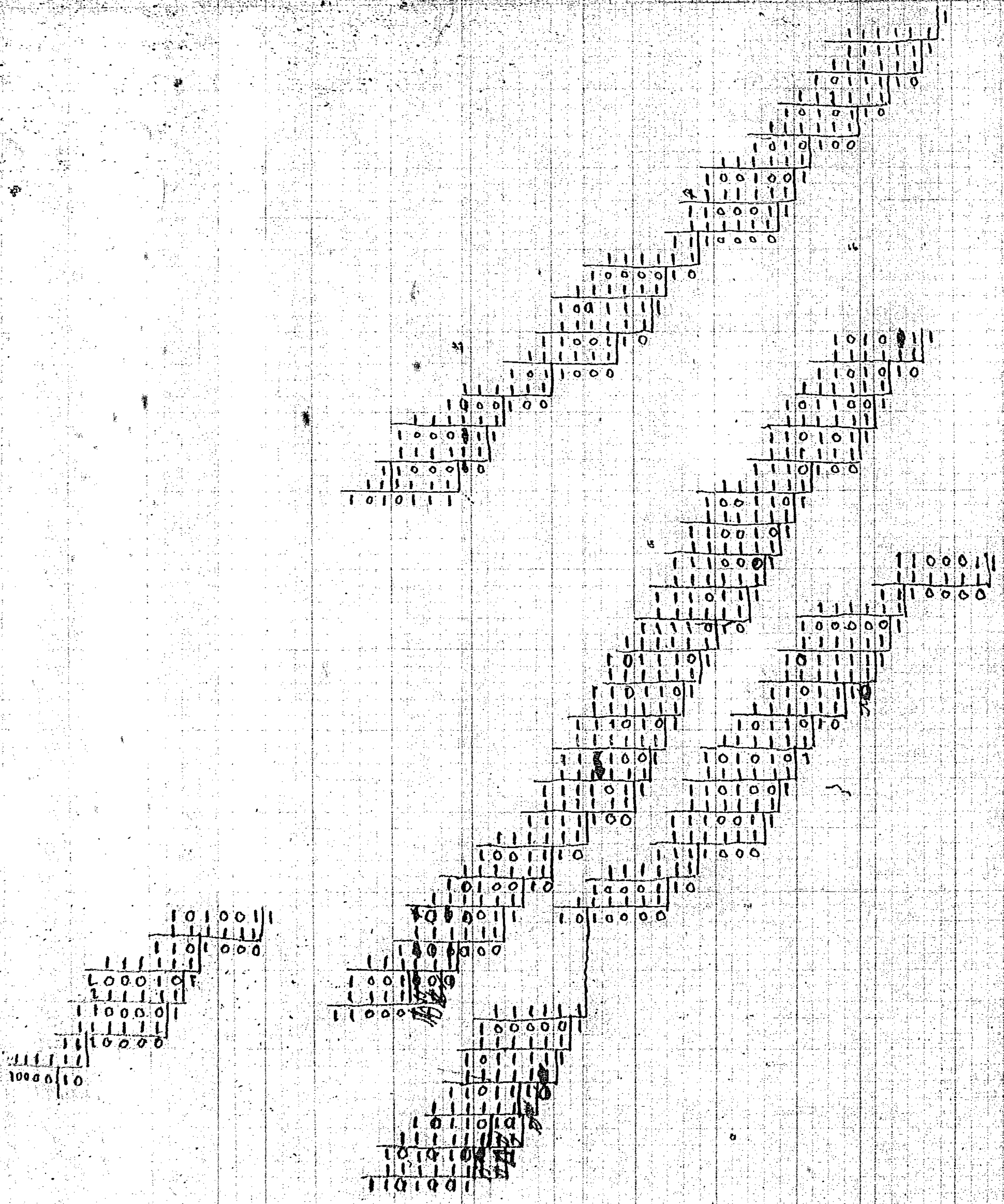
$\underset{24}{1}\,\underset{23}{1}\,\underset{21}{1}\,\underset{17}{1}\,\underset{9}{0}\,\underset{18}{1}\,\underset{11}{0}\,\underset{22}{1}\,\underset{13}{1}\,\underset{13}{1}$

$(101)^3 = \frac{\begin{array}{r}11001\\11001\end{array}}{1111101}$ Require its circulant $\frac{111110}{111111}\Big|^1$

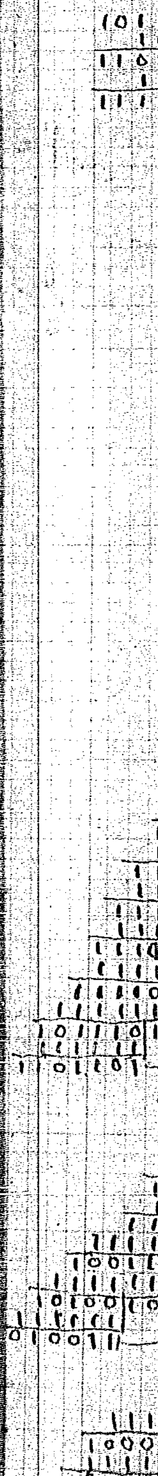


<u>Some mistake!</u>

Turn over

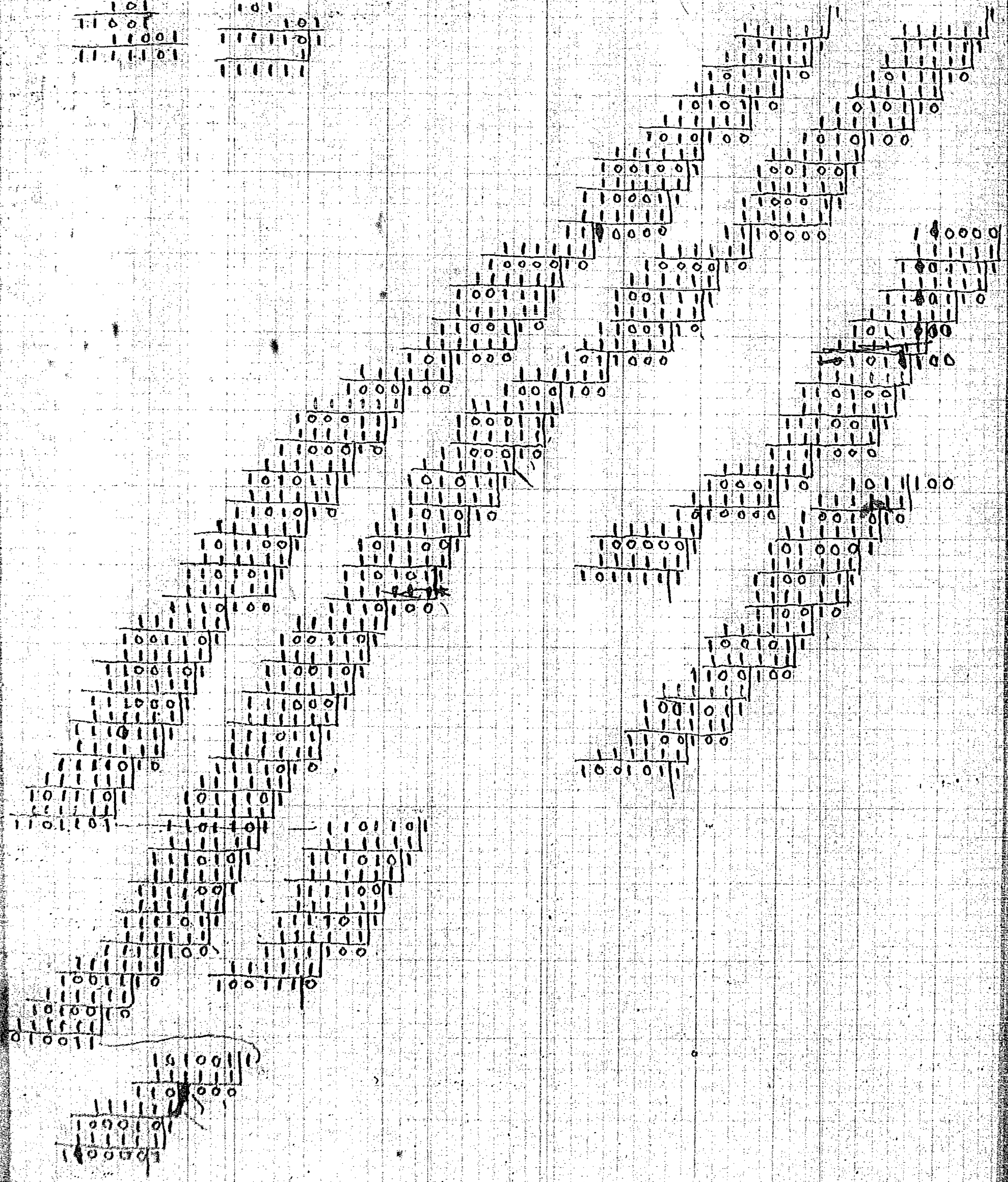


Another



Another try at $\frac{1}{(101)^3}$

```
 101            101
   101           101
 11001             101
   11001        1111110|1
 1111101              1
                111111
```



~~$\frac{1}{101}$ g001~~ 1

$\frac{1}{101}$ g $\dot{0}01\dot{1}0011001100110011001100110011001100110011$

$\frac{1}{(101)^{10}}$ g $\dot{0}000101000111101011\dot{1}00001010001111010111$

$\frac{1}{(101)^{11}}$ g $\dot{0}0000010000011000100100110111010010111100011010100$
$11111101111100111011011001000101101000011100101011$

$\frac{1}{(101)^{100}}$ g $000000000110100011011011100010110110110001110001001$

1 [illegible]2999566
2 06[illegible]05999132
3 090308998698
120411998204
150514997830
180617997396
7 210720996902
8 240823996528
9 270926996094
301029995660

5
2.5
125
0625
03125
015625
0078125
00390625
001953125
0009765625
00048828125
000244140625
0001220703125
00006103515625
000030517578125
0000152587890625
00000762939453125
000003814697265625
0000019073486328125
00000095367431640625
476837158
238418579
119209289
59604644
29802322
14901161
74505806
37252903
18626451+
9313226−
4656613−
2328306+
1164153+
582077−
291038+
145519+

~~6601014322~~
~~3010299913~~
~~3590714409~~
5804144088
3010299957
2793844131

1.58496250074 4
047712125472
030102999566
17609125906
150514997830
25576261230
240823996528
14938615772
12041199826
2897415946
2709269961
188145985
180617997
07527988
06020600
1507388
1505150
002238
2107
131
120

log11 = 11001010111~~011~~0000001~~01~~
1.58496250074
5
0
0625
0224
015625
006837
00390625
~~0029312 5~~
001953125
000978125
0009765625
00001 5632
~~0000009536743164 0625~~
00000060085656835937 5
~~2441406328125 00~~
~~2384185~~
6095656836
4768371532
1327285304
1192092896
135192408
74505806
60686602
37252903
23433739
18626451
4807288
4656613
150675

2.32192809493
0.69897000434
0.60205999132
09691001302
090308998698
06601014322
0.60205999132
05804144088
~~0.60205999132~~
2793844131
2709269961
084574170
060206000
24368170
24082400
0285770
270927
14843
12041
2802
2709
093
1010001110100041
~~1.3219289493~~
1.0101001001110100111101111
1000111 0101
1.3219289493
1.25
07
0625
0094
0078125
0016164
0009765625
000398868
00048828125
00015160555
0001220703125
0000293525 75
0000152587890625
00000142764484375
0000079293939125
000006647053906 25
000003814697265625
0000028343566406 25
0000019073486328125
00000092706800078125
0000004768371532
450170846
2384185766
2117522780
1192092883
0000000009254298 97
596046442
329383455
298023221
00000000031360234
18626457
12733783
9313226
3420557
2328306
1092251
582077
510174
291038

101000111011101100111101000
log11 = 1.1001010111000000000 1~~010001110100001~~
log10 = 11.010100100110100111110111110001101 400

1 030
2 060
3 090
4 120
5 150
6 180
7 210
8 240
9 270
301

.5
25
125
0625
03125
015625
0078125
00390625
00195312
0009765
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0002441
0001220
0000610
0000305
0000152
0000076
0000038
000001 9
0000009
0000004
0000002
1

4656612
2328306
1164153
582076
291038
145519
72759
36379
18189
9094
4547
2273
1136
568

1 0301029995663981
2 0602059991327962
3 0903089986991943
4 1204119982655924
5 1505149978319905
6 1806179973983886
7 2107209969647867
8 2408239965311848
9 2709269960975829
3010299956639810

1·584962500721156
4771212547196 62
3010299956639 81
1760912590556 81
1505149978319 905
255762612236905
240823996531 1848
149386157057202
120411998265592
28974158791610
27092899609758
188145918185 2
1806179973984
075279207868
060205999133
15073298735
15051499783
0021708952
21072100
0636852
0602060
034792
030103
04689
03010
1679
1505
174
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11
1·584962500721156
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1001
11011001110100
00001101000110
110010111100000
1·584962500721156
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00390625
00293125
001953125
0009781 25
0009765625
00000156322115
00000009536743164062 5
0000006095468395937 5
00000047683715820312
0000001327096813906 3
0000001192092895507 8
01350039183985
745058059692
604981124293
372529029846
272452094447
186264514922
86187579524
465661287 30
396214507 94
232830643 65
163383864 29
146415321 83
48968542 44
29103830 40
17864712 09
14551915 23
3312767 7
1818989 40
1493807 37
909494 76
584312 67
454747 35
129565 32
113686 84
15878 48
182 1085
166763

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0001220703125
00006103515625
000030517578125
0000152587890625
00000762939453125
000003814697265625
0000019073486328125
00000095367431640625
000000476837158203125
0000002384185791015625
11920928955078125
59604644775390625
298023223876953125
1490116119384765625
7450580596923828125
37252902984619140625
186264514923095703125
931322574615478515625
465661287 4656612873077392578125
23283064 3 23283064365386962890625
11641532 1 116415321826934814453125
5820766 0 582076609134674072265625
2910383 0 2910383045673370361328125
1455191 5 14551915228366851806640625
7275957 64 72759576141834259033203125
3637978 807 363797880709171295166015625
1818989 403 1818989403545856475830078125
9094947 01 9094947017729282379150390625
4547474 250 45474735088646411895751953125
2273736 754 227373675443232059478759765625
1136868 377 1136868377216160297393798828125
568434 5684361861080801486968994140625
28421709305404007434844970703125
142108546527020037174224853515625
7105427326351001858711242675781 25
3552713663175009293556213378906 25
1·77635683158725

1 030102999566
2 060205999132
3 090308998698
4 120411998204
5 150514997830
6 180617997396
7 210720996902
8 240823996528
9 270926996094
301029995660

1.584962500744
0477121254 72
030102999566
17609125906
150514997830
25576261230
240823996528
14938615772
12041199826
2897415946
2709269961
188145985
180617997
07527988
06020600
1507388
1505150
002238
2107
131
120

2.32192809493
0.69897000434
0.602059991 32
09691001302
090308998698
066010143 22
0.602059991 32
0580414408 8
~~0.6020599913~~
2793844131
2709269961
084574170
060206000
24368170
24082400
0285770
270927
14843
12841
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093
101000111010001

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001953125
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000244140625
0001220703125
00006103515625
000030517578125
0000152587890625
- 00000762939453125
000003814697265625
0000019073486328125
— 00000095367431640625
4768371587
2384185796
1192092895
596046448
298023224
149011612
74505806
37252903
18626451+
9313226-
4656613-
2328306+
1164153+
582077-
291038+
145519+

~~6601014322~~
~~3010299913~~
~~3590714409~~
5804144088
3010299957
2793844131

log11 = 1100101011 ~~011~~ 0000001~~01~~
1.584962500744
5
0
0625
0224
015625
006837
00390625
00293125
001953125
000978125
0009765625
000015632
~~000000095367431640625~~
00000006095656835937 5
~~344108632812500~~
~~2384185~~
6095656836
4768371532
1327285304
1192092896
135192408
74505806
60686602
37252903
23433739
18626451
4807288
4656613
150675

~~1.3219289493~~
1.0101001001110100111110111 11
1.3219289493 1000111010 1
1.25
07
0625
0094
0078125
0016164
0009765625
0006398868
00048828125
00015160555
0001220703125
0000295352375
0000152587890625
000014276448437 5
00000762939453125
00000664705390625
000003814697265625
000002834356640625
0000019073486328125
0000009270080078125
000000476837153 2
4501708546
2384185766
2117522780
1192092883
000000000925429897
596046442
329383455
298023221
0000000003136023 4
18626457
12733783
9313226
3420557
2328306
1092251
582077
510174
291038

101000111011010011101 00
log11 = 1.1001010111000000000 ~~010100011101 00001~~
log10 1.0101001001010011110111110001101000

1	030102999566398 1
2	060205999132796 2
3	090308998699194 3
4	120411998265592 4
5	150514997831990 5
6	180617997398388 6
7	210720996964786 7
8	240823996531184 8
9	270926996097582 9
	301029995663981 0

1·584962500721156
417121254719662
301029995663981
176091259055681
1505149978319905
25576261223690 5
2408239965311848
149386157057202
120411998265592
28974158791610
27092699609758
1881459181852
1806179973984
075279207868
060205999133
15073208735
15051499783
0021708952
21072100
0696852
0602060
034792
030103
04689
03010
1679
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174
181
-7

↓1
1·584962500721156
1·5
0

1001
1110111001110100
000011010001110
↓1001010111100000
1·584962500721156
1·5
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015625
006837
00390625
00293125
001953125
000978125
0009765625
00000156322115
00000095367431640625
00000060954683959375
00000047683715820312
00000013270968139063
00000011920928955078
01350039783985
745058059697
604981124293
372529029846
272452094447
186264514923
86187579524
46566128730
39621450794
23283064365
16338386429
11641532183
4696854246
2910383040
1786471200
1455191523
331279677
181898940
149380737
90949470
58431267
45474735
12956532
11368684
1587848
1421085
166763

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125
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015625
0078125
00390625
001953125
0009765625
00048828125
000244140625
0001220703125
00006103515625
000030517578125
0000152587890625
00000762939453125
000003814697265625
0000019073486328125
00000095367431640625
000000476837158203125
0000000238418579101 5625
11920928955078125
59604644775390625
298023223876953125
1490116119384765625
7450580596923828125
37252902984619140625
186264514923095703125
931322574615478515625

465661287 4656612873077392578125
232830643 23283064365386962890625
116415321 116415321826934814453125
58207660 582076609134674072265625
29103830 2910383045673370361328125
14551915 14551915228366851806640625
727595764 72759576141834259033203125
363797880 363797880709171295166015625
181898940 1818989403545856475830078125
909494701 9094947017729282379150390625
454747350 45474735088646411895751953125
2273736754 227373675443232059478759765625
1136868377 1136868377216160297393798828125
568434 5684341881080801486968994140625
28421709305494007434844970703125
142108548527020087174224853515625
710542732635100185871124267578125
3552713663175500929355621337890625
177635683158725

To calculate $(10)^{100}$

100

$x^4 + 4x^3 + 6x^2 + 4x = 1$

$x^{100} + 100x^{11} + 110x^{10} + 100x = 1$

10110
1011
100001

11101
1101
1010111

1000100101
1000100101
11001101111

1
10000
1000100
111010000
1000100101

1011010101000
1011101010000
10001011111000

100
111000
1011000
1110100

110
10000
10110
1011
1011
10001

0
0
0
1
100
10110
1110100
1000100101
101110101000
~~100010111110000~~
11110101100100
10100010010001000
1000011001100111101
10001010000010111100

1011010100000000
1111010110010100
1111010110010100
1011100000101100

1010001001000100000
1010001001000100
1111001101100110000

1100010110000010111100

1000011001100111101

e
1110100
0

```
       0
       0
       0
       1
     100
   10110
 1110100
00100101
 0101000
 1110000
11100100
10001000
 0111101
 1011100
```

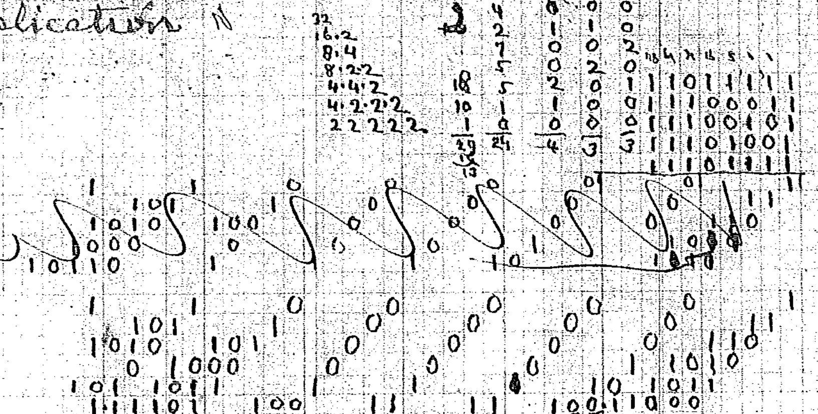
```
        e0011011100001
1110100
-0111011011
  101111011
 -0111011011
    11010001
   -0111011011
      10001111
     -0111011011
             11001
          -0111011011
               111111011
              -01111
```

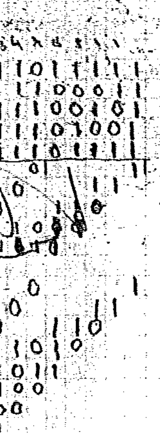
```
                  e001011111
                  00011
                  ~~000100~~
1000100101
-010001011000
  101100000000
  -010001011000
    101001011000

  10110000000
 -010001011000
 -111101011000

  10110000000
   -010001011000
    101001011000
    -010001011000
      100100001000
     -010001011000
        11001101000
       -010001011000
          1100101000
```

Continued Multiplication N

Example: Multiply together 223
227
229
233
239

229	223	128	239
128	128	64	224
101	95	32	15
64	64	3	
37	31	227	
31			
5			





64
32·2
16·4
16·2·2
8·8
8·4·2
8·2·2·2
4·4·4
4·4·2·2
4·2·2·2·2

5
10
2
1
3
10
1
1
2
1
36
32
4

0
1
0
2
0
1
0
0
2
0
6

0
1
0
2
1
1
0
0
0
6

0
0
2
0
1
2
0
2
0
0
7

0
0
0
0
0
0
4
0
6
1
11

6
24
5
35

128
64·2
32·4
32·2·2
16·8
16·4·2
16·2·2·2
8·8·2
8·4·2·2
8·2·2·2·2
4·4·4·2
4·4·2·2·2

5
6
10
2
2
3
0
5
?
0
1
0
41
32
9

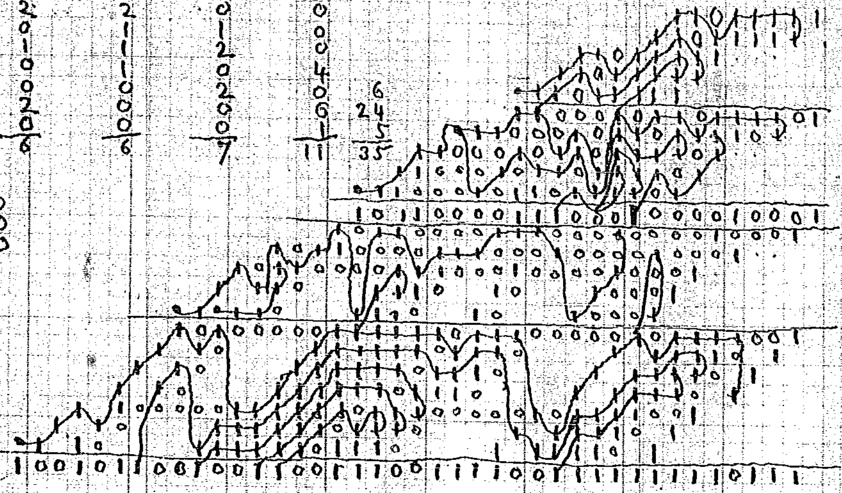
0
1
1
1
0
1
1
0
2
0
1
0
8

0
0
0

16
8
128
4

1 4 5
1 3 5
1 2 5

1325
4125
4312
4315
4325
4572
5312



To Convert Decimals into Secundals

	10^0	10^1	10^2	10^3
0	0000	0		
1	0001	00101		
2	0010	01010		
3	0011	01111		
4	0100	10100		
5	0101	11001		
6	0110	11110		
7	0111	00011		
8	1000	01000		
9 ✓	1001	01101		
10	1010	0010		

1	1	1010	1100100
2	10	10100	11001000
3	11	11110	100101100
4	100	101000	110010000
5	101	110010	111110100
6	110	111100	1001011000
7	111	1000110	1010111100
8	1000	1010000	1100100000
9	1001	1011010	1110000100
10	1010	1100100	1111101000

$10^{91} = 1+x$ $x^{10}+10x=1$

0
1
10
101
1100
11101
1000110
10101001
110011000
111011001
1001010010010
1011001101101

10
110
10
101
1100
11101
1000110
10101001
10011000

1010101001
11100011000
111011001
1001010010010
1011001101101
11011000100100
100000101011010101

0011010100000100111110011

11011000100100
-01111010100011
10101011101011
-01011101010100101
10100100000001
-0111101010100011
100001101001111
-01111110101000011
00000010100001111
-0111110101001011
1111110001011
-0111110101001011
111010110100011
-01111101010100111
110100010001
-01111101010010111
1001110011010
-01111101001001011
011010001001101
-0111110101001011
100111001110011
-0111110101001011
1101000111001

11
100011
01000011
1000101010011
0100001010011
1011111001001

$10^{91} = 11+x$ $x^{11}+11x^{10}+11x=1$

0
0
1
11
1100 ✓
101110 ✓
10110001 ✓
1010101001 ✓
1010001110 ✓
100111011000 ✓
10010111011111101 ✓

1000001

0010100010100010100011
10011101100000
-01101000010000011
011100000000011
-01101000100000011
101000111000011
-01101000100000011
011000000001111
-0110100010000011
1010010111001111
-0110100010000011
01100010011111
-01101000100000011
10011
-0110101
00

0011
101110011
001000000011
1101101001
0011010010100000011

$10^{\frac{1}{91}}$ = 0100000101000101000011
10100001010000100011
1010000101000010001
1010000101000001001

1111
1111
10111011

11101
1100
1011101

111010
111010
10101110
11
1011100
1011111
110011111
1101111
1010011101
11100
101010
1011001001
110110101
101000011110
1010101110
10100011001
110011001101
1100111001001
1001101001011
10111001
10001110100000
1010001111100
1100011001110000
10001100001100
1001010011010100
10101010001
100011011111000

11
1
100
100
1100
11100
11
1111
1111
1011101
1
101111
1100
111010
111010
10101110
11
1011100011
101110
11011111
110111111
1101001111
1100
1010101001
1011001
1111010110
1111010010
1110000001110
101110
1010011111100
1011010101
1100011100101
100010100101
[illegible]

10^{11} = 1100101110010111111101010110

$$10^{9001} = 1 + x\,(x+1)^{1000} = x^{1000} + 10000x^{111} + 11100x^{110} + 111000x^{101}$$
$$+ 1100110x^{100} + 111000x^{11} + 11100x^{10} + 1000x = 1$$

0
0
0
0
0
0
0
0
1
1000
100100
10001111000
1011110110110
1111010000000000
1010010111111101000000

11100
1000
1001100000
1110000000
111000
1000111000000
1111110000
1110000000
1000110
110110
1011110110110000
1000100000000
11111000000
1000110000
11000
1110101000011000000
0000000011111000000
10010111111101000000
110111010100000

1100111000
100111
10011100000
1000100010000000
100100
11
1100000
111111100000
10111110110110000
1111111111
1010010111111010
10001110000
1001111
11110100
1101110101000000

10101101101110000
1111111111
1001011111110100000

$$10^{0001} = 11x\ x^{1000} + 1000x^{111} + 11100x^{110} + 111000x^{101} + 1000110x^{100}$$

$$1 = x\ \{1000x^{[illegible]}\ \{11100\ x^{10}\ \{111000x^{11}\quad 1000110x$$

1000110

```
                  0  0
                  0  0
                  0  0
                  0  0
                     0
                     0
                     0
                     1
                  1000
                100100
            1000111000
         1011110110110
                     1
                  1000
               1011100
           11111111000
```

100

0
0
0
0
0
0
0
1
1000
001000
11000
10110

1
1000
11100
11000

100001100000
1010100001000000

100001100000
111111000100000

1000100100000

1
8
38
56
78
58
8
8

1
8
92
1016
11222
123990
1372448

44
23
92
184
736
224
56
1016
2032
8128
2576
448
11222
22444
89776
28448
3152
560
56
123990
24480
99120
31421
6
59896
6440
448
26
1372448

28
8
224

92
2760
184
2576

56
112
448

30480
2032
28448

336660
22444
314216

50800
9996

460
552
5152

92
6440

560
112
448

256
32
224
512
736

1111
11100
10000000
1110000
111000
1000000000000
110000
11000000
00110
10111011011010000
1010100010000000

1000000
11100
1011100000

127
8
1016

111
111
1000110

101

1000000
11100
1010000000
11000000
111000
11111000000000
1000100100000
11000000000
1000110
1010100101010110

4096
8192
2048
512
64
16
8
10838

1000
0

K 69. 1024 3010299956639812
1025 723865391773
1026 1147360775797
1028 1993114759257
1032 3679697291193
1040 7033339298780
1056 23663918197793
1088 36628895362161

423908951961
423495384034
845753983460

4239008951961
847404135985
1693158119445
3379740651381
6733382658968
13363961557981
26328038722349
51152522447381
96910013008057
176891259055681

7
4
26328938722349

096910013008658
45757490560675
51152522447381

397940008672038
346787486224656
51152522447382

698970004336019
522878745280338
176091259055681

397940008672038
301029995663981
96910013008057

~~(1+1)~~

~~(x+1)~~

~~(1+x)~~

$(x+1)^{10} = x^{10} + 10x + 1$

$(x+1)^{100} = x^{100} + 100x^{11} + 110x^{10} + 100x$

$b^{x+1} - b^{x} = b^{x}(b-1)$

$2^{x+1} - 2^{x} = 2^{x}$

$2^{x+\frac{1}{10}} - 2^{x} = 2^{x}\left(2^{\frac{1}{10}} - 1\right)$

$10^{x+\frac{1}{100}} - 2^{x} = 2^{x}\left(2^{\frac{1}{100}} - 1\right)$

0
1
10
101
1100
11101
1000110

10^{-1}

$10^{\frac{1}{10}} = 1 + x$

1 8 28 0 0 0 0
1 7 21 56
1 6 15 35 70
4 10 20 35
6 10 15 21
1 5 6 7
1 1 1
1

$(1+x)^{n}$

$\frac{32}{1}$

$(1+x)^{n} =$

$\frac{32}{1}\ \frac{31}{2}\ \frac{30}{3}$

$(1+x)$

1
8
1 4 28
2 6 56
1 4 70
1

.000000

1
2
3
12
29
70
169
398
965
2328
5621

1
32

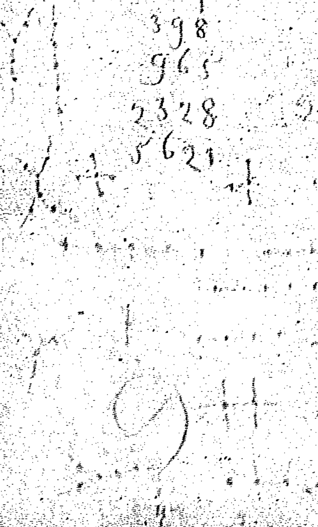
1+1+1 | 1+1+1+1 | 1+1+1+1+1
1+2 | 2+1+1 | 3+1+1
3 | 2+2 | 2+2+1
3+1 | 3+1+1
4 | 3+2
4+1
5

$(1+x)^{1024} = 1 + 1024x$

1+x 1-x

2

$(1+x)^{10000000000} = 1 + 10000000000x$



99 9901
70 9801
4+24
4900
2 +34
9801
3+24
9+4+34
8 +14
4+24
7+20 14
7+4+3+24
24 144
8 144

1 1 0 0
1 1 0 1
1 1 0 1
2 1 0 1
3 1 1
5 2 2
9 4 5 3
18 9 8 6
35 17 14
75 40 27

$= 1 + 1$

$$\Delta^0 + \frac{x}{1}\Delta^1 + \frac{x(x-1)}{2!}\Delta^2 + \frac{x(x-1)(x-2)}{3!}\Delta^3$$

Put $x = \frac{1}{2}$ $\qquad 1 \cdot \frac{1}{2} \cdot \frac{1}{2} \; \frac{1}{3}$

$$\Delta^0 + \frac{1}{2}\Delta^1 + \frac{1}{8}\Delta^2 + \frac{1}{16}\Delta^3 + \frac{5}{128}\Delta^4$$

1876 Paris Pend. swings. Diffs of transits right & left.

Arc	Jan 25 up	Calc	O−C	down	Calc	O−C	Jan 26 up	
2	+016			−006			−027	−010
1½	+ 4			000			−008	−013
1	− 3			−008			−022	−020
0½	− 9			+037			−028	−041
Sum							85	

2

Paris Pond sweeps Diff. of Arc.

1876 Jan 26 up.

Direct obs. W 1′(2′)

Deduced from times

Arc	W-E	Col diff arc
75′	16^s	~~(357)~~ (.4)
70	22	.94
55	37	(1.45)
50	57	1.56
45	53	(1.28)
40(?)	74	1.56
35	69	

Concluded 1.56

Arc at time transit	Δt	2φ/A	2φ
116′.9	−027	.085	9′.9
90.0	−008	.025	2.25
60.9	−022	.069	4.2
29.8	−028	.088	2.6

At obs	Calc	O-C
−027	4	23
−008	6	2
−022	8	14
−028	17	11

Down

Obs E 1′(2′)

		E-W	
1	50	57	2.11
	40	63:	2.02
	20	104	2.37
	10	132	2.41
0	40	222	2.04
			10.1

Concluded 2.20

At obs	Calc	O-C
−047	6	−20
+004	8	−4
+040	12	+28
−007	23	30 (16)

Jan 28 Down

obs W ½′ (1′)

		W	
2	10	10:	
1	50	9:	
1	40	11:	
1	20	7:	
	50	33	0.41
	40	13	0.12

Concluded 0.26

up E ¾ (1.5)

1	50	30:	2.50
1	20	60	3.12
1	10	79	3.38

Concluded 3.25

Jan 29 up E 1′(2′)

		E	
1	20	67:	
	10	69::	
1	30	57	3.52
	20	69	3.60
	10	88	3.76
	50	103:	
	40	172	3.63

Concluded 3.63

Feb 2 up W ½′ (1′)

1	10	19	0.81
	50	38	1.06
	40	53	1.12

Concluded 1.09

down E ¾ (1.5)

		E	
1	50	63	1.96
	40	69	2.21
1	50	57	2.11
	40	77(61)	(2.15)
	20	102	2.33
	10	115	2.15
0	50	176	2.19
	40	269	
			5.5

Concluded 2.16