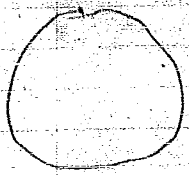
# Notes on Ptolemy

$v = M + 2e\sin M + \frac{5}{4}e^2\sin 2M + \frac{1}{12}e^3(13\sin 3M - 3\sin M)$

$-13$
$-3$
$-16$

Put $M = 90^\circ$ $\sin 90^\circ = 1$ $\sin 2\times 90 = 0$

$v = 90^\circ + 2e - \frac{4}{3}e^3$



$-\sin M\cos M$

~~$v=$~~

$\sin v = \sin M\cos[\cancel{v} - M)M + \frac{5}{2}\quad -2\sin\cos^2 - \sin^3$
$\sin$

$+ \cos M\sin[v - M)$

$\sin v - M$

$x$

$= 2e\sin M + \frac{5}{4}e^2\sin 2M + \frac{1}{12}e^3(13\sin 3M - 3\sin M)$

$-\frac{1}{6}x^3$

$- \frac{16}{12}e^3\sin^3 M$

$= 2e\sin M + \frac{5}{2}e^2\sin M\cos M + \frac{1}{12}e^3(26\sin M\cos^2 M - 3\sin^3 M - 3\sin M)$

$\cos v - M = 1 - 2e^2\sin^2 M + 5e^3\sin^2 M\cos M$

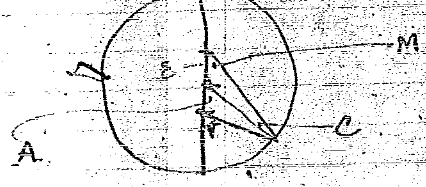
$-\frac{1}{2}x^2$

$\sin v = \sin M($ ~~$1 + 2e\cos M - 5e^3\sin^2 M\cos M$~~

$1 + 2e\cos M - 2e^2\sin^2 M - 5e^3\sin^2 M\cos M + \frac{5}{2}e^2\cos^2 M + \frac{1}{12}e^3(26\cos^3 M - 3\sin^2 M\cos M - 3\cos M)$

$= \sin M\ (1 + 2e\cos M + e^2(\frac{5}{2} - 4\frac{1}{2}\sin^2 M))$

# Notes on Ptolemy



$\sin C = \varepsilon \sin M$

$\arcsin x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5$

~~arcsin~~

$C = \arcsin \varepsilon \sin M = \varepsilon \sin M + \frac{1}{6}\varepsilon^3 \sin^3 M + \frac{3}{40}\varepsilon^5 \sin^5 M$

$A = M + \varepsilon \sin M + \frac{1}{6}\varepsilon^3 \sin^3 M + \frac{3}{40}\varepsilon^5 \sin^5 M$

$r^2 = 1 + \varepsilon^2 - 2\varepsilon \cos \cancel{M}\, A$

$\sin^2 C = \varepsilon^2 \sin^2 M$

$(1+x)^{\frac{1}{2}}$

$\frac{1}{2}(1+x)^{-\frac{1}{2}}$

$-\frac{1}{8} \quad -\frac{1}{4}(1+x)^{-\frac{3}{2}}$

$+\frac{1}{16} \quad +\frac{3}{8}(1+x)^{-\frac{5}{2}}$

$-\frac{5}{128} \quad -\frac{35}{16}(1+x)^{-\frac{7}{2}}$

$\frac{1}{2}\sin^2 - \frac{1}{2}\frac{1}{2}\sin^4$

$A = M + C$

$\cos A = \cos M \cos C - \sin M \sin C$

$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4$

$\sqrt{1-x} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4$

$\frac{1 + 3\sin^2}{-1 + \sin^2}$

$\cos C = 1 - \frac{1}{2}\varepsilon^2 \sin^2 M - \frac{1}{8}\varepsilon^4 \sin^4 M$

$\cos A = \cos M - \varepsilon \sin^2 M - \frac{1}{2}\varepsilon^2 \sin^2 M \cos M \quad - \frac{1}{8}\varepsilon^4 \sin^4 M \cos M$

$\varepsilon \cos A = \varepsilon \cos M - \varepsilon^2 \sin^2 M - \frac{1}{2}\varepsilon^3 \sin^2 M \cos M - \frac{1}{8}\varepsilon^5 \sin^4 M \cos M$

$-2\varepsilon \cos A = -2\varepsilon \cos M + 2\varepsilon^2 \sin^2 M + \varepsilon^3 \sin^2 M \cos M + \frac{1}{4}\varepsilon^5 \sin^4 M \cos M$

$r^2 = 1 - 2\varepsilon \cos M + \varepsilon^2(1 + 2\sin^2 M) + \varepsilon^3 \sin^2 M \cos M + \frac{1}{4}\varepsilon^5 \sin^4 M \cos M$

$r = 1 - \varepsilon \cos M + \frac{1}{2}\varepsilon^2(1 + 2\sin^2 M) + \frac{1}{2}\varepsilon^3 \sin^2 M \cos M + \frac{1}{8}\varepsilon^5 \sin^4 M \cos M$

$-\frac{1}{2}\varepsilon^2 \cos^2 M \quad + \frac{1}{2}\varepsilon^3 \cos M (1 + 2\sin^2 M) + \frac{1}{4}$ [illegible]

$-\frac{1}{8}\varepsilon^4(1 + 2\sin^2 M)^2 + \frac{1}{4}\varepsilon^5(1 + 2\sin^2 M)$ [illegible]

$-\frac{1}{2}\varepsilon^3 \cos^3 M \quad -\frac{13}{4}\varepsilon^4 \cos M (1 + 2\sin^2 M) - \frac{13}{4}\varepsilon^5$ [illegible]

$\frac{1}{16}$ [illegible]

$r = 1 - 2\varepsilon \cos M + \frac{3}{2}\varepsilon^2 \sin^2 M + 2\varepsilon^3 \sin^4 M \cos M$

$\sin 3M$

$= \sin 2M \cos M + \cos 2M \sin M$

$= 3 \sin M \cos^2 M + \cancel{\cos^2 M \sin M}$

$- \sin^3 M \qquad - \frac{13}{3}$

$= 3 \sin M - 4 \sin^3 M$

$13 \sin 3M = \underset{26}{39} \sin M - 52 \sin^3 M$

$v = M + (v - M)$

$\sin v = \sin M \cos(v - M) + \cos M \sin(v - M)$

$v - M = 2e \sin M + \frac{5}{2} e^2 \sin M \cos M + e^3 \left(3 \sin M - \frac{13}{3} \sin^3 M\right)$

$(v - M)^2 \qquad 4 e^2 \sin^2 M \qquad + e^3 (10 \sin^2 M \cos M)$

$(v - M)^3 \qquad e^3 (8 \sin^3 M)$

$\sin(v - M) = 2e \sin M + \frac{5}{2} e^2 \sin M \cos M) + e^3 \left(3 \sin M - \frac{17}{3} \sin^3 M\right)$

$\cos(v - M) = 1 - 2 e^2 \sin^2 M - 5 e^3 \sin^2 M \cos M$

$\sin v = \sin M \qquad - 2 e^2 \sin^3 M - 5 e^3 \sin^3 M \cos M$

$+ 2e \sin M \cos M + \frac{5}{2} e^2 (\sin M - \sin^3 M) + e^3 \left(3 \sin M \cos M - \frac{17}{3} \sin^3 M \cos M\right)$

$= \sin M \left(1 + 2e \cos M + e^2 \left(\frac{5}{2} - 4\frac{1}{2} \sin^2 M\right) + e^3 \left(3 - \frac{32}{3} \sin^2 M\right) \cos M \right)$

$$r^2 = 1 + \varepsilon^2 - 2\varepsilon\cos A$$

$$= 1 - 2\varepsilon\cos M + \varepsilon^2(1 + 2\sin^2 M) + \varepsilon^3 \sin^2 M \cos M$$

$$1 - \tfrac{1}{2}x + \tfrac{3}{8}x^2 - \tfrac{5}{16}x^3$$

$$\cancel{1 - x + x^2 - x^3}$$

$$\begin{array}{ll}
x & -2\varepsilon\cos M + \varepsilon^2(1+2\sin^2 M) + \varepsilon^3 \sin^2 M\cos M \\
x^2 & \qquad\qquad \varepsilon^2(4 - 4\sin^2 M) + \varepsilon^3(-4 - 8\sin^2 M)\cos M \\
x^3 & \qquad\qquad\qquad\qquad + \varepsilon^3(-8 + 8\sin^2 M)\cos M
\end{array}$$

$$\begin{array}{ll}
-\tfrac{1}{2}x & 1 + \varepsilon\cos M + \varepsilon^2\left(-\tfrac{1}{2} - \sin^2 M\right) + \varepsilon^3\left(-\tfrac{1}{2}\right)\sin^2 M\cos M \\
 & \qquad + \varepsilon^2\left(\tfrac{3}{2} - \tfrac{3}{2}\sin^2 M\right) + \varepsilon^3\left(-\tfrac{3}{2} - 3\sin^2 M\right)\cos M \\
 & \qquad\qquad + \varepsilon^3\left(+\tfrac{5}{2} - \tfrac{5}{2}\sin^2 M\right)\cos M
\end{array}$$

$$\frac{1}{r} = 1 + \varepsilon\cos M + \varepsilon^2(1 - 2\sin^2 M) + \varepsilon^3(1 - 6\sin^2 M)\cos M$$

$$\begin{array}{ll}
\sin V & = \sin M\Big(1 + \varepsilon\cos M + \varepsilon^2(1 - 2\sin^2 M) + \varepsilon^3(1 - 6\sin^2 M)\cos M \\
 & \qquad\qquad \varepsilon\cos M + \varepsilon^2(1 - \sin^2 M) + \varepsilon^3(1 - 2\sin^2 M)\cos M \\
 & \qquad\qquad\qquad + \varepsilon^2\left(-\tfrac{1}{2}\sin^2 M\right) + \cancel{\varepsilon^2\left(\tfrac{1}{2} + \sin^2\right)} \\
 & \qquad\qquad\qquad\qquad + \varepsilon^3\left(-\tfrac{1}{2}\sin^2 M\right)\cos M
\end{array}$$

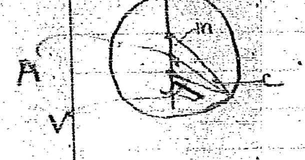
$$\sin V = \sin M\left[1 + 2\varepsilon\cos M + \varepsilon^2\left(2 - 2\tfrac{1}{2}\sin^2 M\right) + \varepsilon^3\left(2 - 8\tfrac{1}{2}\sin^2 M\right)\cos M\right]$$

$(1+x)^{\frac12}$

$\frac12(1+x)^{-\frac12}$

$\frac18 \quad -\frac14(1+x)^{-\frac32}$

Repeated calculation $\quad \frac1{16} \quad \frac38(1+x)^{-\frac52}$



$\sin C = \varepsilon \sin M \quad \frac{5}{2.8.4.16} \quad \frac{3.5}{16}(1+x)^{-\frac72}$

$\sin^2 C = \varepsilon^2 \sin^2 M$

$\sqrt{1-x} = 1 - \frac12 x - \frac18 x^2 - \cancel{x}$

$\cos C = 1 - \frac12 \varepsilon^2 \sin^2 M - \frac18 \varepsilon^4 \sin^4 M$

$A = M + C$

$\sin A = \sin M\left(1 + \varepsilon \cos M - \frac12 \varepsilon^2 \sin^2 M - \frac18 \varepsilon^4 \sin^4 M\right)$

$\cos A = \cos M\left(1 - \varepsilon \frac{\sin^2 M}{\cos M} - \frac12 \varepsilon^2 \sin^2 M - \frac18 \varepsilon^4 \sin^4 M\right)$

$\cos^2 A = \cos^2 M\left(1 - 2\varepsilon \frac{\sin^2 M}{\cos M} + \varepsilon^2\left(\frac{2\sin^4 - \sin^2)}{\cos^2 M}\right) + \varepsilon^3 \frac{\sin^4 M}{\cos M}\right.$

$r^2 = 1 + e^2 - 2e \cos A$

$\frac{\sin^4}{\cos^2} - \sin^2$

$\sqrt{1+x} = 1 + \frac12 x - \frac18 x^2 + \frac1{16} x^3 - \frac5{128} x^4$

$x \quad -2e\cos A + e^2 \quad \frac12 x \quad -e\cos A + \frac12 e^2$

$x^2 \quad 4e^2\cos^2 A - 4e^3 \cos A \quad -\frac18 x^2 \quad -\frac12 e^2\cos^2 A + \frac12 e^3 \cos A$

$x^3 \quad -8e^3\cos^3 A \quad \frac1{16} x^3 \quad \rightarrow \frac12 e^3 \cos^3 A$

$+\frac12 \varepsilon^3 \sin^2 A \cos A$

$r = 1 - e\cos A + \frac12 e^2 \sin^2 A$

$x \quad -2\varepsilon\cos A + \varepsilon^2 \quad -\frac12 x \quad \varepsilon\cos A - \frac12 \varepsilon^2$

$x^2 \quad 4\varepsilon^2\cos^2 A - 4\varepsilon^3 \cos A \quad \frac38 x^2 \quad \frac32 \varepsilon^2\cos^2 A - \frac32 \varepsilon^3 \cos A$

$x^3 \quad -8\varepsilon^3\cos^3 A \quad -\frac5{16} x^3 \quad \frac52 \varepsilon^3 \cos^3 A$

$\sin V = \frac{1}{r} \sin A$

$\frac{1}{1+x} = 1 - x + x^2 - x^3$

$-x \quad +e\cos A - e^2 \sin^2 A \quad -\frac12 \varepsilon^3 \sin^2 A \cos A$

$x^2 \quad e^2\cos^2 A \quad -2\varepsilon^3 \sin^2 A \cos A \quad -\sin^2 \cos$

$-x^3 \quad +e^3 \cos^3 A$

$\frac1r = 1 + \varepsilon \cos A + e^2 \overset{1-\sin^2 A}{\cos^2 A} + \varepsilon^3 \overset{2\frac12}{\left(\sin^2 A \cos A + \cos^3 A\right)}$

$= 1 + \varepsilon \cos M - \varepsilon^2 \sin^2 M - \frac12 \varepsilon^3 \sin^2 M \cos M$

$+ \varepsilon^2 \cos^2 M - 2\varepsilon^3 \sin^2 M \cos M$

$= 1 + \varepsilon \cos M + \varepsilon^2(1 - 2\sin^2 M) - 3\frac12 \varepsilon^3 \sin^2 M \cos M$

$\sin V = \frac{1}{r} \sin A$

$\sin A = \sin(M+C) = \sin M \cos C + \cos M \sin C$

$= \sin M + \varepsilon \sin M \cos M - \frac{1}{2}\varepsilon^2 \sin^3 M - \frac{1}{8}\varepsilon^4 \sin^4 M$

$= \sin M \left(1 + \varepsilon \cos M - \frac{1}{2}\varepsilon^2 \sin^2 M - \frac{1}{8}\varepsilon^4 \sin^3 M\right)$

$\frac{1}{r} = 1 - r + r^2 - r^3$

$$\begin{array}{l|l}
\frac{1}{r} = & 1 \\
-r & +2\varepsilon \cos M - \frac{3}{2}\varepsilon^2 \sin^2 M - 2\varepsilon^3 \sin^4 M \cos M \\
+r^2 & \qquad +4\varepsilon^2 \cos^2 M - 6\varepsilon^3 \sin^2 M \cos M \\
-r^3 & \qquad\qquad +8\varepsilon^3 \cos^3 M
\end{array}$$

$= 1 + 2\varepsilon \cos M + \varepsilon^2 (4 - 5\tfrac{1}{2}\sin^2 M) + \varepsilon^3 \cos M (8 - 14 \sin^2 M - 2 \sin^4 M)$

$\sin V = \sin M \left(1 + 3\varepsilon \cos M + \varepsilon^2 (\underset{2\,-\,2}{6 - \overset{8}{6} \sin^2 M}) + \varepsilon^3 \cos M \left(\underset{4\;-\tfrac{1}{2}\;-1}{\overset{12\quad 20\frac{1}{2}}{8 - 14 \sin^2 M}} - 2\sin^4 M\right.\right.$

$\sin V = \frac{1}{r}\sin A$

$= \sin M\Big(1 + 2\varepsilon\cos M + \varepsilon^2\big(-\frac{1}{2}\sin^2 M \quad + \varepsilon^3\big(-2\frac{1}{2}\sin^2 M\cos M$

$1 - 2\sin^2 M \qquad + \cos M - 2\sin^2 M\cos M$

$1 - \sin^2 M \qquad -\frac{1}{2}\sin^2 M\cos M$

$= \sin M\Big(1 + 2\varepsilon\cos M + \varepsilon^2\big(2 - 3\frac{1}{2}\sin^2 M\big) + \varepsilon^3\big(\cos M\big)\Big)$

## Partial repetition

$(1+x)^{-\frac{1}{2}}$

$-\frac{1}{2}(1+x)^{-\frac{3}{2}}$

$+\frac{3}{4}(1+x)^{-\frac{5}{2}}$

$-\frac{15}{8}(1+x)^{-\frac{7}{2}}$

$r^2 = 1 + \varepsilon^2 - 2\varepsilon\cos A$

$= 1 - 2\varepsilon\cos M + \varepsilon^2(1 + 2\sin^2 M) + \varepsilon^3\sin^2 M\cos M$

$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{\cancel{48}\;16}x^3$

$\frac{1}{r} = x$

~~$x \quad -2\varepsilon\cos M + \varepsilon^2$~~

$x \quad -2\varepsilon\cos M + \varepsilon^2(1 + 2\sin^2 M) + \varepsilon^3\sin^2 M\cos M$

$x^2 \quad 4\varepsilon^2\cos^2 M \quad -4\varepsilon^3(\cos M + 2\sin^2 M\cos M)$

$x^3 \quad -8\varepsilon^3\cos^3 M$

$-\frac{1}{2}x \quad \varepsilon\cos M + \varepsilon^2\big(-\frac{1}{2} - \sin^2 M\big) - \frac{1}{2}\varepsilon^3\sin^2 M\cos M$

$+\frac{3}{8}x^2 \quad \varepsilon^2\big(+\frac{3}{2} - \frac{3}{2}\sin^2 M\big) - 3\varepsilon^3\sin^2 M\cos M - \frac{3}{2}\varepsilon^3\cos M$

$-\frac{5}{16}x^3 \quad -\frac{5}{2}\varepsilon^3\sin^2 M\cos M + \frac{5}{2}\varepsilon^3\cos^3 M$

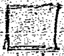
$\frac{1}{r} = 1 + \varepsilon\cos M + \varepsilon^2(1 - 2\sin^2 M) + \varepsilon^3\big(-6\sin^2 M\cos M +$ [illegible] $\cos M\big)$

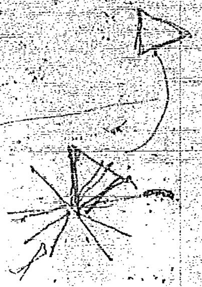
$\frac{(x^4-1)}{(x^2-1)} = (x^2+1)$

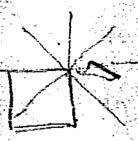
$x^2-1\,)\,x^8-1\;(x^6+x^4+x^2+1$

$x^6-x^4$

$x^4-1$

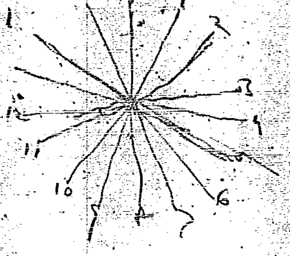






$x^3-1\,)\,x^7-1\;(x^6+x^3+1$

$\frac{(x^{10}-1)(x-1)}{(x^5+1)(x^2-1)}$ $x^7-x^5+x^2+1$



4

$7)\overline{180}$

25 $\frac{5}{7}$

5, $\frac{5}{7}$

77

$x^4-x^3+x^2-x+1$

$x^{11}-x^{10}+x+1$

$x^{11}-x^9+x^6+x^4$

~~$x^6-x^4+x-1$~~

$-x^{10}+x^9+x^6+x^4+x+1$

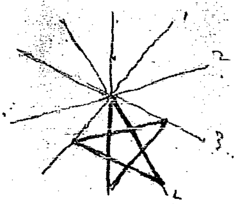
$-x^{10}+x^8+x^5+x^3$

$x^9-x^8+x^6+x^5+x^4+x^3+x+1$

$x^9-x^7+x^4+x^2$

$-x^8+x^7+x^6+x^5+x^3+x^2+x+1$

$-x^8+x^6+x^3+x$



$\frac{9}{7}$ 6

Regular 7 points
8 star.

$x^9-x^7-x^2+1$

$x^{15}-x^{14}-x+1\;(x^6-x^5+x^4-x^3+x^2-x+1$

$x^{15}-x^{13}-x^8+x^6$

$-x^{14}+x^{13}+x^8-x^6-x+1$

$-x^{14}+x^{12}+x^7-x^5$

$x^{13}-x^{12}+x^8-x^7-x^6+x^5-x+1$

$x^{13}-x^{11}-x^6+x^4$

$-x^{12}+x^{11}+x^8-x^7+x^5-x^4-x+1$

$-x^{12}+x^{10}+x^5-x^3$

$x^{11}-x^{10}+x^8-x^7-x^4+x^3-x+1$

$x^{11}-x^9-x^4+x^2$

$-x^{10}+x^9+x^8-x^7+x^3-x^2-x+1$

$-x^{10}+x^8+x^3-x$

$\frac{(x^{10}-1)(x^2-1)}{(x^6-1)(x^4-1)}$

$x^{10}-x^6-x^4+1$

$x^{12}-x^{10}-x^2+1\;(x^2$

$x^{12}-x^8-x^6+x^2$

~~$-x^{10}+x^8+x^6-2x^2+1$~~

$x^{14}-x^{12}-x^2+1\;(x^4-x^2+1$

$x^{14}-x^{10}-x^8+x^4$

$-x^{12}+x^{10}+x^8-x^4-x^2+1$

$-x^{12}+x^8+x^6-x^2$

$x^{10}-x^6-x^4+1$

~~3 4~~ , 1 -3

~~5 2~~ 2 -5

## Prismatoid

$\frac{1}{6} h(B+4\beta+b)$

$$\begin{array}{c|ccc} & a & 2b & c \\ \hline a & (1 & -6 & 9) \\ 2b & (2 & -11 & 15) \\ c & 4 & -20 & 25 \end{array}$$

$$\begin{array}{ccc} a & -6a & 9a \\ 4b & -22b & 30b \\ 4c & -20c & 25c \\ & -2b & +5b \\ & +4c & -10c \\ \hline 1 & -2 & -1 \end{array}$$

$b = -c$

~~$a = -2b$~~

$b = -2a$

$a = 1 \quad b = -2 \quad c = 2$

$x^2 - 2xy + 2y^2$

$$\begin{array}{ccc} 1 & -6 & 9 \\ -8 & +44 & -60 \\ 8 & -40 & 50 \\ \hline 1 & -2 & -1 \end{array}$$

$x^2 - 2xy - y^2$

$\xi^2 - 2\xi\eta - \eta^2$

$b = \frac{2}{\frac{1}{a} + \frac{1}{c}}$

$b = 2\frac{1}{\frac{1}{a} + \frac{1}{c}}$

$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

$\frac{1}{c} = \frac{2}{b} - \frac{1}{a}$

$c = \frac{1}{\frac{2}{b} - \frac{1}{a}}$

$\frac{2}{b} = \frac{1}{a} - \frac{1}{c}$

$c = \frac{1}{\frac{1}{a} - \frac{2}{b}}$

$\frac{1}{a} > \frac{2}{b}$

$b > 2a$

2 3

1 3

$\frac{1}{1} - \frac{2}{3} = \frac{1}{3}$

$\frac{1}{1} - \frac{2}{5} = \frac{3}{5}$

1 5 $\frac{5}{3}$

$\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

$\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

$\frac{8}{5}$

2 3 6 ~~∞~~

6 7 $\frac{42}{5}$ $\frac{21}{2}$ 14 21 42

60 70 84 105 140 210 420

$\frac{7}{7} - \frac{1}{6}$

$\frac{8}{42} - \frac{5}{42} = \frac{3}{42} = \frac{1}{14}$ $\frac{5}{21} - \frac{3}{21}$

$\frac{3}{21} - \frac{2}{21} = \frac{1}{21}$

$\frac{2}{21}$

A

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

B

1
2
3
4
5
6
7
8
9
10
11
12
13
14

C

1
2
3
4
5
6
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D

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