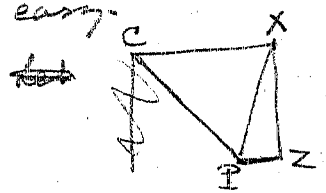
A simple pendulum is a figment of the mathematicians. It is supposed to consist of an immaterial line terminated by a material point. ~~The other end is fixed~~ The other end is fixed. A force along the line is supposed to retain the material point upon the arc of a certain circle having its centre at the fixed end, while ~~the pendulum~~ oscillated backward & forward on the arc under the influence of a constant ~~vertical~~ force.
The circle in which the point oscillates is not necessarily in a vertical plane.

A pendulum is an apparatus designed to oscillate from a fixed centre, under the influence of gravity ~~or some other force~~.
A simple pendulum is an imaginary body without magnitude which oscillates under the influence of a vertical force on the arc of a circle ~~~~ ~~away~~ upon which a second force compels it to remain.

The mathematical analysis of the problem of the simple pendulum is easy.



Let C be the fixed centre
P the pendulum
PX a perpendicular let fall from P onto the ~~~~ intersection of

the plane of ~~motion~~ the circle with the horizontal plane through C.

XZ is the intersection of the vertical plane through CX with the vertical plane through PX.

Let λ be the fixed length CP, which is called the length of the pendulum

Let θ be the complement of the angle PCX

Let φ be the angle PXZ which is the inclination of its plane of oscillation ~~to the~~ vertical.

~~~~

~~The~~ motion of the point P, is subject to two simultaneous & superposed accelerations, the first is a constant
~~~~

vertical acceleration due to the vertical force. We may denote this by ηg. ~~The other~~ ~~is an acceleration along~~ This tends to diminish ϑ and increase λ. The other accele-ration is toward C and is of such magnitude as to keep λ constant.

~~Denote CX by x, and XZ~~

~~by z. Then we have~~

~~We have first to find an equation expressing the value of $D_t^2 s$ where s is the length of the arc traversed by the point P in the time t. In other words, $D_t^2 s$ is the acceleration of P along its path.~~

$$-\frac{1}{y^2 z^2 + \frac{x^2}{\cancel{y^2+x^2}}} \times z \; \frac{1}{\cancel{y^2+z^2}} \; \frac{1}{\sqrt{y^2+z^2}}$$

$$(y^2+z^2)^{-\frac{1}{2}}$$

$$-\varphi = \arctan \frac{z}{y}$$

$$-\tfrac{1}{2}(y^2+z^2)^{-\frac{3}{2}} z$$

$$\frac{1}{y} \; \frac{1}{1+\frac{z^2}{y^2}}$$

$$= \frac{y}{y^2-z^2} \; \frac{xz}{x^2+y^2+z^2} \; \frac{1}{\sqrt{y^2+z^2}}$$

$$\frac{\lambda \sin\varphi \; \cancel{\ }}{(\lambda^2 \sin^2\varphi + z^2 \tan^2\psi + z^2)} = -\frac{y}{\cos\psi}$$

$$(\qquad + z^2 \sec^2\psi) \quad y = z \tan\psi$$

$$= -\frac{\lambda z \tan\psi}{z^2 \tan^2\psi + z^2}$$

$$-\frac{1}{z} \; \frac{1}{2} \sin 2\psi$$

If we reckon s from the lowest point of the path, we have

(1) $s = \lambda\varphi$

and as λ is constant

(2) $D_t^2 s = \lambda D_t^2 \varphi$

From these data we have to find the relation between φ and t, the time past any fixed epoch.

Now of the two accelerations only the first ~~produces~~ has any effect upon φ. We have therefore ~~first~~ to inquire what acceleration of the change in the value of φ is produced by the vertical acceleration of the motion of P equal to g; and this acceleration of the change of φ will be the ~~complete~~ $D_t^2 \varphi$.

Now to find the effect upon φ of a ~~ver~~ tical acceleration we must

~~To determine the~~

This is a purely geometrical problem. Denote CX by x and XZ by z. Then we have by the figure

$$XZ \sec PXZ = PX = CX \, \cancel{\tan} \, \text{cotan} \, PCX$$

or

$$z \sec \psi = x \, \text{cotan} \, \varphi$$

or

$$\text{cotan} \, \varphi = \cancel{\frac{x}{z \sec \psi}} \; \frac{z}{x} \sec \psi$$

Differentiate, letting z vary while x remains constant, and we have

$$-\frac{1}{(\sin \varphi)^2} \delta \varphi = \frac{\sec \psi}{x} \delta z$$

We may here replace x by the given quantity λ by means of the equation

$$\lambda \sin \varphi = x$$

$D_y (D_x y)^2$

$2 \cancel{(\ )} D_x^2 y$

$D_y (D \quad 2 (D_x y)^2$

$2 (D_x y) \, d (D_x y)$

and this gives us

$$(2) \quad \delta \varphi = - \frac{1}{\lambda \cos \psi} \sin \varphi . \delta Z$$

Accordingly, we have

$$(3) \quad D_t^2 \varphi = - \frac{1}{\lambda \cos \psi} \sin \varphi \; g$$

This is the differential equation of the motion of a simple pendulum. In order to integrate it, it is to be observed that, in general,

$$D_y . (D_x y)^2 = D_y x \, D_x (D_x y)^2$$

$$= D_y x . D_x (D_x y) \, D_{D_x y} (D_x y)^2$$

$$= D_y x . D_x^2 y . (2 D_x y)$$

$$= 2 D_x^2 y$$

Hence

$$D_\varphi (D_t \varphi)^2 = - \frac{2}{\lambda \cos \psi} \sin \varphi . g$$

where $\pm A$ the extreme values ~~value values~~ ~~of φ~~. For if $\cos\varphi < \cos A$, $D_t\varphi$ should be imaginary, which cannot happen for any real value of t.

$$D_\varphi t = \frac{1}{\sqrt{\tfrac{1}{2}\lambda\cos\psi}} \sqrt{\frac{\tfrac{1}{2}\lambda\cos\psi}{\cos\varphi - \cos A}}$$

Integrating this we have

$$(D_t\varphi)^2 = \frac{2}{\lambda\cos\psi}(\cos\varphi - \cos A) \quad (4)$$

where $\pm A$ are the values of φ for which $D_t\varphi$ vanishes.

This equation might have been reached, at once, by the principle of living forces.

Taking φ as our independent variable we have by a transformation of (4)

$$D_\varphi t = \frac{1}{\sqrt{C + \frac{2}{\lambda\cos\psi}\cos\varphi}}$$

The integral of this may be expressed as an elliptic function, but it is more useful to develope it into

* A series ascending to powers of φ is much to be preferred to those ascending to powers of $\sin\varphi$ or $\tan\varphi$ —

a series of ascending * powers of φ -

~~D~~

$$\frac{1}{\sqrt{\cos\varphi - \cos A}}$$

$$D_\varphi (\cos\varphi - \cos A)^{-\frac{1}{2}}$$

$$= +\tfrac{1}{2}(\cos\varphi - \cos A)^{-\frac{3}{2}} \sin\varphi$$

$$D^2 = +\tfrac{3}{4}(\cos\varphi - \cos A)^{-\frac{5}{2}} \sin^2\varphi$$

$$+\tfrac{1}{2}(\cos\varphi - \cos A)^{-\frac{3}{2}} \cos\varphi$$

$$D^3 = +\tfrac{15}{8}(\cos\varphi - \cos A)^{-\frac{7}{2}} \sin^3\varphi \;\cancel{\cos\varphi}$$

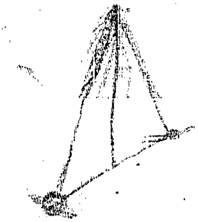
$$+\tfrac{9}{4}(\cos\varphi - \cos A)^{-\frac{5}{2}} \sin\varphi\cos\varphi$$

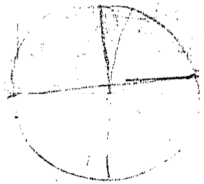
$$-\tfrac{1}{2}(\cos\varphi - \cos A)^{-\frac{3}{2}} \sin\varphi$$

$$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots$$

$$\cos\varphi - \cos A = \frac{1}{2}(\varphi^2 - A^2) + \frac{1}{4!}(\varphi^4 - A^4) - \frac{1}{6!}(\varphi^6 - A^6) + \ldots$$

$$-\frac{d\cos A}{\sin A} = dA$$



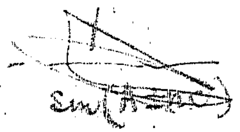


$$\frac{1}{\cos x - \cos A} = \infty$$

$$x = \pm A + 2n\partial$$

$$f(x) = -\frac{1}{\sin x}$$

$\log(x - A)$



$$-\frac{1}{\sin(\pm A + 2n\partial)} \quad \frac{1}{x \mp A - 2n\partial}$$

$$-\frac{1}{\sin A}\frac{1}{(x-A)} + \frac{1}{\sin A}\frac{1}{x+A}$$

$$-\frac{1}{\sin A}\frac{1}{x-A-2\partial} + \frac{1}{\sin A}\frac{1}{x+A-2\partial}$$

$$-\frac{1}{\sin A}\frac{1}{x-A+2\partial} + \frac{1}{\sin A}\frac{1}{x+A+2\partial}$$

Compound Pendulum

A number of masses $m_1\ m_2\ m_3$ etc each receives the same vertical acceleration, g. Also forces towards the centre which make $l_1\ l_2\ l_3$ etc their radii constant.

Also forces which make the angular velocity of all the same. These forces are such as to produce by themselves no rotation (Action & Reaction constants)

The forces acting on the masses of a compound pendulum are

1st, The equal vertical acceleration of all the masses

2nd, The action of the masses on each other. If ~~these~~ components of the consequent accelerations be denoted by ~~$x_1''\, x_2''$ etc~~ $x_1''\ x_2''$ etc $y_1''\ y_2''$ $z_1''\ z_2''$ we have

$$\cancel{\Sigma_i x_i''} = \sum_i m_i x_i'' = 0 \quad \text{etc}$$

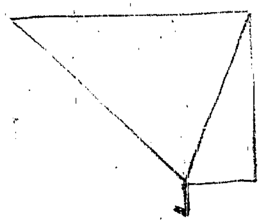
And these forces are such as to make ~~the~~ angular velocity the same for all the masses & to make their distances from the fulcrum ~~the same~~ constant

3rd An external force on the
fulcrum which keeps it constant.

$-\dfrac{y}{z^2}$

$1 + \tan^2\psi$

$\lambda\,\delta\varphi \sec\psi = \delta z$



Simple Pend.

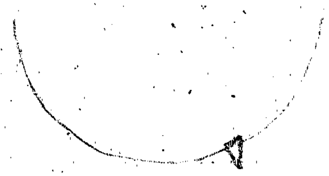
$$\delta\psi = -\frac{\sin 2\psi}{2z}\,\delta z$$

$$\delta\lambda = \frac{2z}{\lambda}\,\delta z \qquad z = \lambda\cos\varphi$$

$$\cos\varphi\,.\,\delta z$$

$$\delta\psi = -\frac{\sin 2\psi}{2\lambda\cos\varphi}\,\delta z$$

$$\delta\varphi = -\frac{1}{\lambda}\,\frac{\lambda\sin\varphi\cos\psi}{\sin^2\varphi + \cos^2\varphi\sec^2\psi}$$



$\frac{g}{\ell}\cos\varphi \sin\varphi = 0\varphi$

$\ddot{\varphi} = g\cos\varphi \sin\varphi$

$\frac{1}{F'}\,dx = \frac{1}{F}\frac{dx}{dy}\,dy$

$F\,D_x y \qquad (D_\varphi \psi)^2$

$\cos\varphi = 1 - 2\sin^2\frac{\varphi}{2}\varphi$

$\cos\varphi - \cos A = 1 - K^2 x^2$

$-\sin\varphi\, d\varphi =$

$\cancel{\cos\varphi = 1 + \cos A}$

$-\cos\varphi + \cos A + 1 = K^2 x^2$

$z - C = k^2(\varphi z)$

$\cos\varphi - \cos A = 1 - K^2\sin^2\psi$

$\cancel{\cos\varphi}\ \frac{1 - \cos\varphi + \cos A}{k}$

$\arcsin\sqrt{\frac{1+\cos A}{k} - \frac{1}{k}\cos\varphi} = \psi$

cos

$$\sqrt{\cos\varphi - \cos A}$$

$$-2\sin^2\tfrac{1}{2}\varphi - \cos A$$

$$\cos A$$

$$d\varphi = 2\, d(\tfrac{1}{2}\varphi)$$

$$(\cos\varphi - \cos A)\,(D_\varphi \psi)^2 =$$

$$\frac{1 - k^2 \sin^2\psi \qquad x = y\sqrt{\ }}{\qquad\qquad \cos x = \cos y}$$

$$\frac{1}{\sqrt{\cos x - \cos A}} = \sqrt{\ }\int \frac{1}{\sqrt{\cos y - \cos A}}$$

$$\cos y = \sqrt{1+z^2}$$

$$\sqrt{\ }\int \frac{1}{\sqrt{\sqrt{1+z^2} - \cos A}}\ \frac{1}{\sqrt{1+z^2}}$$

$$= \int \frac{1}{\sqrt{\left(\sqrt{1+z^2} - \cos A\right)\left(1+z^2\right)}}\, dz$$

$$d\sqrt{1+z^2} = \frac{z}{\sqrt{(1+z^2)}}\, dz$$

$$\sqrt{1+z^2} = w \qquad \cancel{1+}z^2 = w^2 \quad \sqrt{w^2-1}$$

$$= \int \frac{1}{\sqrt{(w - \cos A)}\ \ \frac{1}{(\sqrt{w^2-1})}}$$

$$= \int \frac{1}{\sqrt{w^3 - \cos A\, w^2 - w + \cos A}}$$

$$\int F\,d\varphi = F\frac{d\varphi}{d\theta}\,d\theta$$

$$\sin^2\tfrac{1}{2}\varphi = \sin^2\tfrac{1}{2}\psi\,\sin^2\tfrac{1}{2}A$$

$$\cos^2\tfrac{1}{2}\varphi = 1-\sin^2\tfrac{1}{2}\psi\,\sin^2\tfrac{1}{2}A$$

$$\cancel{\sqrt{1-\frac{\sin\varphi}{\sin i}}}$$

$$\frac{\sin\varphi}{\sin i} = \sin\theta$$

$$d\sin\theta = \cos\theta\,d\theta$$

$$= \frac{\cos\varphi}{\sin i}\,d\varphi$$

$$\frac{d\theta}{d\varphi} = \frac{\cos\varphi}{\cos\theta}\,\frac{1}{\sin i}$$

$$\int\frac{1}{\sqrt{1-\frac{\sin^2\varphi}{\sin^2\theta}}} = \frac{1}{\sqrt{1-\sin^2\theta\,\frac{\cos^2\varphi}{\cos^2\theta}\,\frac{1}{\sin^2 i}}}\,d\theta$$

$$= \frac{1}{\sqrt{1-\frac{1-\sin^2\theta\sin^2 i\,\cancel{\sin^2\theta}}{\cos^2\theta\,\sin^2 i}\sin^2\theta}}$$

$$\sqrt{1-\frac{\sin^2\frac{1}{2}\varphi}{\sin^2\frac{1}{2}A}}$$

$$\frac{\sin\frac{1}{2}\varphi}{\sin\frac{1}{2}A} = \sin\tfrac{1}{2}\psi$$

$$d\sin\tfrac{1}{2}\psi = \frac{\cos\frac{1}{2}\varphi}{\sin\frac{1}{2}A}\,d\tfrac{1}{2}\varphi$$

$$d\sin\tfrac{1}{2}\psi = \cos\tfrac{1}{2}\psi\;d\tfrac{1}{2}\psi$$

$$\frac{d\frac{1}{2}\psi}{d\frac{1}{2}\varphi} = \frac{\cos\frac{1}{2}\varphi}{\sin\frac{1}{2}A}\,\frac{1}{\cos\frac{1}{2}\psi}$$

$$\frac{\cos^2\frac{1}{2}\varphi}{\cos^2\frac{1}{2}\psi\;\sin^2\frac{1}{2}A}$$

$$= \frac{1-\sin^2\frac{1}{2}\psi\sin^2\frac{1}{2}A}{\cos^2\frac{1}{2}\psi\,\sin^2\frac{1}{2}A}$$

$$=$$

$$\int F\,d\varphi = F\,\frac{d\varphi}{d\theta}\,d\theta$$

$$\sin^2 \tfrac{1}{2}\varphi = \sin^2 \tfrac{1}{2}\psi \sin^2 \tfrac{1}{2}A$$

$$\cos^2 \tfrac{1}{2}\varphi = 1 - \sin^2 \tfrac{1}{2}\psi \sin^2 \tfrac{1}{2}A$$

$$\cancel{\sqrt{1 - \frac{\sin\varphi}{\sin i}}} \qquad \frac{\sin\varphi}{\sin i} = \sin\theta$$

$$d(\sin\theta) = \cos\theta\,d\theta$$
$$= \frac{\cos\varphi}{\sin i}\,d\varphi$$

$$\frac{d\theta}{d\varphi} = \frac{\cos\varphi}{\cos\theta}\,\frac{1}{\sin i}$$

$$\int \frac{1}{\sqrt{1 - \frac{\sin^2\varphi}{\sin^2 i}}} = \frac{1}{\sqrt{1 - \sin^2\theta}\ \frac{\cos\varphi}{\cos^2\theta}\ \frac{1}{\sin^2 i}}\ d\theta$$

$$= \frac{1}{\sqrt{1 - \frac{1 - \sin^2\theta \sin^2 i\ \cancel{\sin^2\theta}}{\cos^2\theta\ \sin^2 i}\ \sin^2\theta}}$$

$$\sqrt{1 - \frac{\sin^2 \frac{1}{2}\varphi}{\sin^2 \frac{1}{2}A}}$$

$$\frac{\sin \frac{1}{2}\varphi}{\sin \frac{1}{2}A} = \sin \tfrac{1}{2}\psi$$

$$d \sin \tfrac{1}{2}\psi = \frac{\cos \frac{1}{2}\varphi}{\sin \frac{1}{2}A}\ d\tfrac{1}{2}\varphi$$

$$d \sin \tfrac{1}{2}\psi = \cos \tfrac{1}{2}\psi\ d\tfrac{1}{2}\psi$$

$$\frac{d\frac{1}{2}\psi}{d\frac{1}{2}\varphi} = \frac{\cos \frac{1}{2}\varphi}{\sin \frac{1}{2}A}\ \frac{1}{\cos \frac{1}{2}\psi}$$

$$\frac{\cos^2 \frac{1}{2}\varphi}{\cos^2 \frac{1}{2}\psi\ \sin^2 \frac{1}{2}A}$$

$$= \frac{1 - \sin^2 \frac{1}{2}\psi \sin^2 \frac{1}{2}A}{\cos^2 \frac{1}{2}\psi\ \sin^2 \frac{1}{2}A}$$

$$=$$

His wife is in the family way

$$\begin{array}{r} 4484 \\ \underline{3632} \\ 852 \end{array}$$

$$1 - \frac{1 - \sin^2\theta \sin^2 i}{\cos^2\theta \sin^2 i} \sin^2\theta$$

$$\int_\varphi \sec\theta \, d\varphi = \int \sec\theta \frac{d\varphi}{d\theta} \, d\theta$$

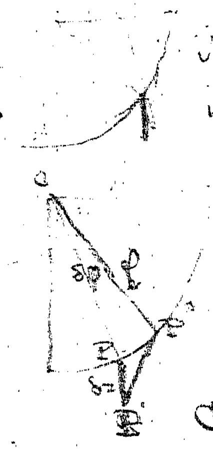
$$= \int \sec\varphi \, d\theta = \cancel{\int \frac{1}{\sqrt{1 - \frac{\sin^2\varphi}{\sin^2 i}}}}$$

$$= \int \frac{1}{\sqrt{1 - \sin^2 i \sin^2\theta}} \, d\theta$$

$$\int$$

$$\int \frac{1}{\sqrt{1 - \frac{\sin^2\varphi}{\sin^2 i}}} \, d\varphi$$

Take any material point of the compound pendulum it receives first ~~an~~ a vertical acceleration $= g$. ~~It also receives~~ What z'' and x'' accelerations must it receive besides, in order on the whole to retain the value of l unchanged and to produce ~~a change~~ an acceleration φ'' in the change of φ.



To ascertain this, vary the position of P by a vertical alteration $\delta_1 z$ and ~~then also vary~~ by such an oblique variation that

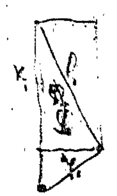
it receives on the whole a ~~varia~~ change of its position along ~~the~~ its path equal to $l\,\delta\varphi$. Required to find $\delta_2 z$ and $\delta_2 x$ the components of the second change.

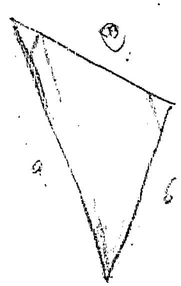
Since $z = l \sin\varphi$
$x = l \cos\varphi$

$$\delta_1 z + \delta_2 z = l \cos\varphi\, \delta\varphi$$
$$\delta_2 x = - l \sin\varphi\, \delta\varphi$$

Hence

$$(D_t^2 z)_2 = l \cos\varphi \cdot \varphi'' - g$$
$$(D_t^2 x)_2 = - l \sin\varphi \cdot \varphi''$$





$\frac{g}{\ell} \sin\varphi$

$\frac{\sin\varphi}{\ell}\,\delta z = \delta\varphi$

$\ell \sin\varphi\,\delta z = \ell^2\delta\varphi$

$\cancel{m\ell \sin\varphi\,\delta z}$

$\left(\varphi'' - \frac{g}{\ell}\sin\varphi\right)\ell^2 m = 0$

$\varphi''\sum m\ell^2 = \sum g\ell m \sin\varphi$

$\varphi'' = g\,\frac{\sum \ell m \sin\varphi}{\sum(m\ell^2)}$

$\sin\varphi = \sin\Phi\cos\varphi + \cos\Phi\sin\varphi$

$D_t \quad D_t\varphi \, D_\varphi x$

$D_t^2 x = D_t^2\varphi \, D_\varphi x + (D_t\varphi)^2 D_\varphi x.$

Simple Pendulum

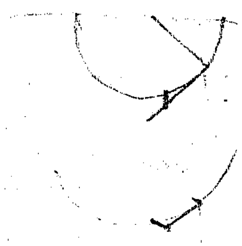
on a [illegible] support

$x^2 + y^2$

$2(x\,dx + y\,dy)$

$x^2 + \sin^2\varphi \, x^2$

$2x(1 + \sin^2\varphi)\,dx$



$-\frac{g}{\lambda} \sin\varphi = D_\varphi (D_t\varphi)^2$

$\frac{g}{\lambda}(\cos\varphi - \cos A)\,dt = \delta\varphi$

$x = \lambda \sin\varphi \quad y = \lambda\cos\varphi$

$\delta x = \lambda \cos\varphi . \delta\varphi$

$y = -\lambda \sin\varphi . \delta\varphi$

$g\cos\varphi(\cos\varphi - \cos A) = dx$

$-g\sin\varphi(\cos\varphi - \cos A) = dy$

$$\cancel{a\,\Theta^{\varphi\sigma} AA + b\,\Theta^{\psi\sigma} AB + c\,\Theta^{\chi\sigma} BA + d\,\Theta^{\omega\sigma} BB}$$

$$\cancel{(a^2\,\Theta^{a\varphi\sigma} + bc\,\Theta^{(\psi+\chi)\sigma})\,AA}$$

$$\cancel{A:A + B:B + C:C + D:D}$$

$$\cancel{A:A + A:B + B:A + B:B}$$

$$(a+e\sigma)A{:}A + (b+f\sigma)A{:}B + (c+g\sigma)B{:}A + (d+h\sigma)B{:}B$$

$$2ae + bg + cf = 0$$

$$af + be + bh + df = 0 \qquad a:d = b:c$$

$$ce + ag + dg + ch = 0 \qquad b:f = c:g$$

$$ab = -bd$$

$$bg + cf + 2dh = 0 \qquad ah = dc$$

$$bg = cf = -dh = -ae$$

$$\cancel{af = bh}$$

$$ae = dh \qquad dh \neq bg$$

$$(a+d)g + c(e+h) = (a+d)f + b(e+h) \qquad \frac{c}{a} = \frac{b}{d}$$

$$g = -\frac{c}{a+d}(e+h) \qquad f = -\frac{b}{a+d}(e+h)$$

$$bg = cf = -\frac{bc}{a+d}(e+h) \qquad bg = cf \qquad cdeg$$

$$\left(d - \frac{bc}{a+d}\right)h - \frac{bc}{a+d}e = 0$$

$$\left(a - \frac{bc}{a+d}\right)e - \frac{bc}{a+d}h = 0$$

$$e = \left(\frac{a+d}{bc}d - \frac{bc}{a+d}\right)h$$

$$h = \left(\frac{a+d}{bc}a - \frac{bc}{a+d}\right)e$$

$$\frac{a^2d + 2ad^2 + d^3 - b^2c^2}{abc + bcd}$$

$$\frac{a^3 + 2a^2d + ad^2 - b^2c^2}{abc + bcd}$$

$$\left(\frac{a+d}{bc}ad - a - d + \frac{b^2c^2}{(a+d)^2}\right) = 1$$

$$\frac{a+d}{bc} = x \quad ad = y \quad bc = z$$

$$xz - xy + \frac{1}{x^2} = 1$$

$$x^3(z-y) - x^2 = -1$$

$$(a+d)(f-g) + (b-c)(e+h) = 0$$

$$\cancel{(a+d)f = b(e+h)}$$

$$a\left(\cancel{h + \frac{e}{h}}\right) f = b\,\cancel{(e+h)}\,h$$

$$af = bh$$

$b(e+h) - (a+d)f = 0$ ∴

$c(e+h) + (a+d)g = 0$

$g\,b(e+h) - (a+d)gf = 0$

$+ f\,c(e+h) + (a+d)gf = 0$

$\cancel{bg + cf = 0}$ Or $e+h=0$ when unless $f=g$ $a+d=0$

$bg = cf$ and a

4 cases none of them $= 0$

neither $a+d=0$ nor $e+h=0$

$bg = cf = -ae = \cancel{dh} - dh$

$ab = -cd$

$a+d=0$ $e+h \neq 0$ $(a=0 \quad d=0)$ see next page

$b = c = 0$

$a+d \neq 0$ $e+h=0$ $(e=0 \quad h=0)$

$f = g = 0$

$(a+d)=0$ $e+h=0$

$bg + cf = -dh$

~~Also~~

$ae = dh$

∴ If $a=-d$ $e=-h$ unless $a=0$

If $e=-h$ $a=-d$ unless $e=0$

$ae + ah = dh + ah$

$a(e+h) = (a+d)h$

$ai + bj + ck + dl$

$i+j$

$\cancel{i+j}$

$i+l$ $(i-l)\surd$ $(j+k)\surd$

$\cancel{i+l}$ $j-k$

D

$i+l$ $i\surd - l\surd$

$(1-\surd)i + (1+\surd)l$

$2(1-\surd)i + 2\surd l$

$i \quad l$

$i \quad l \quad l^2$

$i + \sqrt{l}$

$i + l$

$i + r e^{\theta \sqrt{-}} l$

$i + r^2 e^{2\theta\sqrt{-}} l$

$ai + bj + ck + dl$

$(a^2 + bc) = 0$

$d^2 + bc = 0$

~~ad~~

$b(a+d) = 0$

$c(a+d) = 0$

$a^2 = d^2$

~~a~~ d

1st Case $a = 0 \quad d = 0 \quad b = 0$

2nd Case $a = 0 \quad d = 0 \quad c = 0$

3rd Case $a = -d = \pm\sqrt{bc}$

~~4th Case $a = d \quad b = 0 \quad c = 0$~~

~~Symbol of ... is real~~

~~$\pm$~~

Two ~~$\sqrt{bc}\,i + bj + ck - \sqrt{bc}\,l$~~

$ai + a^2 j + k - al$

$ai + j + a^2 k - al$

$i + aj + \frac{1}{a} k - l$

$i + bj + \frac{1}{b} k - l$

$\frac{i+j}{2} - \frac{i-j}{2}$

$i + j + k - \ell$

$i - j + k - \ell$

$i + j + k + \ell$

$i - \ell$

$(j \mp k)$ $\frac{i-j}{2}$

$\sqrt{2}(\cos 45 + \sin 45\,\ell)$

$\sqrt[4]{2}(\cos 22\tfrac{1}{2} + \sin 22\tfrac{1}{2}\,\ell)$

$\sqrt[4]{2}\, e^{22\frac{1}{2}\, v}$

$\frac{1}{\sqrt{2}}(\cos 45\, v + \sin 45\, v)$

$i + \ell$

$(i + aj + \frac{1}{a}k - \ell)(i + bj + \frac{1}{b}k - \ell)$

$(1 + \frac{a}{b} + \cancel{\ })i + (b-a)j + (\frac{1}{a} + \frac{1}{b})k$

$+ (\frac{b}{a} + 1)\ell$

$\times \quad AiB = \varphi AiB + BiB$

$\frac{b}{a} = \frac{a}{b} \qquad b^2 = a^2$

$b = -a$

$\frac{b}{a} = -1$

$i + \ell$

$j + k$

$i + \ell$

$j + k$

$i_1 = i \quad j_1 = \sqrt{j}$
$k_1 = -\sqrt{k} \quad l_1 = l$

Same multiplication

	i	j	k	l
i	i	j	0	0
j	[illegible]	0	[illegible]	j
k	k	l		
l	[illegible]	[illegible]	k	l

$i = A:A \quad j = A:B$
$k = B:A \quad l = B:B$

$l_1 = -l$

$j_1 = aj + \frac{1}{a}k \quad k_1 = aj - \frac{1}{a}k$

	i	j	k	l
i	i	j	k	l
j	j	i	$-l$	$-k$
k	k	l	$-i$	$-j$
l	l	k	j	i

$j = \{A:B - B:A\} + \{C:D - D:C\}$
$\{A:C + C:A\} - \{B:D + D:B\}$
$\{A:D + D:A\} + \{B:C + C:B\}$

~~$i = A:A + B:B + C:C + D:D$~~
~~$l = A:A - B:B + C:C - D:D$~~

$j = A:B + B:A + C:D + D:C$
$k = A:C - C:A - B:D + D:B$
$l = A:D + D:A - B:C - C:B$
$i = A:A + B:B + C:C + D:D$

$ab = 1 \quad cd = 1 \quad ag = j \quad ce = i$
$bc = -k \quad af = l$
$ef = -1 \quad gh = -1 \quad cb = -l \quad dg = k$
$ij = 1 \quad kl = 1 \quad df = -j \quad bh = j$

$-A:D \; -D:A$
$-B:C \; - C:B$

$-A:C \; - C:A$
$+B:D \; + D:B$

$i_1 = ai + bl$

$j_1 = cj + dk$

$k_1 = ej + fk$

$l_1 = gi + hl$

$i_1^2 = a^2 i + b^2 l$

$l_1^2 = g^2 i + h^2 l$

$i_1 l_1 = agi + bhl$

$\cancel{l_1 i_1 = agi + bhl}$

$i_2 = i + l$

$l_2 = i - l$

$\cancel{j_2 = j + e^{\theta v} k}$

$j_2 = e^{\theta v} j + e^{-\theta v} k$

$k_2 = e^{\theta v} j - e^{-\theta v} k$

$D_t r = 0$

$\cancel{D_t^2 \rho =}$

$C r = -2m$

$D_t^2 r = -\frac{m}{r^2}$

$D_r (D_t r)^2 = \cancel{D_r} D_t^2 r \; 2 \cancel{D_t r}$

$D_r (D_t r)^2 = -2 \frac{m}{r^2}$

$(D_t r)^2 = +2m \frac{1}{r} + C$

$\cancel{(D_r t)^2 = -\frac{1}{\sqrt{2m}}}$

$D_r t = \frac{1}{\sqrt{C + 2m \frac{1}{r}}}$

$$\cancel{\frac{c}{c^2 - A}}$$

$$\arccos\left(\frac{c^2-A^2}{cr} - \frac{A}{c}\right) = \varphi$$

$$\cancel{\sqrt{\frac{c^2-A^2}{cr}} = D_r \varphi}$$

$$\frac{1}{\sqrt{1 - \frac{A^2}{c^2}r^2 + 2\frac{A}{c}\frac{c^2-A^2}{cr} - \frac{(c^2-A^2)^2}{c^2}}}\;\frac{c^2-A^2}{cr} = D_r \varphi$$

$$D_t r^2 = 2C + 2\frac{a}{r} - \frac{E^2}{r^2}$$

$$(D_t\theta)^2 = \frac{E^2}{r^4}$$

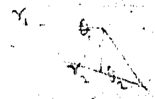
$$(D_\theta r)^2 = \frac{2C}{E^2} r^4 + \frac{2a}{E^2} r^3 - r^2$$

$$D_r\theta = \frac{1}{r}\,\frac{1}{\sqrt{\frac{2C}{E^2}r^2 + \frac{2a}{E^2}r - 1}}$$

$$\frac{A}{c^2-A^2} = \frac{a}{E^2}$$

$$\cancel{\frac{1}{2}(D_t v)^2 = \frac{a}{r^2}} + C$$

$$\frac{1}{2} v^2 = \frac{a}{r} + C$$



$$(ds)^2 = r^2(d\theta)^2 + dr^2$$

$$r^2\, d\theta = C$$

$$\frac{1}{2} r^2 (D_t\theta)^2 + \frac{1}{2}(D_t r)^2 = \frac{a}{r} + C$$

$$r^2 D_t\theta = E$$

$$r^4 (D_t\theta)^2 = E^2$$

$$\frac{1}{2} r^2 (D_t\theta)^2 = \frac{1}{2}\frac{E^2}{r^2}$$

$$(D_t r)^2 = 2C + 2\frac{a}{r} - \frac{E^2}{r^2}$$

$$D_r t = \frac{1}{\sqrt{2C + 2\frac{a}{r} - \frac{E^2}{r^2}}}$$

$$ds^2 = r^2 d\theta^2 + dr^2$$

$$\theta = \arccos\left(\frac{c^2-A^2}{cr} - \frac{A}{c}\right)$$

$$D_t\theta = \mp \frac{1}{\sqrt{\frac{A^2}{c^2} - 2\frac{A}{c^2}(c^2-A^2)\frac{1}{r} + \frac{(c^2-A^2)^2}{c^2r^2}}} \frac{(c^2-A^2)}{cr^2} D_t r$$

$$\cancel{r^2(D_t\theta)^2} = \frac{c^2-A^2}{r\sqrt{A^2r^2 - 2A(c^2-A^2)r + (c^2-A^2)^2}} D_t r$$

$$r^2 D_t\theta^2 = \frac{(c^2-A^2)^2}{A^2r^2 - 2A(c^2-A^2)r + (c^2-A^2)^2}(D_t r)^2$$

$$\frac{(Ar - c^2 + A^2)^2 + (c^2-A^2)^2}{(Ar - c^2 + A^2)^2}(D_t r)^2 = e + \frac{a}{r}$$

$$r = \frac{c^2 - A^2}{A \mp c\cos\varphi}$$

$$D_t r = \mp \sin\varphi\, D_t\varphi \frac{c^2-A^2}{(A + c\cos\varphi)^2}$$

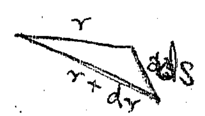
$$=$$

$$= \frac{a}{r} = e$$

$$\frac{1}{2}(D_t r)^2 = e + \frac{a}{r}$$

$$D_r(D_t r)^2 = 2\,\cancel{D_t r \cdot D_t r}\, D_t^2 r$$

$\cancel{D_t r}$



$\cancel{2r + dr + ds}$

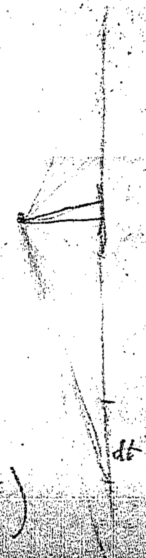
$r + \frac{1}{2}dr + \frac{1}{2}ds$

$r + \frac{1}{2}dr - \frac{1}{2}ds$

$\frac{1}{2}dr + \frac{1}{2}ds$

$-\frac{1}{2}dr + \frac{1}{2}ds$

$\frac{1}{2}r^2 d\theta$



$$\frac{1}{4}r^2((ds)^2 - (dr)^2) = \frac{1}{4}r^2((D_t s)^2 - (D_t r)^2)$$

$$r^2 D_t \theta = E$$

$$r^2 (D_t \theta)^2 + (D_t r)^2 = \frac{a}{r} + b$$

when $D_t r = 0$ r min...

$$r = c - A \qquad \theta = 0$$

$$r^2 (D_t \theta)^2 = \frac{a}{c-A} + b$$

$$r^4 (D_t \theta)^2 = a(c-A) + b(c-A)^2 = E^2$$

$$r^4 (D_t \theta)^2 + r^2 (D_t r)^2 = a r + b r^2$$

$$r^4 (D_t r)^2 = b r^4 + a r^3 - E^2 r^2$$

$$r^4 (D_t \theta)^2 = E^2$$

$$(D_\theta r)^2 = \frac{b}{E^2} r^4 + \frac{a}{E^2} r^3 - r^2$$

$$b(c^2 - A^2) = a(c-A) + b(c-A)^2$$

$$\frac{a(c^2 - A^2)}{2A} = a(c-A) + b(c-A)^2$$

$$c(a - 2bA) = A(a - 2bA)$$

$$ac - aA - 2bcA + 2bA^2$$

$$r = \frac{c^2 - A^2}{A + c\cos\varphi}$$

$$D_\theta r = \frac{c^2 - A^2}{(A + c\cos\varphi)^2}\, c \sin\varphi$$

$$= r^2 \frac{c}{c^2 - A^2} \sin\varphi$$

$$(D_\theta r)^2 = r^4 \frac{c^2}{(c^2 - A^2)^2} \sin^2\varphi$$

$$\cos\varphi = \frac{c^2 - A^2}{c r} - \frac{A}{c}$$

$$\cos^2\varphi = \frac{(c^2 - A^2)^2}{c^2 r^2} - 2\frac{A}{c^2}\frac{(c^2 - A^2)}{r} + \frac{A^2}{c^2}$$

$$\sin^2\varphi = \left(\frac{c^2 - A^2}{c^2}\right) + 2\frac{A}{c^2}\frac{(c^2 - A^2)}{r} - \frac{(c^2 - A^2)^2}{c^2 r^2}$$

$$(D_\theta r)^2 = r^4 + 2\frac{A}{c} r^3 - \frac{c^2 - A^2}{c} r^2$$

$$(D_\theta r)^2 = \frac{1}{c^2 - A^2} r^4 + 2\frac{A}{c^2 - A^2} r^3 - r^2$$

$$b = \frac{a}{2A} \qquad A = \frac{1}{2}\frac{a}{b}$$

$$b\left(c^2 - \frac{1}{4}\frac{a^2}{b^2}\right) = a\left(c - \frac{1}{2}\frac{a}{b}\right) + b\left(c - \frac{1}{2}\frac{a}{b}\right)^2$$

$$r^2(D_t\theta)^2 + (D_t r)^2 = \frac{a}{r} + b$$

$\frac{1}{dr/d\theta}$

$iA:A = iB:B$ etc

Hence, if i contains $A:A$ it contains $B:B$ etc. and is the scalar unit.

If i contains $B:A$

$iA = B$

iB is not zero

Suppose then i contains

$C:B$

Each combination can occur only once

$i = A:A + B:B$

$j = \cancel{a}A:B + \cancel{a}B:A$

$i^2 = {}^aA:\cancel{B} + aB:B$

$A:A + B:B + C:C$

$A:B + {}_aB:C + bC:A$

${}_aA:C + {}_{ab}B:A + bC:B$

~~$pA + qB$~~

$i(pA + qB) = qA + apB$

$q = p \quad q = ap$

$\underline{a}$ cannot be 1. Can be anything else.

$i = A{:}A + B{:}B + C{:}C$

$j = A{:}B + aB{:}C + bC{:}A$

$k = aA{:}C + abB{:}A + bC{:}B$

$j = (pA + qB + rC)$

$= qA + arB + pbC$

$p = q \quad q = ar \quad r = bp$

$p = ar$

Cannot have $a = \frac{1}{b}$

$k(pA + qB + rC)$

$= arA + abpB + bqC$

$p = ar \quad q = abp \quad r = bq$

$q = a^2br$

$= \frac{1}{b}r$ Cannot have $a^2 = \frac{1}{b^2}$

$ar = q \quad abp = ar \quad bq = bp$

$p = \frac{1}{b}r \qquad q = p$

General conclusion

Cannot have $a^2 = \frac{1}{b^2}$

$i = A{:}A + B{:}B + C{:}C$

$j = A{:}B + \surd B{:}C - C{:}A$

$k = 1A{:}C - \surd B{:}A - C{:}B$

$j^2 = k \quad j^3 = -j \quad k^2 = -1$

$i = A{:}A + B{:}B + C{:}C$

$j = A{:}B + 1^{\frac{1}{3}}B{:}C + 1^{\frac{2}{3}}C{:}A$

$k = 1^{\frac{2}{3}}A{:}C + B{:}A + 1^{\frac{1}{3}}C{:}B$

$j^2 = k \quad j^3 = 1 \quad k^3 = 1$

$1 = A:A + B:B + C:C + D:D$

$i = A:B + \surd B:C - C:D - \surd D:A$

$j = \surd A:C - \surd B:D + \surd C:A - \surd D:B$

$k = -\surd A:B - B:A + \surd C:B + D:C$

$i^2 = k \quad \cancel{jk=} \quad i^3 = \ell \quad i^4 = 1$

$1 = A:A + B:B + C:C + D:D$

$i = A:B - B:A + C:D - D:C$

$j = -A:C + C:A + B:D - D:B$

$k = A:D - D:A + B:C - C:B$

1 = A:A − B:B + C:C + D:D + E:E + F:F

i = A:B + B:A + C:D − D:C + E:F − F:E

~~j = A:D − D:A~~ + C:F − F:C + E:B − B:E

k − A:D + D:A + D:A = B:C + C:B

D:F − D:E − E:C − F:B

− A:C · B:D − C:A

$ij = k \quad jk = i$

$\cancel{\Rightarrow}$

$ji = -k \quad kj$

$A{:}A + B{:}B + C{:}C + D{:}D + E{:}E + F{:}F + G{:}G + H{:}H$

$\cancel{A{:}B - B{:}A}$

$\cancel{C{:}D - D{:}C}$

$\cancel{E{:}F - F{:}E}$

$\cancel{G{:}H - H{:}G}$

1 $A{:}B - B{:}A + C{:}D - D{:}C - E{:}F + F{:}E - G{:}H + H{:}G$

$\cancel{A{:}C - C{:}A}$

i $B{:}C - C{:}B + A{:}D - D{:}A - E{:}H + H{:}E + G{:}F - F{:}G$

k $A{:}C - C{:}A - B{:}D + D{:}B - E{:}G + G{:}E - H{:}F + F{:}H$

$A{:}E - E{:}A + B{:}F - F{:}B + C{:}G - G{:}C + D{:}H - H{:}D$

$-B{:}E + E{:}B - F{:}A + A{:}F + G{:}D - D{:}G - H{:}C + C{:}H$

$-D{:}E + E{:}D + B{:}G - G{:}B - C{:}F + F{:}C + A{:}H - H{:}A$

$-C{:}E + E{:}C + A{:}G - G{:}A - B{:}H + H{:}B + D{:}F - F{:}D$

M:M + N:N
M:N − N:M

1 = R1 I1 = Q
R I R
R I m
k = Rk Ik = N

A:A − B:B

A:A + B:B + C:C + D:D + E:E + F:F + G:G

~~A:B − B:A + C:D − D:C + E:F − F:E + G:H − H:G~~

A:E − E:A + B:F − F:B + C:G − G:C + D:H − H:D

A:B − B:A − C:D + D:C

− E:B + F:A + G:D − H:C

$A{:}B - B{:}A + D{:}C - C{:}D - E{:}F + F{:}E + G{:}H - H{:}G = i$

$A{:}C - C{:}A + B{:}D - D{:}B - E{:}G + G{:}E - F{:}H + H{:}F = j$

$A{:}D - D{:}A - B{:}C + C{:}B + E{:}H - H{:}E - F{:}G + G{:}F = k$

$A{:}E - E{:}A$ [illegible]$:B + B{:}F + C{:}G - G{:}C + D{:}H - H{:}D = l$

$A{:}F - F{:}A - B{:}E + E{:}B + D{:}G - G{:}D - C{:}H + H{:}C = m$

$A{:}G - G{:}A - C{:}E + E{:}C - D{:}F + F{:}D + B{:}H - H{:}B = n$

$A{:}H - H{:}A - D{:}E + E{:}D - B{:}G + G{:}B + C{:}F - F{:}C = o$

$A{:}B - B{:}A - C{:}D + D{:}C + E{:}F - F{:}E + G{:}H - H{:}G = i$

$A{:}C - C{:}A + D{:}B - B{:}D + E{:}G - G{:}E - F{:}H + H{:}F = j$

$+A{:}D - D{:}A - B{:}C + C{:}B - E{:}H + H{:}E - F{:}G + G{:}F = k$

$A{:}E - E{:}A + F{:}B - B{:}F - C{:}G + G{:}C - D{:}H + H{:}D = l$

$-A{:}F + F{:}A - B{:}E + E{:}B - D{:}G + G{:}D + C{:}H - H{:}C = m$

$-A{:}G + G{:}A - C{:}E + E{:}C + D{:}F - F{:}D + B{:}H - H{:}B = n$

$+A{:}H - H{:}A - D{:}E + E{:}D - C{:}F + F{:}C - B{:}G + G{:}B = o$

$ij = k$

i

$k \quad j$

$il = m$

$jl = n$

$ij = k \quad jk = i \quad ki = j$

$ji = -k \quad kj = -i \quad ik = -j$

$ij = k \quad kl = o \quad ijl = o$

$jl = n \quad in =$

$s + xi + yj + zk$

$s^2 + 2sx + 2sy + 2sz$

$-x^2$

$-y^2$

$-z^2$

$S = s^2 - x^2 - y^2 - z^2$

$X = 2sx \qquad x = \frac{X}{2s}$

$Y = 2sy \qquad y = \frac{Y}{2s}$

$Z = 2sz \qquad z = \frac{Z}{2s}$

$S = s^2 - \frac{1}{4}(X^2 + Y^2 + Z^2)\frac{1}{s^2}$

$4s^4 - 4Ss^2 = X^2 + Y^2 + Z^2$

$2s^2 - S = \sqrt{X^2 + Y^2 + Z^2 + S^2}$

$s = \frac{1}{\sqrt{2}}\sqrt{S + \sqrt{X^2 + Y^2 + Z^2 + S^2}}$

$q^2 = ai$

$S = 0 \quad X = a \quad Y = 0 \quad Z = 0$

$s = \sqrt{\frac{a}{2}} \qquad 2s = \sqrt{2a}$

$x = \sqrt{\frac{a}{2}} \quad y = 0 \quad z = 0$

$q = \sqrt{\frac{a}{2}}(1 + i)$

$q^2 = a(i + j)$

$S = 0 \quad X = a \quad Y = a \quad Z = 0$

$s = \frac{1}{\sqrt{2}}\sqrt{\sqrt{2a^2}} = \frac{\sqrt{a}}{\sqrt[4]{2}} = a^{\frac{1}{2}}2^{-\frac{1}{4}}$

$2s = 2^{\frac{3}{4}}a^{\frac{1}{2}}$

$x = 2^{-\frac{3}{4}}a^{\frac{1}{2}} \quad y = 2^{-\frac{3}{4}}a^{\frac{1}{2}} \quad z = 0$

$q =$

$q^2 = ai + bj$

$S = 0 \quad X = a \quad Y = b \quad Z = 0$

$s = \frac{1}{\sqrt{2}}\sqrt[4]{a^2 + b^2} \qquad 2s = \sqrt{2}\sqrt[4]{a^2 + b^2}$

$x = \frac{a}{\sqrt{2}\sqrt[4]{a^2 + b^2}} \qquad y = \frac{b}{\sqrt{2}\sqrt[4]{a^2 + b^2}}$

$q = \frac{1}{\sqrt{2}\sqrt[4]{a^2 + b^2}}\left(\sqrt{a^2 + b^2} + a + b\right)$

$$F(i,j,\theta) = \cos\theta \cdot i + \sin\theta \cdot j$$

$$F^2(i,j,\theta) = \cos^2\theta - \sin^2\theta$$

$$= -1$$

$$F(i,j,\theta_1)\,F(i,j,\theta_2)$$

$$= (\cos\theta_1 i + \sin\theta_1 j)(\cos\theta_2 i + \sin\theta_2 j)$$

$$= -\cos(\theta_2 - \theta_1) + \sin(\theta_2 - \theta_1)k$$

$$= F(-1, k, \theta_2 - \theta_1)$$

$$F(i,j,\theta)\,F(i,k,\theta)$$

$$= (\cos\theta i + \sin\theta j)(\cos\theta i + \sin\theta k)$$

$$= -\cos^2\theta + \sin^2\theta i - \cos\theta\sin\theta j$$
$$- \cos\theta \sin\theta k$$

$$(x-a)(x-b) = m$$

$$x^2 - (a+b)x + ab = m$$

$$\cancel{x^2 - (a+b)x + \frac{a^2}{4} + \frac{1}{2}ab + \frac{1}{4}b^2 = m + a^2 + b^2 + \frac{1}{2}ab}$$

$$x$$

$$4x^2 - 4(a+b)x + 4ab = 4m$$

$$4x^2 - 4(a+b)x + a^2 + 2ab + b^2 = 4m + (a-b)^2$$

$$2x - a - b = \sqrt{4m + (a-b)^2}$$

$$x = \frac{1}{2}(a+b) + \frac{1}{2}\sqrt{4m + (a-b)^2}$$

$$x = 6^{\nu - \theta} \qquad y = 6^{\nu - \frac{1}{2}\theta}$$

$$D_\theta x = -\nu - 6^{\nu-\theta} \qquad \int y\, d\theta = -\sqrt{2}\, 6^{\nu - \frac{1}{2}\theta}$$

$$\cancel{\int_0^{2\theta} y\, d\theta =}$$

$$\int y\, dx =$$

$\int \frac{1}{x}\,dx = \log x$

$x = \mathcal{E}^{\iota\theta}$

$dx = \iota\,\mathcal{E}^{\iota\theta}\,d\theta$

$\frac{1}{x} = \mathcal{E}^{-\iota\theta}$

$\frac{1}{x}\,dx = \iota\,d\theta$

$\int \frac{1}{x}\,dx = \iota\theta = \log x$

$T_{x=0}$ $\quad z = \text{Const}$

$x = U_x = F'(\theta,\varphi)$

$dx = D_\varphi F'(\theta,\varphi)\,d\varphi$

$\frac{1}{x} = F(-\theta,\varphi)$

$\frac{1}{x}\,dx = F'(-\theta,\varphi)\,D_\varphi F'(\theta,\varphi)\,d\varphi$

$\cancel{dF\,x^{\frac{1}{2}} = dF}$

$dy^{\frac{1}{2}} = \frac{1}{2}y^{-\frac{1}{2}}\,dy$

$w^2 + x^2 + y^2 + z^2 = 1$

$\sqrt{1-x^2-y^2-z^2} + xi + yj + zk$

$q =$

$\varphi\ \psi\ \chi\ \omega$

$\sqrt{1-\cos^2\varphi-\cos^2\psi-\cos^2\chi} + \cos\varphi\, i + \cos\psi\, j + \cos\chi\, k$

$= \cos\omega + \sqrt{1-\cos^2\psi-\cos^2\chi-\cos^2\omega}\; i + \cos\psi\, j + \cos\chi\, k$

$dq = \frac{\mp\cos\psi\sin\psi}{\sqrt{\;}}\, i - \sin\psi\, j$

$\frac{1}{q}$

$d uv = du\, dv + v\, du$

$uv = \int u\, dv + \int v\, du$ $\quad \cos 2\theta = \cos^2 - \sin^2$

uv' $\quad = 1 - 2\sin^2$

$\frac{1 - \cos 2\theta}{2}$

$(S_q - V_q)(S_q + V_q)$ $\quad u\, du$

$= S_q^2 - V_q^2 = T_q$ $\quad \frac{1}{2}u^2$

$\cos\theta + \sin\theta\,(ai + bj + ck)$

$\cos^2\theta + \sin^2\theta\,(a^2 + b^2 + c^2) = 1$

$a + b + c = \pm 1$ $\quad f_2\varphi$

$df_1 + df_2 + df_3$

$q = \cos\theta + \sin\theta\,(ai + bj + ck)$

$\frac{1}{q} = \cos\theta - \sin\theta\,(ai + bj + ck)$

$a = f_1\varphi \quad b = f_2\varphi \quad \cancel{a = 1 - f_1\varphi - f_2\varphi} \quad c = f_3\varphi$

$dq = \sin\theta\,(f_1'\varphi \cdot i + f_2'\varphi \cdot j + f_3'\varphi \cdot k)\, d\varphi$

$\frac{1}{q}\, dq$

$\cancel{= \frac{1}{2}\sin 2\theta\,(f_1'\varphi \cdot i + f_2'\varphi \cdot j + f_3'\varphi \cdot k)\, d\varphi}$

$\sin^2\theta\,(f_1\varphi \cdot f_1'\varphi + f_2\varphi \cdot f_2'\varphi + f_3\varphi \cdot f_3'\varphi)\, d\varphi$

$+ i \sin\theta\,(\cos\theta\, f_1'\varphi - \sin\theta\, f_2\varphi \cdot f_3'\varphi + \sin\theta\, f_3\varphi \cdot f_2'\varphi)$

$+ j \sin\theta\,(\cos\theta\, f_2'\varphi - \sin\theta\, f_3\varphi\, f_1'\varphi + \sin\theta\, f_1\varphi \cdot f_3'\varphi)$

$+ k \sin\theta\,(\cos\theta\, f_3'\varphi - \sin\theta\, f_1\varphi\, f_2'\varphi + \sin\theta\, f_2\varphi\, f_1'\varphi)$

$\int \frac{1}{q}\, dq = \frac{1}{2}\sin^2\theta\,\left((f_1\varphi)^2 + (f_2\varphi)^2 + (f_3\varphi)^2\right)$

$+ i \sin\theta\,(\cos\theta \cdot f_1\varphi$

$+ f_2\, df_1 + f_3\, df_2 + df_2 + f_1\, df_3 + f_3\, df_3$

$+ f_3\, df_1 - f_3\, df_3 + df_2 - f_1\, df_2 - f_2\, df_2$

$- df_1 + f_1\, df_1 + f_2\, df_1 \quad f_1\, df_1 \quad f_1\, df_2$

$q = \cos\theta + i \sin\theta \sqrt{-1}$

$\frac{1}{q} = \cos\theta - i \sin\theta \sqrt{-1}$

$dq = -\sin\theta \, d\theta + i \cos\theta \sqrt{-1} \, d\theta$

$\frac{1}{q} dq = \cancel{\ldots}$
$\sqrt{-1} \, d\theta$

$q = \cos\theta + \sin\theta . v$

$\frac{1}{q} = \cos\theta - \sin\theta . v$

$dq = -\sin\theta \, d\theta + \cos\theta . d\theta . v$

$\frac{1}{q} dq = d\theta . v$

$\int \frac{1}{q} dq = \theta v$

$q \quad \cos\theta + \sin\theta \, f\varphi \, i$

$\frac{1}{q} \quad \cos\theta - \sin\theta \, f\varphi \, i$

$dq \quad \sin\theta \, f'\varphi \, i \, d\varphi$

$\frac{1}{q} dq = \sin\theta\cos\theta \, f'\varphi \, d\varphi . i + \sin^2\theta \, f\varphi f'\varphi \, d\varphi$

$\int \frac{1}{q} dq = \sin\theta\cos\theta \, f\varphi . i +$

$v = \cos\varphi . i + \sin\varphi \, j$

$q = \cos\theta + \sin\theta (\cos\varphi \, i + \sin\varphi \, j)$

$\frac{1}{q} = \cos\theta - \sin\theta (\cos\varphi \, i + \sin\varphi \, j)$

$dq = + \sin\theta (-\sin\varphi \, d\varphi \, i + \cos\varphi \, d\varphi \, j)$

$\frac{1}{q} dq = \cancel{\sin\theta\cos\theta\sin\varphi \, d\varphi} \; \cancel{- \sin^2\theta \sin\varphi\cos\varphi \, d\varphi} + \sin^2\theta \sin\varphi\cos\varphi \, d\varphi$

$\sin\theta\cos\theta (-\sin\varphi \, d\varphi \, i + \cos\varphi \, d\varphi \, j)$

$- \sin^2\theta \, d\varphi \, k$

$\int \frac{1}{q} dq = \sin\theta\cos\theta (\cos\varphi . i + \sin\varphi \, j)$
$- \sin^2\theta . \varphi k$

Condition of monogeneity

$$\frac{dW}{dw} + \frac{dX}{dw} i + \frac{dY}{dw} j + \frac{dZ}{dw} k$$
$$= \frac{dW}{dx} i + \frac{dX}{dx} + \frac{dY}{dx} k - \frac{dZ}{dx} j$$
$$= -\frac{dW}{dy} j - \frac{dX}{dy} k + \frac{dY}{dy} + \frac{dZ}{dy} i$$
$$= -\frac{dW}{dz} k + \frac{dX}{dz} j - \frac{dY}{dz} i + \frac{dZ}{dz}$$

$ki = j$

~~$ki = j$~~

$$\frac{dW}{dw} = \frac{dX}{dx} = \frac{dY}{dy} = \frac{dZ}{dz}$$
$$\frac{dX}{dw} = -\frac{dW}{dx} = \frac{dZ}{dy} = -\frac{dY}{dz}$$
$$\frac{dY}{dw} = -\frac{dW}{dy} = \frac{dX}{dz} = -\frac{dZ}{dx}$$
$$\frac{dZ}{dw} = -\frac{dW}{dz} = \frac{dY}{dx} = -\frac{dX}{dy}$$

$$v = \cos\phi\, i + \sin\phi\cos\psi\, j + \sin\phi\sin\psi\, k$$

$\cancel{\frac{1}{v}\ v}$

$$q = \cos\theta + \sin\theta . v$$
$$\frac{1}{q} = \cos\theta - \sin\theta . \cos\phi . i - \sin\theta\sin\phi\cos\psi\, j - \sin\theta\sin\phi\sin\psi\, k$$
$$dq = \cancel{-}\sin\theta\sin\phi\, d\phi . i + \sin\theta\cos\phi\cos\psi\, d\phi\, j + \sin\theta\cos\phi\sin\psi\, d\phi\, k$$

$$\frac{1}{q}dq = -\sin\theta\cos\theta\sin\phi\, d\phi\, i$$
$$(+\sin\theta\cos\theta\cos\phi\cos\psi + \sin^2\theta\sin\psi)\, d\phi\, j$$
$$+ (\sin\theta\cos\theta\cos\phi\sin\psi - \sin^2\theta\cos\psi)\, d\phi . k$$

$$\int\frac{1}{q}dq = \sin\theta\cos\theta\cos\phi\, i + (\sin\theta\cos\theta\sin\phi\cos\psi$$

$$q = a\left(\cos\theta + \sin\theta\left(\cos\varphi\, i + \sin\varphi\,(\cos\psi\, j + \sin\psi\, k)\right)\right)$$

$$\frac{1}{q} = \frac{1}{a}\left(\cos\theta - \sin\theta\left(\cos\varphi\, i + \sin\varphi\,(\cos\psi\, j + \sin\psi\, k)\right)\right)$$

$$dq = da.\frac{q}{a} + d\theta\, a\left(-\sin\theta + \cos\theta\left(\cos\varphi\, i + \sin\varphi\,(\cos\psi\, j + \sin\psi\, k)\right)\right)$$

$$\cancel{+ a(\cos\theta + \sin\theta(\cos}$$

$$d\varphi.\, a\sin\theta\left(-\sin\varphi\, i + \cos\varphi\,(\cos\psi\, j + \sin\psi\, k)\right)$$

$$+ d\psi\, a\sin\theta\, \sin\varphi\,(-\sin\psi\, j + \cos\psi\, k)$$

$$- \sin^2\theta\left(\sin\varphi\cos\varphi + \sin\varphi\cos\varphi(-\cos^2\psi - \sin^2 \ldots\right.$$

$$(s+v)(s-v) = s^2 \ldots$$

$$-\sin^2\theta\sin^2\varphi$$

$$q^{-1} = \frac{1}{a}\left(\cos\theta - \sin\theta \ldots\right)$$

$$\frac{1}{q}dq = \frac{1}{a}da + d\theta\{-\sin\theta\cos\theta - \sin\theta\cos\theta\,(-\cos\varphi + \sin^2\varphi\,(-\cos^2\psi - \sin^2\psi))\ldots$$

$$\cos^2\theta\cos\varphi\, i + \sin^2\theta\cos\varphi\, i - \sin\theta\cos\theta\,(\sin^2\varphi\,(\cos\psi\sin\psi - \sin\psi\cos\psi))\, i$$

$$+ \cos^2\theta\sin\varphi\cos\psi\, j + \sin^2\theta\sin\varphi\cos\psi\, j - \sin\theta\cos\theta\,(\cos\varphi\sin\varphi\,(\sin\psi - \sin\psi))\, j$$

$$+ \cos^2\theta\sin\varphi\sin\psi\, k + \sin^2\theta\sin\varphi\sin\psi\, k$$

$$+ d\varphi\sin\theta(\cos\theta\sin\varphi\, i - \cancel{\sin\theta\sin\varphi} + \cos\theta\cos\varphi\cos\psi\, j + \sin\theta\sin^2\varphi\sin\psi\, j + \sin\theta\cos^2\varphi\sin\psi\, j$$

$$= da.\frac{1}{a} + d\theta(\cos\varphi\, i + \sin\varphi\cos\psi\, j + \sin\varphi\sin\psi\, k)$$

$$+ d\varphi\sin\theta\{-\cos\theta\sin\varphi\, i + (\cos\theta\cos\varphi\cos\psi + \sin\theta\sin\psi)\, j$$
$$+ (\cos\theta\cos\varphi\sin\psi - \sin\theta\cos\psi)\, k\}$$

$$+ d\psi\{\sin^2\theta\sin^2\varphi\, i + \sin\theta\sin\varphi\,(\cos\theta\sin\psi + \sin\theta\cos\varphi\cos\psi)\, j$$
$$+ \sin\theta\sin\varphi\,(\cos\theta\cos\psi - \sin\theta\cos\varphi\sin\psi)\, k)$$

$$\cancel{\cos\theta\cos\varphi\sin\psi \ldots \sin\theta\cos\varphi\cos\psi \ldots \sin\theta\sin\psi\cos\psi}$$

$$\cancel{d\sin\varphi = \cos\varphi\, d\varphi}$$

$$\cancel{d\varphi^2 = \varphi\, d\varphi}$$

$$q = a\left\{\cos\theta + \sin\theta\left[F_1(\varphi,\psi)\,i + F_2(\varphi,\psi)\,j + F_3(\varphi,\psi)\,k\right]\right\}$$

$$\frac{1}{q} = \frac{1}{a}\left\{\cos\theta - \sin\theta\left[F_1(\varphi,\psi)\,i + F_2(\varphi,\psi)\,j + F_3(\varphi,\psi)\,k\right]\right\}$$

$$dq = da\,\frac{q}{a} + d\theta\, a\,(-\sin\theta + \cos\theta\, V)$$
$$+ d\varphi\, a\sin\theta\left(D_\varphi F_1\, i + D_\varphi F_2\, j + D_\varphi F_3\, k\right)$$
$$+ d\psi\, a\sin\theta\left(D_\psi F_1\, i + D_\psi F_2\, j + D_\psi F_3\, k\right)$$

$$\frac{1}{q}\,dq = \frac{1}{a}\,da + d\theta\, V + d\varphi\,\sin\theta\left(\sin\theta\,(F_1 D_\varphi F_1 + F_2 D_\varphi F_2 + F_3 D_\varphi F_3)\right) \quad V D_\varphi V$$

$$+ \cos^2\varphi\,\cos^2\psi$$

$$\sin^2\varphi\,\sin^2\psi = (1-\cos^2\varphi)(1-\cos^2\psi) \qquad \cos^2\varphi\,\cos^2\psi + \cos\varphi\sin\varphi\,\cos\psi\sin\psi \qquad \sin^2\varphi\,\sin^2\psi$$
$$1 - \cos^2\varphi - \cos^2\psi + \cos^2\varphi\,\cos^2\psi$$
$$\cos^2\varphi + \cos^2\psi$$

$$q = a(\cos\theta + \sin\theta . V)$$
$$\frac{1}{q} = \frac{1}{a}(\cos\theta - \sin\theta . V)$$
$$dq = da\,\frac{q}{a} + d\theta\, a(-\sin\theta + \cos\theta . V) + d\varphi\, a\sin\theta\, D_\varphi V + d\psi\, a\sin\theta\, D_\psi V$$

$$\frac{1}{q}\,dq = da\,\frac{1}{a} + d\theta\, V + \cancel{d\varphi\,\sin\theta\,(\cos\theta\, D_\varphi V - \sin\theta\, D_\varphi V . V}$$
$$+ d\varphi\,\sin\theta\,\cancel{D_\varphi V}\,(\cos\theta - \sin\theta\, V)\, D_\varphi V$$
$$+ d\psi\,\sin\theta\,\cancel{D_\psi V}\,(\cos\theta - \sin\theta\, V)\, D_\psi V$$

$$\frac{1}{q}\,dq = da\,\frac{1}{a} + d\theta\, V + d\varphi\,\sin\theta\left(\cos\theta\, F_1'\, i + \cos\theta\, F_2'\, j + \cos\theta\, F_3'\, k\right)$$
$$- d\varphi\,\sin^2\theta\left((F_2 F_3' - F_3 F_2')\, i + (\quad\right. \qquad D_{\varphi - \psi} V$$

$$\cos\theta\,\frac{F_2 F_2'}{\sqrt{1}} \qquad F_2 F_3' = F_3 F_2' \qquad \cancel{D_\psi V = D_{\varphi+\psi} V} \qquad F_1' = \sqrt{1 - F_2^2 + F_3^2}$$

$$d\log F_3 - d\log F_2 \qquad D_\varphi V = \cancel{D_{(\varphi+\psi)}} \qquad F_1^2 = 1 - F_2^2 + F_3^2$$

$$\log F_3 \qquad F_3 = F_2\,\text{const} \qquad \frac{D_\varphi V}{D_\varphi(\varphi+\psi)} \qquad F_1 F_1' = F_2 F_2' + F_3' F_3$$

$q = w + xi + yj + zk$

$\frac{1}{q} = w - xi - yj - zk$

$dq = dw + dx.i + dy.j + dz.k$

$\frac{1}{q}dq = \cancel{dw\ dw}$
$(w - xi - yj - zk)\,dw$
$+ (x + wi - zj + yk)\,dx$
$+ (y + zi + wj - xk)\,dy$
$+ (z - yi + xj + wk)\,dz$

$w^2 + x^2 + y^2 + z^2$

$\int \frac{x}{x^2+b} = \frac{1}{2}\log(x^2+b)$ $\qquad \int x^2+b = -\frac{1}{\sqrt{b}}\arctan\frac{\sqrt{b}}{x}$

$\int \frac{1}{q} dq =$

$\int \frac{1}{q} dq = \frac{1}{2}\log T^2 + \frac{xi+yi+zk}{\sqrt{x^2+y^2+z^2}} \arctan \frac{\sqrt{x^2+y^2+z^2}}{w} + F_1(x, y, z)$

$= \frac{1}{2}\log T^2 + \frac{\cdots}{\sqrt{w^2+y^2+z^2}} \arctan \frac{\sqrt{w^2+y^2+z^2}}{x} + F_2(w, y, z)$

$\frac{y}{\sqrt{1-y^2-z^2}}\, y' \cos\theta + \frac{z}{\sqrt{1-y^2-z^2}}\, z' \cos\theta$ $\qquad w + xi$

$+ z\ y' \sin\theta - y\ z' \sin\theta$ $\qquad \frac{1}{q}dq = \frac{w - xi}{w^2+x^2}\,dw$

$= \frac{1}{2}\log T^2 + \frac{-zi - wj + xk}{\sqrt{w^2+x^2+z^2}} \arctan \frac{\sqrt{w^2+x^2+z^2}}{y}$ $\qquad + \frac{x+wi}{w^2+x^2}\,dx$

$\int \log(w+xi) = \frac{1}{2}\log(w^2+x^2) + \arctan\frac{x}{w} \cdot \sqrt{-}$

$\frac{1}{2}\log(w^2+x^2) - \arctan\frac{w}{x}\ \sqrt{-}$

$\frac{1}{2}\ \frac{1}{w^2+x^2}$

$q = a(\cos\theta + \sin\theta\, v)$

$\frac{1}{q} = \frac{1}{a}(\cos\theta - \sin\theta\, v)$

$dq = da\,\frac{q}{a} + a(-\sin\theta + \cos\theta\, v)\,d\theta$

$\frac{1}{q}dq = da\,\frac{1}{a} + v\,d\theta$

$\int \frac{1}{q}dq = \log a + \theta v$

$q = A + Bv$

$\frac{1}{q} = \frac{A - Bv}{A^2 + B^2}$

$dq = dA + dB\,v$

$\frac{A - B}{A^2+B^2}$ $\quad A + Bv$

$\frac{1}{q}dq = \frac{(A - Bv)}{A^2+B^2}dA + \frac{B + Av}{A^2+B^2}dB$

$\int \frac{1}{q}dq = \frac{1}{2}\log(A^2+B^2) + \arctan$

$\int_1^q \frac{1}{q}dq = \log T + \cancel{\tfrac{\theta}{\theta}\arctan\theta}$

$\cancel{+\log\ \tfrac{1}{w}i\arctan\frac{w}{x} + \frac{1}{w}i(2n+\tfrac{1}{2})\theta}$

$-\frac{1}{w}i\arctan\frac{w}{x} + \frac{1}{w}i(2n+\tfrac{1}{2})\theta$

$+\frac{-wj+xk}{\sqrt{w^2+x^2}}\arctan\frac{\sqrt{w^2+x^2}}{y} - \frac{-wj+xk}{\sqrt{w^2+x^2}}(2n+\tfrac{1}{2})\theta$

$+\frac{+yi \pm xj \mp wk}{\sqrt{w^2+x^2+y^2}}\arctan\frac{\sqrt{w^2+x^2+y^2}}{z}$

$\int_1^q \frac{1}{q}dq = \log T + i\arctan\frac{x}{w} + \frac{wj-xk}{\sqrt{w^2+x^2}}\arctan\frac{y}{\sqrt{w^2+x^2}} + \frac{-yi+xj+wk}{\sqrt{w^2+x^2+y^2}}\arctan\frac{z}{\sqrt{w^2+x^2+y^2}}$

$= \log T + i\,\frac{\log(w+xv)}{\log\sqrt{x^2+w^2}\ v} + \frac{wj-xk}{\sqrt{w^2+x^2}}\log\frac{(\sqrt{w^2+x^2}+yv)}{\sqrt{w^2+x^2}}\frac{1}{v} + \frac{-yi+xj+wk}{v\sqrt{w^2+x^2+y^2}}\log\frac{\sqrt{w^2+x^2+y^2}+z}{\sqrt{w^2+x^2+y^2}}$

$= \log T + \frac{i}{v}\left(\log\frac{w+xv}{\sqrt{x^2+w^2}} - \frac{y}{\sqrt{w^2+x^2+y^2}}\log\frac{\sqrt{w^2+x^2+y^2}+zv}{\sqrt{w^2+x^2+y^2}}\right)$

$+\frac{j}{v}\left(\frac{w}{\sqrt{w^2+x^2}}\log\frac{\sqrt{w^2+x^2}+yv}{\sqrt{w^2+x^2}} + \frac{x}{\sqrt{w^2+x^2+y^2}}\log\frac{\sqrt{\ }+zv}{\sqrt{\ }}\right)$

$w + x + y + z = \omega$ $\omega^2 = w^2 + x^2 + y^2 + z^2 + 2wx + 2wy + 2wz + 2xy + 2xz + 2yz$

$w - x + y + z = \xi$	ξ^2	−	−	+	+	−	−
$w + x - y - z = \eta$	η^2	+	−	−	−	−	+
$w + x + y - z = \zeta$	ζ^2	−	+	−	−	+	−

$x^2 + w^2 = d\zeta$ $\dfrac{\omega^2 + \xi^2 + \eta^2 + \zeta^2}{4} = w^2 + x^2 + y^2 + z^2$

x

$w\,dw + x\,dx + y\,dy + z\,dz$

$\frac{1}{2}dw^2 + \frac{1}{2}dx^2 + \frac{1}{2}dy^2 + \frac{1}{2}dz^2$

$-x\,dw + w\,dx + z\,dy - y\,dz$

Put $\dfrac{w}{\sqrt{w^2+x^2}} = \cos\theta$ $\sin d\cos - \cos d\sin$ $-\sin^2 - \cos^2$ $d\dfrac{y}{x} = \dfrac{dy \cdot x - dx \cdot y}{x^2}$

$w\,dx - x\,dw$ $\dfrac{x}{y}\,d\dfrac{y}{x} = \dfrac{dy}{y} - \dfrac{dx}{x}.$

$= (x^2+w^2)\left(\dfrac{w}{\sqrt{w^2+x^2}}\dfrac{dx}{\sqrt{\;}} - \dfrac{x}{\sqrt{w^2+x^2}}\dfrac{dw}{\sqrt{\;}}\right)$ $xy\,\dfrac{x}{y}\,d\dfrac{y}{x} = x\,dy - y\,dx$

$= (x^2+w^2)(\cos\theta\, d\sin\theta - \sin\theta\, d\cos\theta.$

$$\int_1^q \frac{1}{q}\,dq$$

$$q = T_q\left(S U_q + V U_q\right)$$

$$q = a\left(\cos\theta + \sin\theta \cdot \upsilon\right)$$

$$\frac{1}{q} = \frac{1}{a}\left(\cos\theta - \sin\theta \cdot \upsilon\right)$$

$$dq = da\,\frac{q}{a} + d\theta\left(-\sin\theta + \cos\theta \cdot \upsilon\right)$$

$$\frac{1}{q}\,dq = \frac{da}{a} + d\theta \cdot \upsilon$$

$$\int \frac{1}{q}\,dq = \log a + \upsilon\theta = \log T_q + \angle q\, U V_q$$

$$\log q = \log T_q + \angle q\, U V_q$$

$$q = T_q\, e^{\angle q\, U V_q}$$

A B C D

$\frac{1}{2}A + \frac{1}{2}C$

$C + x(D - C)$

$A + x(B - A)$

$\frac{1}{4}(A + B + C + D)$

$(2-x)A + xB + (2-x)C + xD$

$(1-x)A + (x+1)B + (1-x)C + (x+1)D$

$y(2-x) + (1-y)(1-x) = 1$

$yx + (1-y)(1+x) = 1$

$2y - xy + \cancel{y} - x = \cancel{x}$

$-y + xy$

$y - x = 0$

$\cancel{x^2} + 1 - \cancel{x^2} = 1$

$$B - \ A \ \ \alpha C$$

$$B - M + x \ldots (C - M)$$

$$\ldots = M + x \ldots$$

$$A = \alpha a$$
$$B = \beta b$$
$$C = \gamma c$$

$$A + \gamma B - \alpha$$

$$\cancel{\alpha a + X \beta b = 0}$$

$$X \alpha a + (1 - \cancel{X}) \beta b = x a + (1 - x) b$$

$$(X\alpha - x) a = \ldots + (\beta - x) b$$

$$X\alpha = x \qquad \cancel{(1-\beta) + x(\alpha\beta - 1) = 0}$$

$$X = \alpha \frac{\beta - 1}{\alpha\beta - 1} \qquad x = \frac{\beta - 1}{\alpha\beta - 1}$$

$$(1-\beta) + X(\beta - \alpha) = 0$$

$$x = \alpha \frac{1-\beta}{\alpha-\beta} \qquad X = \frac{1-\beta}{\alpha-\beta}$$

$$\alpha \frac{1-\beta}{\alpha-\beta} a + \beta \frac{\alpha-1}{\alpha-\beta} b$$

$$\alpha \frac{1-\beta}{\alpha-\beta} a + \beta \frac{\alpha-1}{\alpha-\beta} b$$

$$\beta \frac{1-\gamma}{\beta-\gamma} b + \gamma \frac{\beta-1}{\beta-\gamma} c$$

$$\alpha \frac{1-\gamma}{\alpha-\gamma} a + \gamma \frac{\alpha-1}{\alpha-\gamma} c$$

$$\frac{1-\gamma}{\alpha-\gamma} \frac{\alpha-\beta}{1-\beta} =$$

$$\frac{(\alpha-\gamma)(1-\beta) - (1-\gamma)(\alpha-\beta)}{(\alpha-\gamma)(1-\beta)}$$

$$\cancel{\alpha} - \gamma - \alpha\beta + \cancel{\beta\gamma} + \beta + \gamma\alpha$$

$$\frac{(\beta-\gamma)(1-\alpha)}{(\alpha-\gamma)(1-\beta)}$$

$$\frac{(1-\gamma)(\alpha-1)\cancel{(\alpha-\beta)}}{\cancel{(\alpha-\beta)}(\alpha-\gamma)(1-\beta)} \quad \frac{(1-\alpha)\cancel{(\beta-\alpha)}}{(\beta-\gamma)\cancel{(1-\beta)}}$$

A $\quad mA$

$B \quad nB$

$x B + (1-x) A = y\, mA + (1-y)\, nB$

$ym + x = 1$

$yn + x = n$

$\cancel{y(m-n)} = 0$

$y(n-m) = n-1$

$y = \frac{n-1}{n-m}$

$x = 1 - m \frac{n-1}{n-m}$

$= \frac{n - \cancel{m} - mn + \cancel{m}}{n-m}$

$x = n \frac{1-m}{n-m}$

$m \frac{n-1}{n-m} A + n \frac{1-m}{n-m} B \;\Big|\; m \frac{o-1}{o-m} A + o \frac{1-m}{o-m} C$

$B \qquad C$

$nB \qquad oC$

$x B + (1-x) C = y\, nB + (1-y)\, oC$

$x = yn$

$1 - x = o - yo$

$1 - yn = o - yo$

$y(o-n) = o-1$

$y = \frac{o-1}{o-n}$

$n \frac{o-1}{o-n} B + o \frac{1-n}{o-n} C$

$\frac{o-1}{o-n} \frac{n-m}{1-m} \;\Big|\; 1 - \frac{o-1}{o-n} \frac{n-m}{1-m}$

$(o-n)(1-m) - (o-1)(n-m)$

$o \cancel{-om} \cancel{-n} + nm - on - m$

$m \frac{n-1}{o-n} \frac{o-1}{1-m}$

$m \frac{o-1}{o-n} \frac{1-n}{1-m} \;\Big|\; \frac{(o-m)(1-n)}{(o-n)(1-m)}$

$A \quad B \quad C \quad D \quad E$

$$\begin{array}{c|c|c|c|c} A+C & A+C & A+D & A+D & B+D \\ & B+E & B+E & C+E & C+E \end{array}$$

$xA + xC + (1-x)B + \cancel{(1-x)D} =$

$yA + yC + (1-y)B + (1-y)E$

$(x-y)(A + C - B) = xD - yE$

$uA + uC + (1-u)B + (1-u)D =$

$vA + (1-v)B + vD + (1-v)E$

$\cancel{(u-v)(A-B) + uC}$

$A \quad B \quad C \quad D \quad E$

$\cancel{O \quad B \quad C \quad D \quad bB + bC + aD}$

$O \quad A \quad B \quad C \quad aA + bB + cC$

$$\begin{array}{c|c|c} O+B & O+B & O+C \\ A+C & (1-a)A + bB + cC & (1-a)A + bB + cC \end{array}$$

$$\begin{array}{c|c} O+C & A+C \\ aA+(1+b)B + cC & aA + (1+b)B + cC \end{array}$$

$\cancel{xO + xB}$

$\cancel{xO + (1-x)A + xB + (1-x)C}$

$xA + (1-x)B + xC =$

$y(1-a)A + (1-y+by)B + cyC = 0$

$y(1-a) = x$

~~A~~+

$\cos(A+B) + \sin(A+B)\,\checkmark$

$= (\cos A + \sin A\,\checkmark)(\cos B + \sin B\, j)$

$= \cos A \cos B - \sin A \sin B$

$+ (\cos A \sin B + \sin A \cos B)\,\checkmark$

$(\cos A + \sin A\, i)(\cos B + \sin B\, j)$

$= \cos \cancel{A} \cos B + \sin A \cos B\, i$

$+ \cos A \sin B\, j + \sin A \sin B\, k$

$\cos C = \cos A \cos B.$

~~$\sin C = \sin A \cos B +$~~

$$\cancel{\frac{1}{q}dq = c}$$

$$q = \cos\theta + \sin\theta\, v$$

$$\frac{1}{q} = \cos\theta - \sin\theta\, v$$

$$\cancel{dq =}$$

$$q = \cos\theta + \sin\theta(\cos\varphi\, i + \sin\varphi\, j)$$

$$\frac{1}{q} = \cos\theta - \sin\theta(\cos\varphi . i + \sin\varphi . j)$$

$$dq = \sin\theta(-\sin\varphi\, i + \cos\varphi\, j)\, d\varphi$$

$$\frac{1}{q}dq = \cancel{\sin^2\theta}\ \cos\theta\sin\theta(-\sin\varphi . i + \cos\varphi . j)\, d\varphi$$

$$-\sin^2\theta(\cos^2\varphi + \sin^2\varphi)\, k\, d\varphi$$

$$\int \frac{1}{q}dq = \sin\theta\cos\theta(\cos\varphi\, i + \sin\varphi\, j)$$

$$-\sin^2\theta . \varphi . k$$

$$\cancel{q = Sq + Vq}$$

$$\cancel{q^2 = (Sq)^2 + Vq}$$

$$q = Tq(\cos\theta + \sin\theta . v)$$

$$q^2 = (Tq)^2(\cos^2\theta - \sin^2\theta + 2\sin\cos\theta\, v)$$

$$q^2 = (Tq)^2(\cos 2\theta + \sin 2\theta . v)$$

$$dq^2 = \cancel{2(Tq)}\ \frac{2q^2}{Tq}\, dTq$$

$$+ 2(Tq)^2(-\sin 2\theta + \cos 2\theta . v)\, d\theta$$

$$+ (Tq)^2 \sin 2\theta . dv$$

$$dq = \frac{q}{Tq}\, dTq +$$

$$q = T_q\left(\cos\varphi\, i + \cos\chi\, j + \cos\psi\, k + \sqrt{1-\cos^2\varphi-\cos^2\chi-\cos^2\psi}\right)$$

$$\frac{1}{q} = \frac{1}{T_q}\left(-\cos\varphi\, i - \cos\chi\, j - \cos\psi\, k + \ldots\right)$$

$$dq = \frac{q}{T_q}\, dT_q + T_q\left(-\sin\varphi\, i + \sin\varphi\cos\varphi\sqrt{1-\cos^2\chi-\cos^2\psi-\cos^2\varphi}\right) d\varphi$$
$$+\, T_q\left(-\sin\chi\, j + \sin\chi\cos\chi\right) d\chi$$
$$+\, T_q\left(-\sin\psi\, k + \sin\psi\cos\psi\right) d\psi$$

$$\frac{1}{q}dq = \frac{dT_q}{T_q} + \left(\cancel{-\sin\varphi\cos\varphi + \sin\varphi\cos\varphi\left(1-\cos^2\varphi-\cos^2\chi-\cos^2\psi\right)}\right)$$
$$- \sin\varphi\cos\varphi\left(\cos^2\varphi + \cos^2\chi + \cos^2\psi\right) d\varphi$$

$$-\sin\varphi\sqrt{1-\cos^2\varphi-\cos^2\chi-\cos^2\psi}\; i\, d\varphi$$

$$-\cos\chi\sin\varphi\cos\varphi\sqrt{}\; j\, d\varphi + \sin\varphi\cos\psi\, j\, d\varphi$$

$$-\cos\psi\sin\varphi\cos\varphi\sqrt{}\; k\, d\varphi \frac{1}{2}\ \sin\varphi\cos\chi\, k\, d\varphi$$

$$(a-\cos^2\chi)^{\frac{1}{2}}$$

$$-+\frac{1}{2}(a-\cos^2\chi)^{-\frac{1}{2}}\cos\chi\sin\chi\, d\chi$$

$$\int\frac{1}{q}dq = \log T_q + \tfrac{1}{4}\cos^4\varphi + \tfrac{1}{2}(\cos^2\chi + \cos^2\psi)\cos^2\varphi$$

$$+\frac{1}{2}(a-\cos^2\chi)\cos\chi\sin\chi$$

$$+\frac{1}{2}(a-\cos^2\chi)\cos\chi\sin\chi$$

$\sin x \cos^3 x$

$$(S_q + V_q)(S_q' + V_q')$$
$$+(S_q' + V_q')(S_q + V_q)$$
$$= S_q S_q' +$$
$$(V_q V_q' + V_q' V_q)$$
$$= 2S_q S_q' + (V_q V_q' + V_q' V_q)$$
$$+2(S_q V_q' + S_q' V_q)$$

$(ai + bj + ck)(li + mj + nk)$

$+(li + mj + nk)(ai + bj - ck)$

$ijk = -1$
$jik = +1$

$-2al - 2bm - 2cn$

$jij = jkk = i$
$kik =$

$(S_q + V_q)(S_{q'} + V_{q'})(S_q + V_q)$

$= (S_q)^2 S_{q'} + S$

$(S_q)^2 V_{q'} + 2 S_q S_{q'} V_q$

$S_q (V_{q'} V_q + V_q V_{q'}) + S_{q'} (V_q)^2$

$A +$

$V_q V_{q'} V_q$

$\sqrt{\omega_1^2 + \omega_2^2}$

$\frac{\omega_1}{\sqrt{\omega}}$ $\cancel{\omega_1 + \omega_2}$ $T(v_1 + v_2) \, U(v_1 + v_2)$

$(ai + bj + ck)(li + mj + nk)(ai + bj + ck)$

$\cancel{a} \;\; \cancel{-am}$

$-acm + abn + bcl$

$+acm - abn - bcl$

0

$\cancel{i\,2(amb + anc) + l}$

$\cancel{i\,2a(mb + nc) + l(b^2 + c^2)}$

$\cancel{i\,2al}$

$\cancel{a(al + 2mb + 2nc)}$

$i(-\cancel{2a}^2 l + 2a(mb + nc) + 2l(b^2 + c^2))$

$\log q = \log T_q + \angle q \cdot UV_q$

$\log q_1 q_2 = \log T_{q_1} + \log T_{q_2}$

$+ \angle q_1 q_2 \cdot UV_{q_1 q_2}$

$\cancel{\omega_1 i + \omega_2(\cos\varphi\, j + \sin\varphi\, k)}$

$\omega_1 i + \omega_2(\cos\varphi \cdot i + \sin\varphi \cdot j)$

$q_1 \quad q_2 \qquad e^{\log T + L} \qquad e^{\log q_1 + \log q_2} = q_1 q_2$

$\log q_1 = \log T q_1 + \angle q_1 \, UV q_1$

$\log q_2 = \log T q_2 + \angle q_2 \, UV q_2$

$\log q_1 q_2 = \log (Tq_1 Tq_2)$

$+ \angle q_1 \, UV q_1 + \angle q_2 \, UV q_2$

$= \log T q_1 q_2$

$T\sqrt{\angle q_1^2 + \angle q_2^2 - 2 S . \angle q_1 \angle q_2 \, UV q_1 . UV q_2}$

$\angle q_1 \, UV q_1 + \angle q_2 \, UV q_2$

X_q

$Ax \, q_1 q_2 = Ax \, q_1 + Ax \, q_2$

$q_1 q_2 = Tq_1 Tq_2 (\cos \quad + \sin \quad)$

$\frac{q_2}{q_1 q_2} dq_1 +$

$UV q$

$\angle q_1 (x_1 i + y_1 j + z_1 k)$

$+ \angle q_2 (x_2 i + y_2 j + z_2 k)$

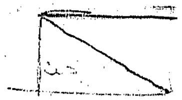
$=$

$(\angle q_1 x_1 + \angle q_2 x_2)^2$

$\angle q_1^2 x_1^2 + 2 \angle q_1 \angle q_2 x_1 x_2 + \angle q_2^2 x_2^2$

$\angle q_1^2 + \angle q_2^2 - 2 \angle q_1 \angle q_2 \cos \angle \frac{UV q_1}{UV q_2}$

$SU \frac{UV q_1}{UV q_2}$



$\int \frac{1}{q_1} dq_1 + \int \frac{1}{q_1} dq_2$

$= \int \frac{1}{q_1} dq_1 + \frac{1}{q_2} dq_2$

$= \int \frac{1}{q_1} dq_1 \frac{q_2}{q_2} + \frac{1}{q_2}$

$\rho = t\alpha + \frac{1}{t}\beta$

$q = (t - t_1)\alpha + \left(\frac{1}{t_1} - \frac{1}{t}\right)\beta$

$\rho = (t - t_1)\alpha + \frac{t - t_1}{t t_1}\beta$

$\rho = (t + x(t-t_1))\alpha + \frac{1}{t} - x\,\frac{t - t_1}{t t_1}\beta$

$\rho = (t + x)\alpha + \frac{t - x}{t^2}\beta$

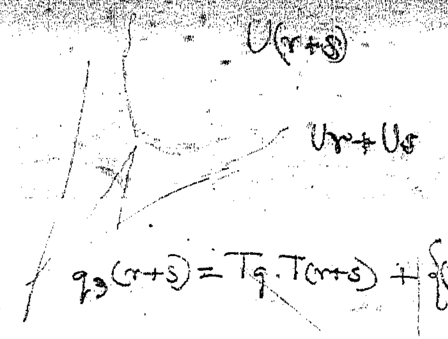
$\rho = (t_1 + y)\alpha + \frac{t_1 - y}{t_1}\beta$

$t + x = t_1 + y$

$t_1^2\frac{t - x}{t^2} = \frac{t_1 - y}{t_1^2}$

$t^3 + t_1^2 x + t_1^2 t + t_1^2 x = 2t_1 t^2$

$x = \frac{2t_1 t^2 - t_1^2 t - t^3}{t^2 + t_1^2} = -t + \frac{2 t t_1}{t^2 + t_1^2}t = -t\,\frac{(t - t_1)^2}{t^2 + t_1^2}$

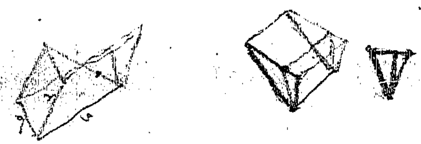


$q_3(r+s) = T_q \cdot T(r+s) + \{(U_q)^{\frac{1}{2}}(U(r+s))^{\frac{1}{2}}\}$

$q_0 r + s$	$q_1 r + q_1 s$
$q_0 s + q_0 s$	$q_1^2 s + q_1 r s$
	$q s (q + r)$

$2 t t_1$

$t^2 t_1^2 - t^3 + 2t^2 t_1 - t t_1^2 t t$
$-t^3 \quad - t t_1^2$



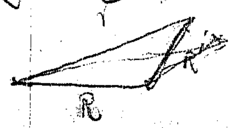
$z = r(\cos\theta + \sin\theta$

$u \neq R(\cos\Theta + \sin\Theta\, j)$

$n = U =$

$u' = u - U + U(-1)^N$

$= u + U((-1) - 1)$

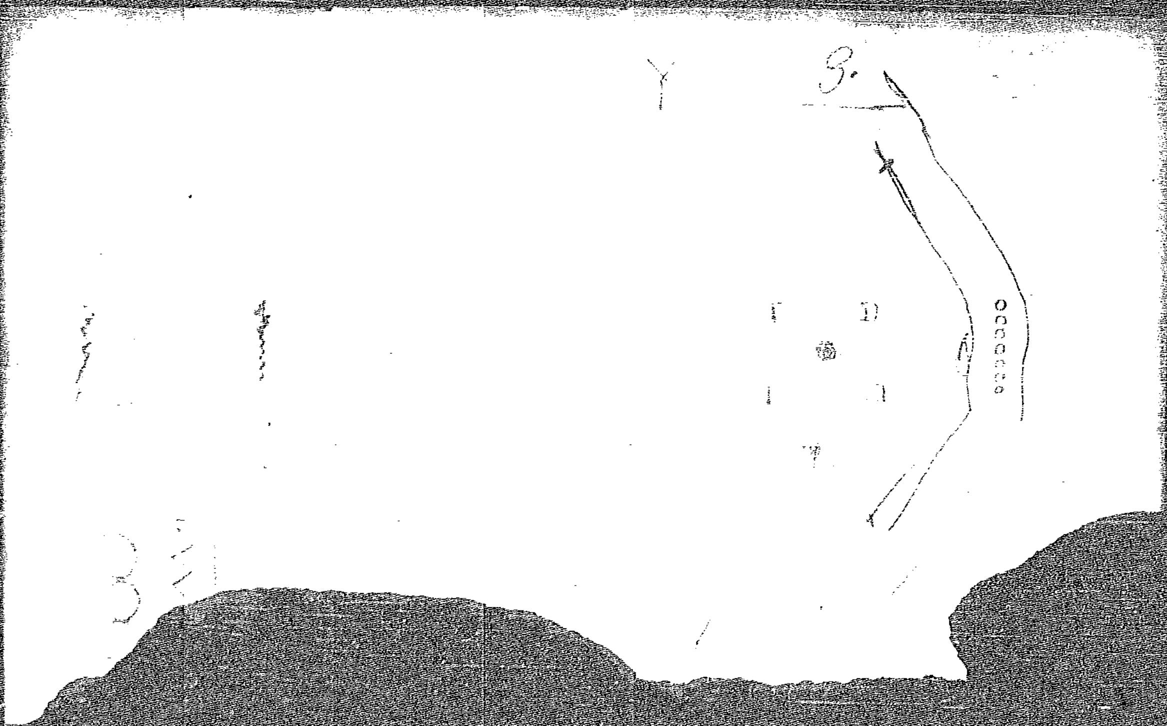


$R'(\cos\Theta' + \sin\Theta'\, j)$

$R'(\cos(\Theta' + \varphi) + \sin(\Theta' + \varphi)\, j)$

$\cancel{R'\cos\Theta'}$

$R\cos\Theta + R'\cos\Theta'$


Y
3.

Velocity of rotation

$$\sqrt{(D_t\varphi)^2 + (D_t\psi)^2 + 2\sin\theta\,(D_t\varphi)(D_t\psi)}$$

Distance of ...

$$\sqrt{\left(k\,\frac{D_t\psi}{\sqrt{(D_t\varphi)^2 + (D_t\psi)^2 + 2\sin\theta\,(D_t\varphi)(D_t\psi)}} + h\sin\varphi\,\frac{D_t\varphi + \sin\theta\,(D_t\psi)}{\sqrt{\ \ }}\right.}$$

$$+ h^2 \sin^2\varphi$$

moment of inertia

$$I\,\frac{(D_t\varphi + \sin\theta\,(D_t\psi))^2}{(D_t\varphi)^2 + (D_t\psi)^2 + 2\sin\theta\,(D_t\varphi)(D_t\psi)} + J\,\frac{(D_t\psi)^2\cos^2\theta}{(D_t\varphi)^2 + (D_t\psi)^2 + 2\sin\theta\,(D_t\varphi)(D_t\psi)}$$

$$+ m\,(\text{Dist of centre of grav})^2$$

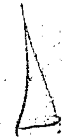
$$T = \tfrac{1}{2}(I + mk^2)(D_t\varphi)^2$$
$$+ (I\sin\theta + mk^2\sin\theta + mkh\cos\varphi)(D_t\varphi)(D_t\psi)$$
$$+ \tfrac{1}{2}\big(I\sin^2\theta + J\cos^2\theta + mk^2 + mh^2(\cos^2\varphi\sin^2\theta + \sin^2\varphi)$$
$$+ mkh\cos\varphi\sin\theta\big)(D_t\psi)^2$$

$\psi =$

$mg \sin\varphi \cos\varphi$

$D_\varphi \Omega = \left(D_t D_{\varphi'} - D_\varphi\right) T'$

s



$\frac{1}{2} I (D_t\varphi)^2 + \frac{1}{2} m (D_t s)^2 = T'$ $\quad mgh\frac{1}{2}\varphi^2$

$D_\varphi \Omega = -mgh\varphi$

$D_s \Omega = -\epsilon s$

$D_{\varphi'} T' = I(D_t\varphi) \qquad D_{s'} T' = m(D_t s)$

~~$I D_t^2 \varphi = -mgh\varphi$~~

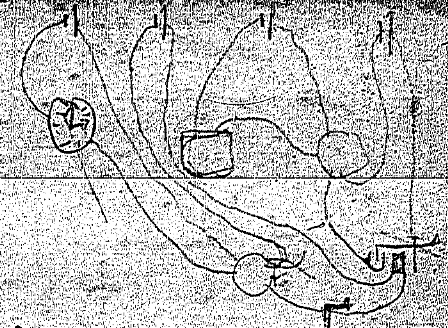
$D_t^2 \varphi = -\frac{mgh'}{I} \varphi$

$D_t^2 s = -\frac{\epsilon}{m} s$

30

30.

$$\frac{0125 \times 3}{375}$$



~~6~~ 9403'

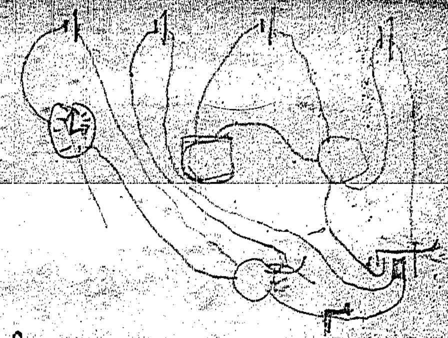
Interference of minute break

	0265		
	1076	0.681	
10	1757	0.893	
30	2650	0.212	
	2650	0.762	
10	3412	0.236	9.474
40	3648		
	3648	.041	.421
00	3689	.462	
80	4151		
	4279	.128	.410
	4817	.538	

30

30.

$$\begin{array}{r} 0125 \\ 3 \\ \hline .375 \end{array}$$



~~26~~ 9403'

	0265		
	1076	0.681	
10	1757	0.893	
30	2650	0.212	
	2650	0.762	
10	3412	0.236	9.474
40	3648		
	3648	.041	9.421
00	3689	.462	
50	4151		
	4279	.128	9.410
	4817	.538	

Interference of minute break

$<$ $[A_i A_j A_k] = -[A_i A_j (a + A_i)]$

$= -[A_i A_j] a - [A_i^2] A_j$

$-a \cdot A_i$

$= +a^2 A_i + a A_i^2 + [A_i^2](a + A_j)$

$= a^2 [A_i] + 2a [A_i^2] + A_i^3$

$= -a^3 + 2a [A_i^2] + A_i^3$

$[A_i B_j B_k] = -[A_i B_j B_i + A_i B_i^2]$

$= -[A_i B_i^2] + A_i (\Sigma B_i^2 - B_i^2)]$

$= [A_i B_i^2] - [A_i](2b - a^2 + \Sigma A_i^2) + [A_i B_i^2]$

$2[A_i B_i^2] + a(2b - a^2) + \Sigma(A_i^3) + \cancel{[A_i B_i^2]}$

$6c = -a^3 + (2a - 2ab + a^3) \Sigma(A_i^2) + [A_i^3]$

$- 2[A_i B_i^2]$

$+64 + (-8 + 48 - 64) \cdot 1 + 4$

96 68 28   72 48 24 4 96

Eq of 4th de

Roots $A_i + B_i \sqrt{-}$

$x^4 + ax^3 + bx^2 + cx + d = 0$

$a = -\Sigma_i (A_i)$ $\Sigma_i B_i = 0$

$2b = \Sigma_{ij} (A + B\sqrt{-})(A + B\sqrt{-})$

$= \Sigma AA - \Sigma BB + \Sigma AB \sqrt{-}$

$\Sigma AB = -\Sigma_i (A_i B_i) = 0$

$\Sigma AA = -\Sigma A_j (a + A_i) = -a \Sigma A_j - \Sigma A_i^2$

$= a^2 - \Sigma A_j^2$

$\Sigma\Sigma BB = -\Sigma B_i^2$

$2b = a^2 - \Sigma A_i^2 + \Sigma B_i^2$

$6c = \Sigma_i \Sigma_j \Sigma_k (A_i + B_i\sqrt{-})(A_j + B_j\sqrt{-})(A_k + B_k\sqrt{-})$

$= \Sigma_i \Sigma_j \Sigma_k (A_i A_j A_k - A_i B_j B_k + (A_i A_j B_k - B_i B_j B_k)\sqrt{-})$

$\Sigma_i \Sigma_j \Sigma_k (A_i A_j B_k - B_i B_j B_k) =$

$- \cancel{A_i A_j^2} A_i A_j B_j + B_i^3$

$+ \cancel{A_j^2} A_j B_i B_j$

$\cancel{2 A_j B_j} \Sigma A_i^2 B_i + B_i^3 = 0$

.055 9
.063 1
.068 4
.071 7
.063 1
Mean .064 22
5
±.004

1/150

1/160

159) 160.064
159
R064

0.02694
2.20140
7.82554
2.17446
1.00669

4817
10 4817 0.222 0.392
40 5039 0.614
5653

5955 .302 .478 0.176 [.318]
6433

6833 .400 .774 0.374
7617

10 8083 .466 .521 .055 38
90 8604
170

8898 .294 .357 .063
9255

0093 838 806 -.032 [.068]
Perhaps 79 899

1681 782 753 -.029 [.071]
2434

2823 .389 .485 .096 24
3308 .063
3856 .548

0.67705
2.87216
7.80489

00518
20140
80378

7.80489
2.17319
9.97808
2.20140
0.00629
2.14301
9.94790

94934
14801
80633

150 149

9.96689
2.17319
7.79370

0.00689
2.20148
9.80549

159

9.95875
2.14301
7.81574 44

150

Calc

149.9508	149	149.9266	1.006219
160.0146	169	162.0160	1.006390
139.2870	159	139.9094	1.006542
160.0141	159	160.0120	1.006365
139.8878	139	139.8899	1.006402
	745	749.7539	Mean 1.006381

or 1 in

0000
42 .02 5725
x = 00 $5724\frac{1}{2}$
02 $6297\frac{1}{2}$
00 4
$0 = -1 + 6672\frac{3}{4}x$ 00

$150y = 8.0 + 6297\frac{1}{2}x$
$149y = 9.00 + 6246x$
$149y = 1.0 + 9321x$

0	3.856	.419	419	
75	4.275	.325	425	+6
150	4.600	.408	408	−17
225	5.008	.479		707
300	5.487	.228		
375	5.715	.435	435	+9
450	6.045		87	
			.422	

6
27
33
17
516

149 150.003

	6.045	556	−19
75	6.601	537	−78
150	7.138	459	
225	7.597		

2

75	3.413	.5816	.0733	
150	8.496	.5083	.0746	−.0373
230	3.739	.5243		
300	8.076	.4337	.0981	−.0490
380	2.533	.4457		
450	5.889	.3356		

$139y = 1 - 1005x$
$159y = 1 + 140x$
$20y = 0 + 1145x$
$10y = 0 + 572\frac{1}{2}x$

149 − 733
159 + 160
159 + 0906
159 + 120
139 − 1101

139 − 1005
159 + 140

INO

1.0 1145
0 064
1.0907
.2552
.0638

1276
7
.8932
64
8868
.1132

.1148

1.0129
0064
1.0193
.2548
0637

1274
7
8918
64
8854
906
18
924
231
1155
1101

1155
1123
78
1139
1101
22
1120

1812

6)160
257
133
120
253
5
258
129

99838
00962
00217
01179 1028
14613
15575 142

139 139.8972
159 160.0143
298 299.9115

0.28137
2.47422
9.80715 6414
2.14301
2.20140
9.95016
0.00855

8916
10199

$\alpha = +(A_1+B_1\sqrt{-})(A_2+B_2\sqrt{-})(A_3+B_3\sqrt{-})(A_4+B_4\sqrt{-})$

$= A_1A_2A_3A_4$

$\sqrt{-}\ + A_1A_2A_3B_4 + A_1A_2B_3A_4 + A_1B_2A_3A_4 + B_1A_2A_3A_4$

$- A_1A_2B_3B_4 -$

$\sqrt{-}\ - A_1B_2B_3B_4$

So that $A_1A_2A_3A_4 \sum \frac{B_4}{A_4}$

$\cancel{A_1\left(\sum A - A_1\right)}$

$A_1A_2A_3A_4 \sum \frac{B_3}{A_3}\left(\sum \frac{B_j}{A_j} - \frac{B_3}{A_3}\right)$

$6abc$

$\Sigma(a^3) + 3\Sigma a^2 b + 6\Sigma abc$

$a^2(\Sigma a - a)$

$3\Sigma a^2 . \Sigma a - 3\Sigma a^3$

$3\Sigma a^2 \Sigma a - 2\Sigma a^3$

Eq. of 4th Deg

$$x^4 + ax^3 + bx^2 + cx + d = 0$$

$$a = -\sum_i (A_i + B_i\sqrt{-}) = -\sum_i \alpha_i$$

$$\sum_i B_i = 0$$

$$b = \frac{1}{2}\left\{\left[\sum_i (A_i + B_i\sqrt{-})\right]^2 - \sum_i (\alpha_i + B_i\sqrt{-})^2\right\}$$

$$= \frac{1}{2}\left\{\left(\sum_i A_i\right)^2 - \sum_i A_i^2 + \sum_i B_i^2\right\}$$

$$= \frac{1}{2}\left(a^2 - \sum_i A_i^2 + \sum_i B_i^2\right)$$

$$\sum_i (A_i B_i) = 0$$

$$c = \frac{1}{6}\left\{\left[\sum_i (\alpha_i + B_i\sqrt{-})\right]^3 - 3\sum_i (A_i + B_i\sqrt{-})^2 \sum_i (\alpha_i + B_i\sqrt{-}) + 2\sum (\alpha_i + B_i\sqrt{-})^3\right\}$$

$$= \frac{1}{6}\left\{\left(\sum_i A_i\right)^3 - 3\sum_i A_i^2 . \sum_i A_i + 3\sum_i B_i^2\left(\sum_i A_i\right) + 2\sum A_i^3 - 6\sum_i (A_i B_i^2)\right\}$$

$$= \frac{1}{6}a^3 - \frac{1}{2}a(a^2 - 2b) - \frac{1}{3}\sum A_i^3 + \sum_i (A_i B_i^2)$$

$$= -\frac{1}{3}a^3 + ab - \frac{1}{3}\sum A_i^3 + \sum_i (A_i B_i^2)$$

$$3\sum A_i^2 B_i - \sum B_i^3 = 0$$

$a + b + c + d$

$a^4 + 4a^3b + 12a^2bc$

$+ 6a^2b^2 + 24abcd$

$abcd = \frac{1}{24}(\Sigma a)^4 - \frac{1}{24}\Sigma a^4 - \frac{1}{6}\Sigma a^3 b$

$- \frac{1}{4}\Sigma a^2 b^2$

$- \frac{1}{2}\Sigma a^2 bc$

$-\frac{1}{6}\Sigma a^3(\Sigma a - a)$

$-\frac{1}{6}\Sigma a^3 \Sigma a + \frac{1}{6}\Sigma a^4$

$-\frac{1}{4}\Sigma a^2(\Sigma a^2 - a^2)$

$-\frac{1}{4}(\Sigma a^2)^2 + \frac{1}{4}\Sigma a^4$

$\frac{1}{6}\Sigma(\Sigma a - d)(abc)$?

$d = \frac{1}{24}\left([\Sigma(A + B\sqrt{-})]^4 - \frac{1}{24}\Sigma(A + B\sqrt{-})^4\right.$

$-\frac{1}{6}\Sigma(A + B\sqrt{-})^3 \cdot \Sigma(A + B\sqrt{-})$

$+\frac{1}{6}\Sigma(A + B\sqrt{-})^4$

$-\frac{1}{4}\left[\Sigma(A + B\sqrt{-})^2\right]^2$

$+\frac{1}{4}\Sigma(A + B\sqrt{-})^4$

$S_1 + S_2 = S$

$S_2 = \rho S_1$

$S = (1+\rho) S_1$

$T = \frac{1}{2} m (1+\rho) \dot{S}_1^2$

$D_s \Omega = \cancel{} - mgS_1 + \epsilon S_2$

$= (-mg - \epsilon\rho) S_1$

$m(1+\rho) D_t^2 S_1 = -(mg+\epsilon\rho) S_1$

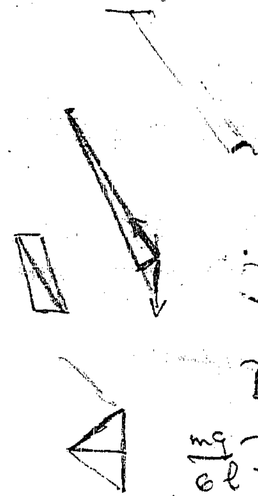
$D_t^2 S_1 = - \frac{mg + \epsilon\rho}{m(1+\rho)} S_1$

$= g(1-\rho) + \frac{\epsilon}{m}\rho$

$= g - (g - \frac{\epsilon}{m})\rho$

$m \frac{g}{\ell}$

$m \frac{g}{\ell} \varphi = \epsilon S$



$S = \frac{mg}{\epsilon \ell} \varphi$

$D_t^2 \varphi = - \frac{mg}{\ell} \varphi$

$D_t^2 S = - \epsilon S$

$\frac{mg}{\epsilon \ell} D_t^2 \varphi = - \frac{mg}{\ell} \varphi$

$\left(1 + \frac{mg}{\epsilon \ell}\right) D_t^2 \varphi = - 2 \frac{mg}{\ell} \varphi$

$D_t^2 \varphi = - \frac{2 \frac{mg}{\ell}}{1 + \frac{mg}{\epsilon \ell}} \varphi$

$= -2 \frac{mg}{\ell + \frac{mg}{\epsilon}}$

Now

The force of restoration is

$v = D_t s + h D_t \varphi$

~~$T' = \frac{1}{2} m (D_t s)^2 + \ldots$~~

$T' = \frac{1}{2} m \left(D_t (s + h\varphi)\right)^2$

$D_{\varphi'} T' = m\, D_t (s + h\varphi)$

$D_{s'} T' = m\, D_t (s + h\varphi)$

$m\, D_t^2 \left(\frac{g}{e\ell} \Sigma mr + h\right) \varphi =$

The force of angular restoration of the pendulum is $m \frac{g}{\ell} \varphi$ ~~and therefore this must be the force applied to the~~ and the linear force is therefore $\frac{g}{\ell} \varphi \Sigma mr$ and this must be ~~the~~ force acting on the fulcrum. Hence

$$\frac{g}{\ell} \varphi \Sigma mr = e s$$

and $s = \frac{g}{e\ell} \Sigma mr \cdot \varphi$

$v = D_t s + D_t \varphi = \left(1 + \frac{g}{e\ell} \Sigma mr\right) D_t \varphi$

$T' = \frac{1}{2} m \left(1 + \frac{g}{e\ell} \Sigma mr\right)^2 (D_t \varphi)^2$

$D_{\varphi'} T' = m \left(1 + \frac{g}{e\ell} \Sigma mr\right)^2 D_t \varphi$

In the ordinary case of the pendulum

$v = r D_t \varphi$

$v^2 = r^2 (D_t \varphi)^2$

$T = \frac{1}{2}(D_t\varphi)^2 (\Sigma m r^2) = \frac{1}{2}(I + mh^2)(D_t\varphi)^2$

$D_{\varphi'} T = (I + mh^2) D_t \varphi$

$D_t D_{\varphi'} T = (I + mh^2) D_t^2 \varphi$

$\Omega = mh \cos\varphi$

$D_\varphi \Omega = -mgh \varphi$

Equation is $D_t^2 \varphi = -\frac{mgh}{I + mh^2}\varphi = -\frac{g}{\frac{I}{mh} + h}\varphi$

When the stand is bent, the force on fulcrum is

$+\frac{g}{\frac{I}{mh} + h}\varphi \Sigma m r$

$\Sigma m r =$ $\qquad n \frac{(\Sigma m r)^2}{\Sigma r^2}$

$\frac{g}{\frac{I}{mh} + h}\varphi \Sigma m r = e s$

$s = \frac{1}{e} \frac{g}{\frac{I}{mh} + h}\varphi \Sigma m r$ $\qquad \frac{M}{T^2}\ s \quad \frac{S}{T^2} M$

$e S = g M$

$e = \frac{M g}{S}$

$v = D_t s + r D_t \varphi$

$v^2 = [D_t(s + r\varphi)]^2$

$T = \frac{1}{2}\Sigma m [D_t(s + r\varphi)]^2$

$D_{\varphi'} T = \Sigma(m r) D_t s + (I + mh^2) D_t \varphi$

$= \left[(\Sigma m r)^2 \frac{1}{e} \frac{g}{\frac{I}{mh} + h} + (I + mh^2)\right] D_t \varphi$

$D_\varphi \Omega = -mgh\varphi$

Hence equations

$D_t^2 \varphi = -\frac{g}{\frac{I}{mh} + h + \frac{g}{e}\frac{(\Sigma m r)^2}{I + mh^2}}\varphi$

and as $\Sigma m r$ is nearly mh

The addition to the length of the pend is nearly

$\frac{mg}{e} h = \frac{S m}{e M} h = \frac{hm}{e M} S$

Another way of treating the problem

$$v = D_t s + r D_t \varphi$$

$$T = \tfrac{1}{2} \Sigma m [D_t s + r D_t \varphi]^2$$

$$D_\varphi T = \Sigma(mr) \cdot D_t s + \Sigma m r^2 \cdot D_t \varphi$$

$$D_s T = \Sigma m \cdot D_t s + \Sigma m r \cdot D_t \varphi$$

$$D_\varphi \Omega = -mgh\varphi \qquad D_s \Omega = -\epsilon s$$

The equations are

$$\Sigma mr D_t^2 s + \Sigma m r^2 D_t^2 \varphi = -\Sigma mgh\varphi$$

$$\Sigma m D^2 s + \Sigma mr D^2 \varphi = -\epsilon s$$

$$\cancel{\Sigma m \Sigma mr D^2 s + \Sigma m \Sigma m r^2 D^2 \varphi = -(\Sigma m)^2 gh\varphi}$$

$$\cancel{\Sigma m \Sigma mr D^2 s + (\Sigma mr)^2 D^2 \varphi = -\Sigma mr \epsilon s}$$

$$(\Sigma m D^2 + \epsilon)\Sigma mr D_s^2 + (\Sigma m D^2 + \epsilon)\Sigma m r^2 D^2 \varphi = -(\Sigma m D^2 + \epsilon)\Sigma mgh\varphi$$

$$(\Sigma m)\Sigma m r^2 D^4 + [(\epsilon - \Sigma m)\Sigma m r^2 + (\Sigma m)^2 gh] D^2 = ((\Sigma m)^2 - \epsilon \Sigma m) gh$$

$$D^4 + \left[\frac{(\epsilon - \Sigma m)}{\Sigma m} + \frac{(\Sigma m)}{\Sigma m r^2} gh\right] D^2 = \left(\frac{\Sigma m}{\Sigma m r^2} - \epsilon \frac{1}{\Sigma m r^2}\right) gh$$

$$D^2\left[(\Sigma mr + x\Sigma m)s + (\Sigma m r^2 + x\Sigma mr)\varphi\right] = -(\Sigma mgh\varphi + x\epsilon s)$$

$$x\epsilon \Sigma mr + x^2 \epsilon \Sigma m = gh(\Sigma m)^2 + x gh \Sigma m \Sigma mr$$

$$x^2 - x\left(\frac{\Sigma mr}{\Sigma m} - \frac{gh}{\epsilon}\Sigma mr\right) = \frac{gh}{\epsilon}\Sigma m$$

$$x = \frac{1}{2}\left(\frac{\Sigma mr}{\Sigma m} - \frac{gh}{\epsilon}\Sigma mr\right) \pm \sqrt{\frac{gh}{\epsilon}\Sigma m + \left(\frac{1}{\Sigma m} - \frac{gh}{\epsilon}\right)^2 (\Sigma mr)^2}$$

$$-x\ldots = (1+x)A_1 = -p(L+hx_2)A_1$$

$$p(L+hx_1)A_2$$

$$\frac{(\ldots) + p(L+hx_2)}{(1+x_1)+p(L+hx_1)} = \frac{A_2}{A_1}$$

$$\frac{(L+xh)\epsilon x}{gm} = x+1$$

$$\epsilon h x^2 + \epsilon L x = gmx + gm$$

$$x^2 + \frac{\epsilon L - gm}{\epsilon h}x = \frac{gm}{\epsilon h}$$

$$x = -\frac{1}{2}\frac{\epsilon L - gm}{\epsilon h} \pm \sqrt{\frac{gm}{\epsilon h} + \frac{1}{4}\left(\frac{\epsilon L - gm}{\epsilon h}\right)^2}$$

$$= -\frac{1}{2}\frac{\epsilon L - gm}{\epsilon h} \pm \frac{1}{2}\frac{L}{h}\sqrt{\frac{4gmh}{\epsilon L^2} + \left(1 - \frac{gm}{\epsilon L}\right)^2}$$

$$= -\frac{1}{2}\frac{L}{h} + \frac{1}{2}\frac{gm}{\epsilon h} \pm \frac{1}{2}\frac{L}{h}\left(1 + \frac{gm}{\epsilon L} + 2\frac{gmh}{\epsilon L^2}\right)$$

$$= -\frac{1}{2}\frac{L}{h} + \frac{1}{2}\frac{gm}{\epsilon h} \pm \left(\frac{1}{2}\frac{L}{h} - \frac{1}{2}\frac{gm}{\epsilon h} + \frac{gm}{\epsilon L}\right)$$

$$= \begin{cases} +\dfrac{gm}{\epsilon L} \\ -\dfrac{L}{h} + \dfrac{gm}{\epsilon h} - \dfrac{gm}{\epsilon L} \end{cases}$$

$$L + hx_1 = L + \frac{gmh}{\epsilon L}$$

$$\frac{g\varphi_0 mh}{L} = \epsilon s_0$$

$$\frac{gm}{\epsilon} - \frac{gmh}{\epsilon L}$$

$$M D_t^2 s + Mh D_t^2 \varphi = -\epsilon s$$

$$Mh D_t^2 s + MLh D_t^2 \varphi = \cancel{g\varphi}\ Mhg\varphi$$

$$D_t^2 s + h D_t^2 \varphi = -\frac{\epsilon}{M}s$$

$$D_t^2 s + L D_t^2 \varphi = -g\varphi$$

$$\cancel{(1+x)D_t^2 s + (h+xL}$$

$$(x+1)D_t^2 s + (L+xh)D_t^2\varphi = -\left(g\varphi + \frac{\epsilon x}{M}s\right)$$

$$= -\frac{g}{L+xh}\left((L+xh)\varphi + \frac{L+xh}{g}\frac{\epsilon x}{M}s\right)$$

$$(L+xh)\varphi + (x+1)s = A_1 \cos\left(\sqrt{\frac{g}{L+xh}}\,t + \eta\right)$$

$$\varphi = (1+x_2)A_1\cos\left(\sqrt{\frac{g}{L+hx_1}}\,t + \eta_1\right) - (1+x_1)A_2\cos\left(\sqrt{\frac{g}{L+hx_2}}\,t + \eta_2\right)$$

$$s = -(L+hx_2)A_1 \qquad +(L+hx_1)A_2$$

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3759
3933 0.169
3760 0.124
3640

145
9855