IMPLICATION AND EXISTENCE IN LOGIC.[1]

MODERN logic has done much, both by precept and example, to inculcate fresh habits of exact and clear thinking. It urges an explicit setting forth of all the premises of your conclusions—a putting into separate categories of those which you can prove and those which you are obliged to assume, a sharp distinction, also, between the terms which you can define and those which you cannot define. The doctrine which stands at the beginning of its method is that (as I have lately pointed out)[2] of "explicit primitives." Modern logic would also highly recommend, to whatever extent it may prove to be convenient, a simple and appropriate symbolism, as a sure cure for the ingrained habit of many reasoners—Euclid, our great exemplar, was not free from it—of letting fresh assumptions slip in surreptitiously. It is in philosophy especially, as the most difficult and perplexing of the sciences, and that in which pure reasoning plays, after mathematics and logic itself, the greatest role, that these good habits ought to prove peculiarly beneficial.[3] But while this more formal Formal Logic is destined, without doubt, to a speedy and wide extension among exact reasoners, it may safely be affirmed that some of the aspects in which it is presented

[1] Read, in brief, before the meeting of the American Philosophical Society of December, 1911.

[2] *Journal of Philosophy, etc.*, VIII, p. 708.

[3] Thus in the admirable representation of propositions by the symbols *SeP*, *MiP*, etc., to be read "No *S* is *P*," "Some *M* is P," etc., which begins to prevail, not only are the terms symbolized (as has been done since the time of Aristotle), but so is also what I have called the "figured copula,"—*i. e.*, not the simple copula *is*, but the copula with all the quantity and quality of the proposition incorporated within it,—'*a* is-wholly *b*,' '*a* is-not-wholly *b*,' etc. This represents a vastly important advance in the right direction, and ought to prepare the way for something more carefully thought out and more detailed. It is a pity that symbol-logic in general is in danger of becoming identified with the system of Peano, in which everything is sacrificed to the modes of thought of the mathematician. For example, the variable, that bugbear to the non-mathematical student, has no proper place in the non-mathematical part of logic, no matter how symbolic that may be.

in the voluminous work of Bertrand Russell leave much to be desired in the way of saneness and sobriety. In particular, there is a phrase to which all those who have read the imposing first chapter of his *Principles of Mathematics* are inclined to attribute a cabalistic meaning, a significance as a picture of the type of reasoning that takes place in the hypothetico-deductive fields of thought, which it does not, in fact, possess,—I mean the phrase 'p implies q.' There are several objections to using this phrase as the diagrammatic representation of reasoning; and that it has so caught the fancy of the outside world is, I believe, much to be deplored. There are many good reasons for dropping it. The word itself, *implies*, is a badly chosen word, for it has, as a word in common use, too strong a connotation of 'implies more or less but not exactly nor rigidly,' and this sense is especially strong in the substantive form, *implication*. It is better not to wrest words from their actual meaning for technical purposes when that can easily be avoided. There are many other words that would answer the purpose better. For the present, however, I shall continue to say 'implies.'

This choice of a term, however, is, to a certain extent, a matter of taste or convenience; the other objections to the formula are of a more fundamental kind. In order the more briefly to discuss them, I permit myself to make use of a simple sign to stand for the logical relation here involved,—namely, the sign $<$, and I shall write $p < q$. Bertrand Russell uses the semi-ellipse of Peano, who objected, very naturally, to the awkward form introduced by Schroeder. My own form has now been adopted by Mally,[1] and I shall hence (on account also of its many advantages) hereafter make no apologies for using it.

There are several objections to making this relation 'p implies q' typical of pure mathematics (and of other subjects of the same kind) which I shall try to set forth. In the first place, it represents a conclusion as following from a premise. It happens, it is true, that a conclusion does, upon occasion, follow from a premise; but the main characteristic of reasoning is that a conclusion follows from several premises,—two, or more. Reasoning may

[1] *Die grundlegenden Beziehungen u. Verknüpfungen der Gegenstände.* Graz, 1912.

be defined as putting This and That together and extracting something Other,—something which has been asserted by the two premises together, but which contains, in the case of the syllogism, only half of what they assert.[1] It may be regarded (in its simpler forms) as the elimination of a common term (or terms) from simple propositions in *is*, or from any of the other transitive relations, as: is-an-ancestor-of, is-a-successor-of, is-an-antecedent-of, is-an-intermediate-between,—the last three being fundamental relations of mathematics. Drawing conclusions from a single premise occurs, it is true, but it is subsidiary to the main work of logic; it has been fully considered by the logician under the name of immediate inference, an existing, but a relatively unimportant, part of the subject. The reasoning-relation then, should rather be written: $p_1p_2 \ldots \lessdot q$, or (to give the conclusion its proper distinctiveness) $p_1p_2 \ldots \lessdot c$. But as soon as we have changed our mystic formula to this extent, it has become nothing more than the common view of the reasoning process,—the premises entail the conclusion. Nothing novel, either, is added by insisting upon the fact that the sequential relation holds (when it does hold) even though the premises are not true, that it has nothing to do with the truth of the premises. This is an old story in logic; there is nothing that all modern logicians have more constantly insisted upon than that the elements of the particular proposition, simple or compound, are affirmed to exist (or to be true), while the universal proposition, in whatever form it is given, is always strictly equivalent to a simple assertion of non-existence, or of non-concurrence, or of incompatibility,—we use different words, in language, according as the elements are terms or propositions (and in the latter case according as the relation is empirical or logical), but the relation continues to be the same. (If the terms or propositions of a universal sequence are, as matter of fact, known to exist, or to be true, and if the fact is relevant, it must be asserted in a separate statement.) But this, as I have said, is an old story in logic, and involves nothing of mystic value.

Bertrand Russell takes up, in a later chapter, this simplifica-

[1] *Journal of Philosophy, etc.*, IX, p. 398.

tion, which he admits will appear objectionable to the logician (this singularity of the premise), and gives his reasons for holding to this procedure. He says that the premises (though consisting of several propositions) can be stated as one,—instead of uttering them separately we can say 'if p_1p_2 . . . are all true, then c (the conclusion) follows,' and 'if p_1p_2 . . . are all true' is one statement. This is true,—it *can* be done. But what is his motive for doing it? It is an amusing one,—he says that the type-phrase looks more symmetrical this way than if we put several propositions into the antecedent while there is only one in the consequent. But surely to give your formula an appearance of symmetry where no symmetry is, is the most fatal of errors; we should do everything in our power to guard the unwary reasoner against the ever-lurking danger of wrong reasoning instead of enticing him into it. It is exactly for the purpose of preventing such confusions as this that symbolic logic was devised. In the inconsistent triad, of course,—$pqr < 0$, 'the constituent propositions cannot possibly all be true at once' (see p. 648),—perfect symmetry is obtained at no cost of incorrectness,—this, indeed, is the purpose for which this mode of reasoning was invented.

But this aspect of the use of $p < q$, while a very dangerous procedure, tempting, perhaps, to the error of Wrong Conversion, is of far less consequence than the error which is involved in setting up this one type of statement as the form of the primitive logic-relation. There are many forms of this relation, and whatever the mathematician may think, in his haste to rush on to mathematics, the logician is bound to study them all, and to choose only after mature consideration the one, if there should be only one, which he will adopt as type. There are eight distinct types of simple statement (all of which can be represented symbolically by modifications of a few simple straight lines), as can readily be seen by noting that there are four possible combinations of two terms,—

$$ab,\ \bar{a}b,\ a\bar{b},\ \bar{a}\bar{b},$$

and that each combination can be stated either to exist or not to exist, and that no statement regarding these two terms (in any form of the simple relation *is*) can be made that is not equivalent to one of these,—*e. g.*, nothing but a is b and not everything but a is b are equivalent, respectively, to $\bar{a} \overline{\vee} b$ and $\bar{a} \vee \bar{b}$. By properly chosen relation-words (and by their equivalent symbols) these may all be expressed in terms of positive elements only.[1] For instance, corresponding to the relation $p < q$ (p is a sufficient condition for q) we shall also have $p \overline{<} q$, 'p is an indispensable condition for q,'—that is, 'if p occurs, q occurs' and 'not unless p occurs does q occur' (the latter relation is a negative one). The lack of the common and facile use of the phrase *indispensable condition* is the cause, I am convinced, of a sad amount of bad reasoning. Thus we cannot infer, from the truth of a state of things, that whatever can be shown to be a *sufficient* explanation of it is a true state of things, but only that what can be shown to be an *indispensable* explanation of it is true. It is only when we can say 'no other explanation is possible' that we have any ground for assuming that a given explanation, though it fully explains, is a true occurrence. We do not infer that a certain noise is made by a railroad train because that would be a *sufficient* ground for it, but because there is *nothing else* which could conceivably happen in my quiet neighborhood which could explain it. I do not infer that the noise in my nursery is being made by *my* children, unless I know that my neighbor's mischievous children have not come in. I am convinced that a great deal of loose reasoning is due to the fact that we have not these correlative phrases, 'sufficient and indispensable,' 'sufficient but not indispensable,' 'indispensable but not sufficient,' etc., in common use. These conceptions the mathematicians make constant use of, and they would find it very hard to carry on their exact trains of reasoning without them. But the name which they give to conditions which have both these characters is 'necessary and sufficient'; sufficient and indispensable is a far better pair of words, for, in the first place, the more important of the two (in practical and also in theoretical matters) is the sufficient condition, and it should therefore stand first,—it is more impor-

[1] See Baldwin's *Dictionary of Philosophy and Psychology* for this complete scheme of Propositions—articles "Syllogism" and "Proposition."

tant that a man should know that a given occupation is sufficient to gain him a living than that nothing else would do, and it is more important to know that we have got a sufficient proof of a thesis than to know that no *other* proof can be found. In the second place, the second condition is really of the nature of a negative (the Latin language expresses it correctly in the phrase *conditio sine qua non*), but its negative characteristic is better expressed in *indispensable* than in necessary. I therefore strongly recommend the introduction, as a fluent form of speech, of the correlative terms 'sufficient and indispensable.' When I said to my little girl, "I will take you down town this afternoon if you are good," she said "And only?"—meaning: That is no doubt a sufficient condition, but is it also indispensable?

The relations just named and their negations (which are particular propositions) are both non-symmetrical; from 'not unless p is true is q true' it does not follow that 'not unless q is true is p true.' But the remaining four relations in 'is—implies' are symmetrical. As soon as we have expressed our propositions in any one of these good forms, all difference between subject and predicate, between antecedent and consequent, between premise and conclusion, has vanished. We have, for example, an inconsistency, an incompatibility (if the elements are propositions),—a non-occurrence, if they are terms. In either case, the fatal error of Wrong Conversion is eliminated automatically, —it is practically impossible to make it. You may inadvertently infer from $p < q$ that also $q < p$,—as who has not done upon some occasion?—but who would infer from the fact that $p \,\overline{\vee}\, q$, that $\bar{p} \,\overline{\vee}\, \bar{q}$,—from the fact that p and q are incompatible that their negations are incompatible? But this is what false conversion is, in terms of the negative relation. You see at once that it is impossible to commit this error. From 'no dancing is moral' it does not follow that 'nothing which is not dancing is immoral,' and it almost makes one dizzy to try to believe that it does. But what it would have meant in the long history of bad reasoning in this world, if we had always been warned against Wrong Conversion by a feeling of dizziness, as we literally should be, if we tried to commit it in terms of the negative copula! The one error in reasoning that people are actually in danger of falling into is this, and a sure safeguard against it ought to be heartily welcomed. The practical rule of reasoning is then: think always in negatives, if you are dealing with universal statements (but in affirmatives, if you are dealing with particular relations). Thus, take the saying of Kant,—"there are no classical philosophical authors," or, what is doubtless just as true (and will give us a *b* for our symbolic term instead of a *p*) 'there are no classical biological authors,'

$$abc \overline{\vee} \infty.$$

We can say, at once, 'no authors are both biological and classical,' 'no biologists are classical authors,' 'no classical biologists are authors,' or any other arrangement you please,—it is impossible to get it wrong no matter what you do, wrong conversion has been eliminated, there is nothing possible but right conversion, —unless, indeed, you drag in statements about non-authors, or non-classicists, or non-biologists, which you are not in the least tempted to do. Compare the simple reversibility of this relation with what we find in the ordinary relation in *is*. Take the familiar judgment of the poet regarding astronomers,—'the undevout astronomer is mad,'

$$\bar{d}a < m.$$

Try to transpose the terms correctly,—you get

$$a < d + m$$
$$\infty < \bar{a} + d + m,$$

any astronomer is either devout or mad, all are either not astronomers or else devout, or else mad,—and so on, eight forms in all— all these are legitimate transpositions,—all these statements are absolutely equivalent, each to each,—but how difficult they are to effect! You must constantly change from *and* to *or*, and from the positive to the negative term,—the rules for procedure are decidedly intricate,—so much so, in fact, that in laying them down we have already passed beyond the field which the ordinary logic ever has attempted to cover. But the transpositions in $abc \overline{\vee} \infty$, on the other hand, are so easy to make that we feel that we are uttering platitudes when we enunciate them. Such are the advantages of the symmetrical copula! Exactly the same state of things holds, of course, when the elements related are premises and conclusion, instead of simple terms. Express everything symmetrically, and temptations to wrong reasoning have practically vanished.

In particular, the syllogism, with its numerous modes and figures, becomes one single form, with one simple rule for validity, when once it is expressed in this way. This 'Inconsistent Triad,' or 'Antilogism' (to use a term which suggests its connection with, and its antitheticalness to, the ordinary syllogism),[1] is the form in which all reasoning-in-transitive-relations should be conducted, so soon as that reasoning becomes at all difficult. (See Schroeder, *Algebra der Logik*, Bd. II (20), § 43, and Baldwin's *Dictionary*, "Syllogism".) Instead of all the complicated rules for testing the fifteen valid modes of syllogism, one has simply this, for every case: express universal propositions with negative copula and particular propositions with affirmative copula, deny the conclusion, and then note conformance to the one simple type,—

(A). *No a is b, no c is non-b, and some a is c cannot all be true at once.*

If any two of these statements are known to be true, the contradictory of the third is a valid conclusion. The advantage of this type-form—the Antilogism, (*A*),—is that not only is the order of terms in the propositions wholly immaterial but so also is the order of the propositions themselves in the triad. Such is the beauty of symmetrical forms of speech![2] That this is a perfectly natural mode of reasoning, my favorite illustration will show; a mother, reproving her child at the table, said, "Nobody eats soup with a fork, Emily," and Emily replied, "But I do, and I am somebody." With this 'but' she said in effect: Here

[1] Royce has adopted one name for it, and Keynes the other. *Formal Logic*, 4th edition, p. 332. I had not yet named it at the time Schroeder wrote his § 43.

[2] Professor de Laguna, in the last number of the *Journal of Philosophy, etc.*, IX, p. 399, recommends for regular use this Inconsistent Triad, but he seems to think that it is desirable to reduce all propositions to the existential form—there is no *ab*, there is some *ac*. There is, of course, no need of this transformation, and they will seem more natural, for practical use, if left in the original subject-predicate form.

is an inconsistent triad of statements, and since mine are patently true, yours must certainly be false. And Emily was four years old! The antilogism (instead of the syllogism) is the natural, the inevitable, form of reasoning in cases of controversy, rebuttal,—in fact, in all cases of discussion between opponents. It is singular that it is yet to be admitted into the logics. (The case of Emily is special, on account of its containing both existence-terms and individual-terms, but by this it is made more difficult, not easier.) Before applying the test of validity, viz., propositions of like quality must have common terms of unlike quality, and conversely,—thus, in (A), b occurs twice, with unlike quality, a and c twice with like quality,—it is of course necessary to remember to make universal propositions begin with *no* and particular propositions begin with *some*.

I add another example to show the naturalness of the Antilogism (which is somewhat obscured when expressed formally), an example which is also taken from real life: 'It is impossible that none of these birds which you shot should be alive, when some of them are breathing and nothing that breathes is dead.' And here is one for the logician who still clings to his s, m, and p: 'No priests are saints.' 'But some priests are martyrs, and there are no martyrs who are not saints.'[1] When propositions have suffered this apotheosis into symmetrical forms, they have lost, as premises, all their right-and-left-ness,—that remains only as a psychological aspect. In speech, it is not possible to preserve this lack of order, but the eye can be trained to take in $a \vee b$, $a \bar{\vee} b$, as a whole, without precedence of either term; so also in the propositional elements of the Antilogism.

In view, then, of the immense advantage, for actual reasoning, of a symmetrical mode of expression, why should we give it up, at the beginning, without any reflection or consideration, in favor of the difficult and dangerous 'p implies q'? I maintain that there exists no even apparent excuse for throwing away, untried, this most useful form of speech.

But there is still another objection to singling out 'p implies

[1] Note that this is something which the traditional logicians have not before devised—a seeming-sensible syllogism in terms of s, m and p.

q' as the sole type of compound (and simple) expression,—two more objections, in fact; one I mention briefly, and one I shall dwell on more fully. To make this one form, which is universal, so exclusively typical of the reasoning relation,—to ignore particular propositions, which are affirmations of existence, of concurrence, of compatibility,—is one-sided in the extreme. When people meet together to discuss things, there is constant occasion on the part of one side to the debate to deny the validity of conclusions drawn by the other side. We need the form of statement

$$p \vee q,$$

'p is-compatible-with q,' or 'p and q are *not* inconsistent,' they *can* occur together, with which to combat the assertion $p \mathbin{\overline{\vee}} q$; or, if we are using the dangerous affirmative form of speech, in order to deny that $p < q$. This corresponds, in propositions, to the particular statement in terms. It is one of the crimes of the recent mathematico-logicians to ignore the existence of the particular, or at most to give it very inadequate discussion. I have given, in *Studies in Logic*, the rules for its treatment; Whitehead (alone among recent writers) returns to the subject (*Universal Algebra*, pp. 83-98). There is no ground whatever for its ever having been neglected; it is one face, or aspect, of logic, and of quite equal voluminousness and importance with that which deals with the universal relation.

My remaining objection to $p < q$ is a more important one still: to take the typical proposition as of this form is fatally to obscure the existence of the existence-term,—an effect which is much to be deplored. I have just used my substitute-relation in the form

$$pq\bar{c} \mathbin{\overline{\vee}} \infty,$$

or

$$pqr \mathbin{\overline{\vee}} \infty,$$

'pqr is-not a possible combination' or 'the concurrence of p, q and r is-excluded-from possible states of things.' I have introduced here an existence term,—and I have, for the moment, represented it by the mathematician's sign for infinity.[1] In the

[1] When writing more voluminously, I use ⦶ and Θ for the logician's everything and nothing; they enable one, when rows and columns are used to represent products

case of terms, this would read

$$apc \; \overline{\vee} \; \infty,$$

'Classical philosophical authors are-not existent.' This term means 'existent things' or 'things which exist.' As the subject of a proposition it will be read, in words, denotatively, as things; in the predicate of a proposition it will be read connotatively, as existent; but for *logic* the full meaning is exactly the same in both cases. (See my doctrine of the four-fold implication of the judgment, *Mind*, October, 1890, pp. 361–2, and Keynes, *Formal Logic*, 4th edition, p. 179.) Thus if a stands for acid things, b for blue things and c for cold things, then

$$\infty \; \overline{\vee} \; abc$$

will be read, 'no things are at once acid, blue and cold,' but its fully equivalent form,

$$abc \; \overline{\vee} \; \infty$$

will be read 'whatever is at once acid, blue and cold *is not* existent,' or (if we like to put the tautologous 'things' into the predicate also) 'is-excluded-from all existent things.' And in the particular statement we shall have $ab \vee \infty$, 'acid-blue things exist,' and $\infty \vee ab$, 'some things are at once acid and blue;' and either of these statements says *no more* than has already been said when we say $a \vee b$ and $b \vee a$, 'some acid things are blue' and 'some blue things are acid,' or, more fully expressed,

$$\infty \, a \vee b$$

$$\infty \, b \vee a,$$

'some *things which are acid* are blue,' and 'some *things which are blue* are acid.'[1] The point is that an existence-term is always involved, in every possible statement, and it is entirely at our discretion whether we make it explicit or not. The usual view is that there are certain 'existential' propositions, as 'diamonds

and sums, to read off all dual forms of statements by rotating the paper through 90°. The terms themselves I have called the Special Terms of logic; unlike the logician's a, b and c, they are never without fixed significance.

[1] I make it a point to make up my illustrative examples out of all nouns or else all adjectives, in the effort gradually to disabuse the mind of logicians of the belief that subjects are necessarily nouns and predicates necessarily adjectives.

exist,' namely, those which contain only one significant term, and that all other propositions have nothing to do with existence. But the true state of things is that *every* proposition is an *existence*-proposition, in the sense of being concerned with existence,—that is, of having existence for one of its terms, and that propositions are of two classes according as they are affirmations of existence or denials of existence,—that is, according as they are particular or universal. When there is only one significant term involved, since every proposition is a relation between two terms, the existence term *must* be present explicitly, as 'some things are accidents,' 'mistakes occur,' $\infty \vee a$, $m \vee \infty$; but in all other cases it is matter of preference whether the existence-term is explicit or implicit.

Now one of the bad consequences of giving to $p < q$ such fictitious prominence as some logicians have done is that the existence of the existence-term is obscured by it. This statement is equivalent to $\infty \vee \bar{p} + q$, and here its true character and import are apparent,—a circumstance which may become of great consequence. Whitehead and Russell say that they have found little need to use propositions in this form. But this is purely a matter of taste. If anyone has a liking for existence rather than for non-existence, these forms of speech are perfectly open to him,—and reasoning will proceed in absolutely parallel courses, whether you use the one form or the other. The only reason for their preference for non-existence over existence is the mathematician's inborn liking for zero.[1] To the philosopher, existence ought to be, of the two, the preferred concept. Keynes, in the admirable last section of his *Formal Logic*, has shown how easy and natural it is to state your premises in the form 'everything is.' And this personal idiosyncrasy of Bertrand Russell's has not been without its consequences; it has led him to develop a theory of types which, if his universe-terms had been more explicitly in his mind, and on his paper, he would doubtless have seen to be (as Dr. H. C. Brown has shown, I believe correctly, *Journal of Philosophy, etc.*, VIII, p. 85) nothing but the good old doctrine of the variable domain of thought.

[1] In the remaining pair of my eight copulae, significant statements are made in terms of the non-existent,—as $o \not\vee m_1 + m_2$, '*all but* mind and matter is non-existent.'

This, then, is the correct and simple function which an existence-term fulfils in logic: it doubles, at once, the number of transpositional forms which a given proposition can appear in, but it changes in no whit the signification which is essential to every judgment. It is always virtually present,—you cannot introduce a fresh existence-term into any statement, because there is always one already there. But its purport, its bearing, its exact extent, remains to be defined. Logic can therefore throw no light upon the particular meaning to be attached to such terms as reality, existence, occurrence, 'things.' They mean, all of them, occurrence within a given domain of thought, and only the character and limits of that domain of thought are not fixed by the proposition. As a general thing, it is something the meaning of which is taken for granted between the interlocutor and the hearer,—just as is the meaning of words. One says: there are criminal actions, there are infinite numbers, there are heroes of novels, there are stones, there are (for purposes of logical discussion) round-squares,—all can be referred by the hearer to the proper domain of occurrence without farther explicification. The term existence (or reality) is the very type and model of the ambiguous, or as Whitehead and Russell say, it is of ambiguous type. While it is a term which is virtually (when not explicitly) present in every sentence which you utter,—while its general character is exactly this,—that it makes no difference whether you say it or not (the *definition* of the term in symbolic logic is $\infty\, a = a$, as the definition of nothing is $a + 0 = a$,—that is, that which is limited by being existent is not limited at all, and that which is increased by the non-existent is not increased at all,—no matter what sort of existence you are talking about), nevertheless its *special* character in any given sentence depends wholly upon the context. If I am talking about ripe apples which exist, I may be thinking simply about existence within my own garden; if I am in the mood of the philosopher, the range of meaning of my existence-terms will have a much wider circumference. The meaning of the term will always depend upon the state of mind of the 'utterer' of the proposition. The one care which logic must have constantly in mind, if it would avoid all the tangle of paradoxes which overwhelm the unthinking reasoner, is not to mix up its domains of thought, —and this it will find distinctly easier to accomplish if its existence-terms are explicitly present in its premises than if they are only implied. They can, in fact, then be tagged with a plain indication of the limits to be kept in mind, in the form of a subscript letter attached to the ∞ or the 0. But to keep them obscure is to invite unnecessarily the fallacy of 'mixed-up fields of thought.'

The several theses that I am here maintaining (1) that $p < q$ has no cabalistic and newly discovered significance, and that as the single representative of all the manifold relations of logic it is a very poorly chosen one;[1] (2) that the symmetrical forms of speech are the only safe ones if one wishes to avoid the fatal danger of wrong conversion, (3) that the 'necessary and sufficient' condition of the mathematician ought to become current with the philosopher (and in common speech as well) under the better name of 'sufficient and indispensable' condition, (4) that the concepts 'existent things' and 'non-existent things' are already existent in every statement that can be made, not simply in the so-called existential proposition, and that therefore the proposition $p < q$ cannot possibly be used as the source of their definition,—all this will have seemed very much in the air,—both very self-evident and very unimportant. But it is a mistake to suppose that errors of this simple kind do not occur among philosophers. It happens that I have at hand a single article[2] which will serve to illustrate more than one of these misconceptions. This article of Professor Marvin's consists in an effort to obtain a definition of the concept 'existence,' or 'reality' (*i. e.*, the totality of all existent things—it is a pleasure to see that Professor Marvin apparently uses the terms as practically synonymous, p. 477). It has been shown by Professor Lovejoy,

[1] Since this was written Dr. Karl Schmidt has advocated the same view, and more; he maintains vigorously that "logic could be developed altogether without even mentioning implication." *Journal of Philosophy, etc.*, IX., p. 436.

[2] "The Existential Proposition," *Journal of Philosophy, etc.*, VIII, pp. 477–490, This term is not taken in its usual signification,—it means here a proposition about terms which are actually existent things.

very acutely, that the effort is unsuccessful, and that any such effort is foredoomed to failure.[1] But there is still room for something more in the way of comment upon the article as an illustration of the many sources of error that lie in wait for the unwary follower of the concepts of Bertrand Russell. I shall mention some of them, without holding to any particular order.

The phrase $p \lessdot q$ has no secret significance beyond the fact that the human mind is capable of reasoning. Instead of using the phrase you may just as well make use of the one word, reasoning, or of the two words, drawing conclusions,—all that $p \lessdot q$ means is that this world is such that conclusions follow upon premises—that reasoning occurs. And non-affirmation of truth or existence for the constituent simple-terms or proposition-terms is nothing that has not always been noticed. What logician has failed to mention that in 'if *a* is *b*, *c* is *d*', it is not said that *a* is *b* is true? To digress for a moment, however, I must say that I cannot pretend to be able to attach a consistent meaning to the '*p* implies *q*' of Bertrand Russell. For instance, in the *Principia Mathematica* the authors regularly speak of *p* as a premise and of *q* as a conclusion, but it is also said that 'every man is mortal' states an implication (formal), though it would not seem that being mortal is a logical conclusion from being a man, unless the proposition is taken as being a verbal proposition,—and this, in fact, is the interpretation of it which is adopted by Dr. H. C. Brown;[2] but 'every man is mortal' seems to be taken as merely typical of *any* relation of inclusion between 'classes,' and surely not every universal proposition is purely verbal? This particular proposition is, no doubt, near the border line between the verbal and significant: the distinction is, in any case, a relative one,—what is verbal to the chemist will be informational to the common man. It is a pity that this is the only example in simple (non-propositional) terms that Mr. Russell ever makes use of. In my corresponding logic-form, $x \lessdot y$, *x entails y*, which I call a sequence (to distinguish it sharply from the mysterious 'implication') the elements (argu-

[1] *Journal of Philosophy, etc.*, VIII, p. 661.

[2] *Journal of Philosophy, etc.*, VIII, p. 87.

ments) may be either simple terms or propositions, $a, b, c, \ldots$ or $p, q, r, \ldots$ (I use x and y to cover explicitly both $a, b, \ldots$ and $p, q, \ldots$,—they are *not* variables). The difference which is supposed to exist between the two forms is wholly removed if one notices that the propositional terms correspond to *individual* (*i. e.*, during the given discussion *indivisible*) terms. The definition which I have given of 'x is an individual' (written as a capital, X) is

$$(x \equiv X) \equiv (x \vee m . \lessdot . x \lessdot m)$$

where m is anything whatever,—that is, whatever x can be said to be in part it can be said to be wholly if, and only if, it is *indivisible*. The relation $p \lessdot q$ covers, of course, not only the relation of logical sequence, but also that in which the truth of p entails the truth of q simply as matter of empirical observation,—as in 'wherever the soil is poor, the inhabitants are of low stature,' a truth which, as matter of fact, was noticed before the intermediate effect-cause (effect of one state of things, cause of the other), 'nutrition is inadequate,' was discovered. But after that we have two logical relations (together with that which results from eliminating the middle one). That is to say, the relation which was at first empirical has become logical. Take also the case of the orphan asylum (well known in the logics): the boys were bad and broke the windows, the girls were good and did not; upon the inset of an epidemic, the girls all died, the boys did not. This coincidence, which was at first purely empirical, became (after science had made farther progress), by the insertion of an intermediate effect-cause, a logical relation. It must be remembered that Bertrand Russell uses *formal* and *material* as applied to implication in totally different senses from those which they bear in logic. Thus (*Principia Mathematica*, p. 22) he says that "every man is mortal" (still his only example of the proposition in simple terms) states a formal implication, and again that "the relation in virtue of which it is possible for us validly to infer is what I call material implication" (whatever this may mean.—*Principles of Mathematics*, p. 338). It appears that this last (formal implication), although it is "the relation in virtue of which it is possible for us validly to infer" is very unimportant, though not so much so that we are justified in completely neglecting it (*Principia Mathematica*, p. 22). A correspondent of mine thinks that formal implication may be identified simply with the universal proposition, in general,—and that the material implication is the same thing as the proposition with an individual subject, in the writings of Bertrand Russell.

After this long digression, I return to the subject of Professor Marvin's article. I shall use, for the moment, the relation 'p implies q' (as he does) as standing simply for the relation 'premises imply conclusion,' or 'the following-relation holds.' Now this relation would not seem in itself to be a particularly hopeful ground on which to look for light upon the nature of existence, and, in fact, no unforeseen results will be found to have been discovered by means of it. But the danger which I have adverted to as possibly resulting from turning the very unsymmetrical relation

$$p_1p_2 \ldots \lessdot c$$

into the seeming-simple

$$p \lessdot q$$

has not been escaped. The phrase, in fact, is used in this article without due regard to its characters of absolute non-convertibleness. Professor Marvin says in plain words, speaking of chemistry, for instance; 'We know q to be true, we discover that p implies q and we therefore assert p as true.' That is, we know the facts of chemistry to be true, we devise a theory to account for them, and straightway we know that theory to be descriptive of a true state of things. Again he says, explicitly, "q being true, p is true, since it implies q." This form of transposition, when p and q stand for terms, is known quite simply as wrong conversion; when p and q are propositions, it is exactly the same thing in form,—it may be described in words as a confusion between the sufficient condition and the indispensable condition. It would add much to safety in reasoning if we could bring ourselves to use freely a simple symbolism for these two relations,

$$p \leqslant q,$$
$$p \mathrel{\overline{<}} q.$$

The second of these statements, it is true, is strictly equivalent to $q \leqslant p$, and to $\bar{p} \leqslant \bar{q}$ (there are, in all, sixteen different forms in which it can be expressed,—see, "The Complete Scheme of Propositions," in article "Proposition," *Dictionary of Philosophy and Psychology*, and "Some Characteristics of Symbolic Logic," *Am. Jour. of Psychology*, Vol. II), but there is only one way in which it can be affirmed directly, *i. e.*, without the transposing or the negating of terms,—viz., in words (these are all the same thing), 'only if p is true is q true,' 'not unless p is true is q true,' 'the truth of p is the *conditio sine qua non* for the truth of q,' or, 'p is the *indispensable* condition of q.' If we wish to deduce the truth of p from the truth of q backwards, it is not sufficient that we establish the truth of $p \leqslant q$,—that has nothing to do with the case,—it is 'indispensable' that we should have proved that p is the *indispensable* condition for q. Suppose we have established it beyond doubt that the atomic hypothesis is a *sufficient* explanation for all the facts of chemistry. Professor Marvin will say that the atomic hypothesis is then known to be both true and existential. But this is not the case,—we are still forced to speak of it as the atomic *hypothesis*. But if we could prove that there is no other conceivable conception that can account for these facts, then and only then could we believe in it as an actually existing state of things, and our ground would then be, *not* that it thoroughly explains, but that *nothing else* can explain. When I say: 'This noise is surely made by a railroad train,' to use another illustration of Professor Marvin's, I base my judgment not upon the fact that a railroad train is sufficient to account for it, but upon the fact that *nothing else* could, under the given circumstances, be its cause. Language is often elliptical in real life, and we may really mean this condition of things when we do not exactly say it, but in the foundations of philosophy we cannot get on with any safety unless our statements are exact. We can, for instance, imagine a pupil of Professor

Marvin's reasoning in this way: 'He certainly looked cross. Fifty reasons occur to me which would have accounted for it,—one is that he had an indigestion. Consequently, I am convinced that he had an indigestion, that the indigestion which could have accounted for his crossness really occurred, was a really existent thing; but also all the other forty-nine things that might have caused it,—for we have learned that our definition of existence "must not imply that the real is unique."' But it is in any case a foregone conclusion that you cannot (even though you reason correctly) use the judgment $p < q$ to define the nature of existence (which is Professor Marvin's contention), because existence is a term which any judgment is already engaged in describing. (This is also Bosanquet's view of the nature of the judgment, but for different reasons.) The meaning of $p < q$ is $\infty < \bar{p} + q$,—that is, existence, or the possible, is characterized by the fact that p is false or else q is true; but also it is characterized negatively by the fact that $\infty \;\bar{\vee}\; p\bar{q}$, that p true and q false does not occur in it,—whether *it* be reality, or truth, or a physical world, or experience, or even that world which the logician has as good a right to as the mathematician has to his domain of the non-Euclidean—the world in which the laws of thought are one and all transcended. In any case, an existence-term is already present,—the conception is so ingrained in the very nature of the judgment (whether simple or compound—in terms or in propositions) that to seek for a philosophical (though non-ontological) definition here is to invite the 'circle-in-definition.' Professor Lovejoy considers that this effort of Professor Marvin's is foredoomed to failure for the reason that logic does not deal with existences. But this, I take it, is because Professor Lovejoy himself has been hypnotized more or less by the Bertrand Russell school into believing that the universal proposition is everything. I should prefer to say the reverse: it is because logic is all compact of existences, because the concept existence is already a part of the warp and woof of logic (and not of the particular—the 'existential'—proposition only, but of the universal as well), because it already exists as one of the terms of every conceivable statement, that no statement (not even $p < q$) can be made use of to define it, if one would avoid the circle in definition.

I am sorry to say that Dr. Bernstein, of the University of California, also takes this view. He writes me, with reference to my brief paper on the Foundations of Philosophy,[1] that it will be impossible to carry out my plan of insisting upon "explicit primitives" for philosophy, because philosophy deals with existences, and Logic has "nothing to do with existences."[2] One might as well say that logic has nothing to do with any real meanings for its symbolic terms, *a*, *b*, *c*, etc.,—that these cannot mean, upon occasion, Absolute, Begriff, consciousness, etc. Dr. Bernstein also has probably been hypnotized by Mr. Russell's $p < q$, and forgets the existence of its denial $p \overline{<} q$, or $p\bar{q} \vee \infty$. The universal proposition, especially when in the form 'everything is $\bar{a}$ or b,' or '$a\bar{b}$ is not existent' would already seem to be concerned with the concept existence, but surely the affirmation of existence is so. When you are reasoning about real things, it is necessary that your symbolic terms, your *a*'s, *b*'s, *p*'s, *q*'s, etc., should preserve the same meaning throughout a given discussion,—your *p*'s cannot mean prunes, prisms, and electric particles all at once. And the same precaution must be observed in regard to your existence-term,—your domains of thought must not be mixed up. But the precaution requires no more acuteness in the carrying out in the one case than in the other.

There is also a material error in the argument of Professor Marvin which does not come exactly under the topic of symbolic logic. He fails, I believe, to distinguish sharply enough between the proposition as true and the proposition as "existential." (By the latter he means a proposition dealing with actually occurring things, and even, in this paper, things occurring in a physical world—'physical objects,' to use the undefined term of the Six Realists, chemical substances, for instance.) Thus Bertrand Russell's definition of pure mathematics does not simply involve (p. 478) that the constituent propositions of $p < q$ need not be true, but also that they need not deal with existent objects—that they need not be 'existential.' (This is not a bad sense in

[1] *Journal of Philosophy, etc.*, VIII, (1911).

[2] And this in spite of the fact that Dr. Bernstein attended my lectures in Baltimore!

which to use the term existential; to denote 'existential proposition' in the usual meaning—'there are occasions,' 'whatever is, is right,'—it is better to say: propositions with only one non-special term, or uni-terminal propositions.) This is all that is involved when Bertrand Russell introduces, to the confusion of the general reader, in the very first sentence of his *Principles of Mathematics*, that uncanny term, the variable (and, more terror-striking still, the real and the apparent variable). Professor Marvin would seem to have forgotten for the moment that for a proposition to be true it is neither sufficient nor indispensable that it should be existential. (Professor Lovejoy has pointed out this oversight, p. 661). The final form of his definition is: 'The existent is the asserted sufficient condition of any true proposition,' that is, of p, when p implies q, and q is known to be true,—*e. g.*, the atomic hypothesis, if the facts of chemistry have been correctly collected, and if the hypothesis really explains them. But, waiving the *non sequitur* of this,[1] Professor Marvin forgets that before you can devise your existential explanation of the facts of chemistry, you must know that your facts themselves are 'existential.' We cannot give physical-world explanations of imaginary states of things. What then is his test for the actuality of the facts which are to be explained by a given theory? Curiously enough, he takes an unexceptionable view of the criterion, in the last analysis, of existent things (in a physical world)—they are the things that can be pointed at; what I express in my doctrine of Histurgy by saying that they are experiences which have the one-time one-place coefficient attached to them. (See *Report of the Congress of Philosophy*, Heidelberg, 1908.) But surely emotions, indifferences, feelings of admiration and of contempt, are quite as 'real' as colors,—Professor Marvin gives no criterion for recognizing them; he speaks as if only the physical world 'existed.' Limiting ourselves, then, to the physical world, not only the truth but also the existentiality of q must be known before you can infer (back-

[1] Our author says, indeed, in one place, p. 479, "*As far as logic is concerned*, q does not imply the truth of p"; what I object to is that he immediately ignores the fact that every case of reasoning about material occurrences even must always continue to be the anxious concern of logic.

wards!) that those qualities are to be found in *p*; hence you must know what existence is, and be able to apply your knowledge, *before* you can define it. This is doubtless the curious circle-in-definition which Professor Lovejoy divines to exist in this argument of Professor Marvin.[1]

The real state of things then is this: if *q* is true—and if *p* accounts completely *and uniquely* for *q*, then *p* is true, but also if the truth of *q* has been got by empirical observation, and hence deals with real objects ("has been experimentally ascertained"—we cannot *experiment* with imaginary test-tubes) then not only is *p* true, but also it deals with really existing objects. That is, if balls made of negative corpuscles enclosed in a positive electric sheath will fully explain matter (with all its qualities thick upon it), and if nothing else will, then these positive-negative balls are really existent objects. But it is so hard to prove that no *other* conception will explain matter,—so many conceptions in the past have had to be given up for better ones, that the right-thinking individual will be very loath to give these conceptions any very firm lodgment in his mind,—he will be more inclined to continue to regard them as hypotheses.

What Professor Marvin accomplishes in the end (if anything) is to add to those real existences which are forced upon us by immediate experience all the hypothetical, ingeniously conceived, objects and events which have been devised to explain them (*e. g.*, side-chains, corpuscles of negative electricity, hollow spheres of positive electricity, vortices, the twisted rubber tubes of Sir William Thomson, etc. It seems to me that we may well hesitate to accept these as existences in the same sense as the sticks and stones which are well known to us,—that we shall do better if we continue to hold, as we have always done, that the figments of the active brain of the scientist are rather inhabitants of the world of *hypothetical physical existences* than of any world more substantial. Why not continue to preserve the distinction? In any case, far from giving us the distinguishing mark of existent objects, which we must first have learned to recognize elsewhere, these hypotheses at most enlarge their field,—but

[1] *Loc. cit.*, p. 663.

that by new objects whose right to admission is certainly questionable. It is true that many of the commonly accepted properties of the world are, in the beginning, of this sort, more or less, but they have acquired their firm lodgment in our thoughts by the fact that they have so long 'held together,'—that interweaving which takes place between the innumerable products of empirical induction, in the way of piecing together, again and again, pairs of fitting premises and deriving fresh conclusions which can then be put to the test of experiment, strengthens enormously the validity of the whole closely connected structure: this is what I have called the doctrine of Histurgy. I have found it necessary to give a distinctive name to this doctrine, in order to mark it out sharply from the vicious doctrine of pragmatism—its nearest foe; things that are unnamed can hardly be said to 'exist.' The erroneous reasoning of Professor Marvin is peculiarly deserving of study because it is the very same fallacy as that upon which pragmatism is built up. Those who desire to see philosophy enumerated among the sciences—that is, among the domains native to those thinkers who strive for truth, not, like Bergson, for romanticism (Professor Lovejoy has called him, very happily, the last of the romantic philosophers) will do well to strive together to exterminate what may be called the Fallacy of the Compound Wrong Conversion.

The question has lately been discussed (in the *Journal of Philosophy, etc.*) by Professor Perry and Dr. Brown whether symbolic logic is likely to be of value to the philosopher,—whether it is calculated to assist him in the tangled mazes of thought through which he is forced to make his way; Professor Perry maintains the affirmative of this question and Dr. Brown the negative. In view of the considerations which I have set forth, I am myself strongly on the side of both of these disputants; a good symbolic logic, kept simple, sufficiently elementary, and thoroughly sane, would be of really incalculable value to the philosopher,—it has become, in fact, an indispensable tool,—but the one-sided and amorphous form of logic which Peano and Russell make use of as prolegomena to mathematics is certain to be terribly injurious to him—as the example of it which I am here discussing will illustrate. A little symbolic logic is a dangerous thing, and the more so if that little is entirely unadapted to its purpose. The great advantage which symbolic logic ought to secure for the actual reasoner is that his premises and conclusions, his equivalences and his under-statements, would be set down so sharply and definitely before him that it would be difficult for him to fail to keep their relationships exactly in mind,—it would be quite impossible, for instance, for him to lay down, at the beginning of his philosophy as *two* principles, what is really only one principle together with the same thing re-stated in its contrapositive form, as some one has lately done in the program of the six realists. Besides exactness, this form of speech secures extreme conciseness, in a material sense,—you can overlook so much of your argument with a single sweep of the eye that obscure odds and ends of error are not likely to escape you. Again, the mere mechanism of the various transpositions that you are constantly called upon to perform,—(especially if you give preference, in your language, to the symmetrical forms of speech, no *a* is *b*, etc.) will become an ingrained habit, and hence a great aid to exactness. But the overloaded and excessively cumbrous symbolism of Mr. Russell —as $\exists ! \alpha$ for 'α exists' and $(\exists (x))$ for 'x exists' (instead of a simple copula and existence-term for both, $\alpha \vee \infty_1$, $x \vee \infty_2$, if it is necessary to distinguish the types of existence)—obscures many things that are really very simple. Consider, for instance, the "very difficult" (!) logical problem discussed in § 38, *Principles of Mathematics.* The limitations to the usefulness of this form of logic are evident, and I shall not dwell upon them here—no unimportant one is the smallness of the number of letters in the alphabet, even when the Greek alphabet has been added to our own. It may become necessary to annex the Chinese alphabet in order to have at hand a greater number of symbols for terms! But besides the difficulties that are inherent in the subject, there remains the fact that the symbolism of Peano and Russell is a badly chosen one,—it is impossible that any one who is not going to make logic his life work should take the trouble to learn to distinguish between $\cap$ and $\cup$, as signs for *and* and *or*, and between $\supset$ and $\subset$, as signs for the two senses of implies.[1] But with the aid of a symbolism which should be chosen for the needs of the non-mathematician, and which should take proper account of the inertia of the human mind, much advantage might be had from these devices. The habits of exact thought which the discipline entails, the custom of setting out your complete chains of deduction all the way back from your explicitly undemonstrable propositions and your explicitly indefinable term, of guarding rigidly against the slipping in of postulates and axioms which have not been distinctly enumerated, would certainly be a gain in any field of intricate reasoning—and especially in philosophy, where foundations are so much in evidence. The chemists would have been sadly handicapped if they had balked at an intricate symbolism. And who knows how long it took the early logicians before they were willing to trust argument to the letters of the alphabet instead of to really significant terms?

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[1] I shall use, for the logic-relation, following Mally, $-\!\!\prec$ when it is necessary /to distinguish them.