ON THE INFLUENCE OF THE FLEXIBILITY OF THE SUPPORT ON THE OSCILLATION OF A PENDULUM.

(Translated from French into English by the author.)

NEW YORK, *July* 13, 1877.

DEAR SIR: On taking charge of the Coast Survey researches upon gravity, I ordered of Messrs. Repsold a reversible pendulum, to be a copy of that of the Prussian Geodetical Institute. But the instrument makers were at that time so taken up with the construction of instruments for the Transit of Venus, that the pendulum was only ready in the spring of 1875. I then went to Hamburg to receive it; and from Hamburg I went on to Berlin, where I found General Baeyer rather dissatisfied with the results obtained with the Prussian instrument. He specially mentioned the flexibility of the tripod, a source of error which pendulum experimenters have surely never overlooked. The pendulum apparatus that I had carried with me from America having been ruined in transportation, I was under the necessity of employing the new instrument, and therefore undertook to measure and take account of the error in question.

A pendulum support might be rickety, so that the pendulum in its oscillations should throw the knife-edge plane from one position to another, without its undergoing any resistance to the motion other than inertia and friction, between two fixed points. This, however, does not happen in the case of any of the supports that I have examined; for, upon observing their behavior under a high-power microscope, I have always found that they spring back exactly to their original position after every flexure that I have applied to them. In short, the movement with which we have to do is the oscillatory flexure of an elastic body. The amplitude of the oscillation is, at most, about $\frac{1}{5000}$ of that of the lower knife-edge of the pendulum, so that its square may be neglected.

The plane of support of the knife is itself undoubtedly bent during the movement; but I neglect this and limit myself to the consideration of the movement of its middle point. When to this middle point is applied a horizontal force perpendicular to the knife edge, the latter describes a movement of revolution around an axis which, in the case of the Repsold apparatus, is situated behind and above the tripod at a distance of about a meter from the knife-edge. We can neglect the difference between this movement and a translation, until we come to measure its amount. There is also a minute variation in the vertical pressure of the pendulum on the support, but this is very far from producing any sensible effect on the period of oscillation.

Let us denote by

m the mass of a particle,

r its distance from the knife-edge,

ω the inclination, at rest, to the vertical of the perpendicular let fall from the particle on to the knife-edge,

M the mass of the pendulum,

l the length of the corresponding simple pendulum,

h the distance of the center of mass from the knife-edge,

T the period of the oscillation,

g the acceleration of gravity,

ε the elasticity of the support,

φ the instantaneous inclination of the pendulum to its position of rest,

s the instantaneous displacement of the middle point of the knife-edge from its position of rest,

t the time.

Then, the horizontal velocity of a particle will be

$$r \cos(\varphi+\omega) \mathrm{D}_t\varphi + \mathrm{D}_t s$$

and its vertical velocity will be

$$r \sin(\varphi+\omega) \mathrm{D}_t\varphi.$$

Its living force will, therefore, be

$$\frac{1}{2}mr^2(D_t\varphi)^2+mr\cos(\varphi+\omega)D_t\varphi.D_ts+\frac{1}{2}m(D_ts)^2,$$

and that of the pendulum will be

$$\frac{1}{2}Mlh(D_t\varphi)^2+Mh\cos\varphi.D_t\varphi.D_ts+\frac{1}{2}M(D_ts)^2.$$

The living force of the motion of the support itself may be left out of account since it involves the square of an excessively small velocity.*

The differential of the potential energy is

$$Mgh\sin\varphi.d\varphi+\varepsilon s.ds.$$

There is really a third term to be added to this expression dependent on the molecular friction of the matter of the support. But I think we may neglect this term; for its effect cannot be very great, and its coëfficient is, at any rate, unknown.†

From the expressions for the living force and potential we deduce the Lagrangian equations

$$lD_t^2\varphi+\cos\varphi.D_t^2s=-g\sin\varphi$$

$$-h\sin\varphi.(D_t\varphi)^2+h\cos\varphi.D_t^2\varphi+D_t^2s=-\frac{\varepsilon}{M}s,$$

or, neglecting terms of the second degree,

$$lD_t^2\varphi+D_t^2s=-g\varphi$$

$$hD_t^2\varphi+D_t^2s=-\frac{\varepsilon}{M}s.$$

[NOTE.—1882, July 24. I omit the solution of these equations as originally given, and substitute the following, which is perhaps less inelegant. Subtracting the second equation from the first, we get

$$(l-h)D_t^2\varphi+g\varphi=\frac{\varepsilon}{M}s$$

or

$$D_t^2s=\frac{M}{\varepsilon}(l-h)D_t^4\varphi+\frac{Mg}{\varepsilon}D_t^2\varphi$$

Substituting this value in the first differential equation, we have

$$\frac{M}{\varepsilon}(l-h)D_t^4\varphi+\left(l+\frac{Mg}{\varepsilon}\right)D_t^2\varphi+g\varphi=0.$$

Separating the operator into factors, and using the abbreviation

$$i=4\frac{Mg}{\varepsilon l}\cdot\frac{1-\frac{h}{l}}{\left(1+\frac{Mg}{\varepsilon l}\right)^2}$$

we get

$$\left[D_t^2+\frac{\varepsilon l+Mg}{2M(l-h)}(1+\sqrt{1-i})\right]\cdot\left[D_t^2+\frac{\varepsilon l+Mg}{2M(l-h)}(1-\sqrt{1-i})\right]\varphi=0.$$

* It is easy to see that the effect of this would be to increase the last term of the living force; this would affect the second of the differential equations just as if M had been multiplied and h divided by the same quantity. But this would not affect the final result. [1882.]

† This is the point to which the greatest objection to my work has been made. [1882.]

The solution of this is

$$\varphi = \mathrm{A}_1 \cos\left(\sqrt{\frac{\varepsilon l+\mathrm{M}g}{2\mathrm{M}(l-h)}(1-\sqrt{1-i})}\,.\,t+\eta_1\right)+\mathrm{A}_2 \cos\left(\sqrt{\frac{\varepsilon l+\mathrm{M}g}{2\mathrm{M}(l-h)}(1+\sqrt{1-i})}\,.\,t+\eta_2\right)$$

where $\mathrm{A}_1, \mathrm{A}_2, \eta_1, \eta_2$ are arbitrary constants. On neglecting the square of $\frac{\mathrm{M}g}{\varepsilon l}$, this reduces to

$$\varphi = \mathrm{A}_1 \cos\left(\sqrt{\frac{g}{l}\left(1-\frac{\mathrm{M}g}{\varepsilon l}\right)}\,.\,t+\eta_1\right)+\mathrm{A}_2 \cos\left(\sqrt{\frac{\varepsilon}{\mathrm{M}}\cdot\frac{1+\frac{\mathrm{M}gh}{\varepsilon l^2}}{1-\frac{h}{l}}}\,.\,t+\eta_2\right)$$

The second term represents a mere tremor, for its period is very short, owing to the large value of ε. The period of the first harmonic constituent is

$$\mathrm{T}=\sqrt{\frac{l}{g}+\frac{\mathrm{M}}{\varepsilon}}$$

From the value of φ and the first equation of this note, we deduce the following value of s:

$$s=\frac{\mathrm{M}g}{2\varepsilon}\left(-\frac{\varepsilon l}{\mathrm{M}g}(1-\sqrt{1-i})+1+\sqrt{1-i}\right)\mathrm{A}_1\cos\left(\sqrt{\frac{\varepsilon l+\mathrm{M}g}{2\mathrm{M}(l-h)}(1-\sqrt{1-i})}\,.\,t+\eta_1\right)$$

$$+\frac{\mathrm{M}g}{2\varepsilon}\left(-\frac{\varepsilon l}{\mathrm{M}g}(1+\sqrt{1-i})+1-\sqrt{1-i}\right)\mathrm{A}_2\cos\left(\sqrt{\frac{\varepsilon l+\mathrm{M}g}{2\mathrm{M}(l-h)}(1+\sqrt{1-i})}\,.\,t+\eta_2\right)$$

It thus appears that the amplitude of the principal constituent of s is nearly

$$h\frac{\mathrm{M}g}{\varepsilon l}\mathrm{A}_1,$$

while that of the other constituent is nearly $-l\mathrm{A}_2$.

To find the best way of starting the pendulum so as to make the ratio of A_2 to A_1 as small as possible, we must consider how to make the initial value of s as nearly as possible $h\frac{\mathrm{M}g}{\varepsilon l}$ times the initial value of φ. Now, it is easy to see that if the pendulum is supported at a point at a distance x from the knife-edge, any yielding of the support will diminish the value of φ as expressed by the equation

$$ds=-x \sec\varphi\,.\,d\varphi.$$

Substituting this in the expression for the differential of the potential energy, this last becomes

$$\mathrm{M}gh \sin\varphi\,.\,d\varphi-\varepsilon s\, x \sec\varphi\,.\,d\varphi.$$

Equating this to zero, we find

$$s=h\frac{\mathrm{M}g}{\varepsilon x}\sin\varphi\,.\,\cos\varphi.$$

In order that this should be equal to $h\frac{\mathrm{M}g}{\varepsilon l}\varphi$, it is only necessary to put $x=l$, so that in starting the pendulum the finger or trigger should be applied at the lower knife-edge or center of gyration.]

The elasticity, ε, may be measured by observing the deflection, S, of the support produced by a horizontal force equal to the unit of weight. For

$$\varepsilon=\frac{g}{\mathrm{S}}$$

Substituting this value, we find

$$\varphi=\frac{A}{h}\cos\left(\sqrt{\frac{g}{\left(l+MS\frac{h}{l}\right)}}\,t\right)$$

Accordingly, the effect on the pendulum is to give it a virtual length greater than what it would have on a rigid support by $MS\frac{h}{l}$.

Let us denote the duration of an oscillation by T, and let $\varDelta$ be used to indicate the effects of flexure. Then, since

$$T^2=\varpi^2\frac{l}{g}$$

we have

$$\varDelta T^2=\frac{\varpi^2}{g}MS\frac{h}{l}.$$

If we distinguish by subjacent letters the two positions of a reversible pendulum, we have

$$\frac{\varpi^2 l}{g}=\frac{T_d^2 h_d-T_u^2 h_u}{h_d-h_u}$$

and

$$\varDelta l=MS,$$

or putting λ for the length of the second's pendulum

$$\varDelta\lambda=MS\frac{\lambda}{l}.$$

To determine the flexure, I fasten in the slot in the plane of suspension of the Repsold apparatus a fish-line passing horizontally in the direction of the pendulum's movement over an Atwood's machine pulley, and on the end of this cord I hang a kilogramme. [With a stronger support, the pendulum itself may conveniently replace the kilogramme.] On the extremity of the plane of suspension, or at the end of an arm attached thereto,* I stick a glass stage micrometer, turned so as to measure in a direction parallel to the impressed force. This scale is looked at by a microscope carrying a filar micrometer, and solidly mounted upon an independent support, the standard of which is a piece of gas pipe about 10 centimeters in diameter.

I now give a brief *résumé* of my results, beginning with the experiments to determine the position of the fixed axis about which the head of the Repsold support rotates during flexure.

A.—*Experiments made on a level with the suspension plane.*

HOBOKEN, *March* 10, 1877. Temperature 13° C.

+ = forward; − = back.

Distance of scale from end of plane.	Flexure in revolutions of the micrometer screw.	
	Observed.	Calculated.
m.		
−0.496	+0.211	+0.209
+0.053	+.0356	+0.358
+0.318	+0.436	+0.431

The calculated quantities suppose that the axis pierces the suspension plane at a distance of $1^m.355$ behind the forward end of the suspension plane.

* This arm is best made of brass tubing, which may be cut out to make it lighter. [1882.]

B.—*Experiments in the vertical of the forward end.*

HOBOKEN, *March* 12, 1877. Temperature 14° C. Observer, Sub-assistant SMITH.

+ = below; − = above.

Position of the scale relative to the suspension plane.	Flexure in revolutions of the micrometer screw.	
	Observed.	Calculated.
m.		
−0.44	+0.196	+0.196
0.000	+0.340	+0.332
+0.395	+0.446	+0.454

The calculated quantities suppose the axis to pierce the vertical of the forward end of the suspension plane $1^{m}.07$ above this plane. It is not at all surprising that the instantaneous axis is above the suspension plane. Let us suppose that the flexure existed exclusively in three feet of the support. In this case the movement of the upper end of each foot would be perpendicular to the general direction of the foot, and at the same time perpendicular to the radius of the circle of revolution, so that the foot would be directed directly towards the fixed axis. The axis is without doubt behind the support, on account of the flexure of the plane itself.

I made experiments at Geneva, Paris, Berlin, and New York, in order to determine S numerically. The experiment at Geneva, made the 13th of September, was only a trial. But I had a good pulley which I had borrowed from the workshop of the Geneva society for the construction of physical instruments, and I got as an approximate value—

$$S = 0^{mm}.034$$

The pulley that I used at Paris had considerable friction, to which can be attributed the fact that the numbers found differ sensibly from those obtained with the aid of better apparatus.

These are the figures—

January 18, 1876, at Messrs. Brünner, Temp. 1° C $S=0^{mm}.0363$
March 7, 1876, at the Paris Observatory, Temp. 9° C $S=0^{mm}.0371$

At Berlin I used a very delicate pulley which turned on friction-whels, in order to diminish the friction. It belonged to the Physical Cabinet of the Institute of Technology of Berlin, and was put at my disposition by the kindness of Professor Paalzow. The micrometric readings were made alternately with and without the weight, making but one reading each time, in order to avoid any error arising from the support of the micrometer, this being made of wood. In the readings made alternately with and without the weight, I ended with the arrangement with which I began (11 for one, and 10 for the other), in order that the mean instant of the observations should be the same for the two arrangements. The value of 1 division of the micrometer screw was measured separately.

Below are the results of the different series—

May 24, 1876, a. m.,	$S=0^{mm}.0340$
Temp. 13° C., p. m.,	$0^{mm}.0339$
	$0^{mm}.0340$
	$0^{mm}.0341$
May 25, 1876, Temp. 13°,	$0^{mm}.0337$
	$0^{mm}.0336$
Mean,	$S=0^{mm}.0339 \pm 0^{mm}.001$

At Hoboken (near New York) I obtained, through the kindness of Professor Morton, an excellent pulley, made in the workshop of the Stevens Institute of Technology. I always made a reading on each one of the lines of the scale before changing the disposition of the weight.

The results of the separate series are—

March 7, 1877, Temp. 15° C.,	$S = 0^{mm}.0342$
March 10, 1877, Temp. 12°.	$0^{mm}.0332$
	$0^{mm}.0337$
	$0^{mm}.0343$
	$0^{mm}.0342$
	$0^{mm}.0339$
	$0^{mm}.0334$
These two series should have double weight in the reduction,	$0^{mm}.0342$
	$0^{mm}.0342$
Mean,	$S = 0^{mm}.0340 \pm 0^{mm}.0001$

In all the experiments made in the different positions of the scale the flexure obtained has been reduced to the center of the knife, and this last is what is called S.

It is to this last value that I give the preference.

It follows, from the experiments described on pages 430–431, made to determine the position of the axis of rotation, that the forward end of the suspension plane is distant from that axis by $\sqrt{1^m.355 \times 1^m.07} = 1^m.20$. And, since the movement of this end with the weight of a kilogramme is $S + 0^{mm}.0008$, the correction $+0.0008$ arising from the reduction from the center of knife to the forward end, it follows that the torsion of the support by that force is $\frac{0^{mm}.0348}{1^m.20} = 0.0000290 = 5.''8$. Although there is nothing to be suspected in this result, I wished to check it by a direct experiment. I attached a mirror at the extremity of the suspension plane, and, with the aid of a telescope, I measured the torsion by the reflection of a scale, and I found it $6''$. This method, of course, is not as exact as the other.

In order to arrive at another confirmation of the theory, I made the following observations on the flexure produced by the oscillation of the pendulum itself in its two positions, using a tolerably high-power microscope (*i. e.*, magnifying 500 diameters). The scale used was made by Mr. Rodgers, of Harvard College Observatory. It is divided with extreme exactness, the interval between two lines being $\frac{1}{4000}$ of an English inch. It was fixed 70 millimeters before the center of the knife, which gives a correction to S of $+.0019$.

If Φ is the amplitude of oscillation on each side of the vertical, the double amplitude of the vibration of the scale should be

$$2\ M\ (S + 0^{mm}.0019) \frac{h}{l} \Phi$$

in which $M = 6.308$ and $\frac{h}{l} = \frac{17}{56}$ or $\frac{39}{56}$ according as the pendulum is suspended by the knife nearest or farthest from the center of gravity. I used this formula in calculating the quantities now given.

DYNAMICAL FLEXURE.

A.—*Pendulum suspended by the knife farthest from the center of gravity.*

HOBOKEN, *March* 20, 1877.

Φ	Amplitude of the movement of the scale. 1 div. = $\frac{1}{4000}$ inch.	
	Observed.	Calculated.
° ′	*Divisions.*	*Divisions.*
2 32	2.2	2.2
2 30	2.1	2.1
2 24	2.0	2.1
2 22	1.9	2.0
2 20	1.9	2.0
2 19	1.95	2.0
1 43	1.5	1.5
0 47	0.8	0.7

B.—*Pendulum suspended by the knife nearest the center of gravity.*

Φ	Amplitude of the movement of the scale. 1 div. = $\frac{1}{4000}$ in. Observed.	Calculated.
° ′	*Divisions.*	*Divisions.*
2 39	1.0	1.0
2 34	0.9	1.0
2 29	0.9	0.9
2 25	0.9	0.9
2 22	0.8	0.9
2 14	0.8	0.8
2 12	0.8	0.8
2 06	0.7	0.8
2 04	0.75	0.8
1 57	0.75	0.7
1 51	0.75	0.7

In making these observations, I saw distinctly the little subsidiary vibration at the end of each oscillation arising from the second term of the formula.

Finally, I swung the pendulum on two supports of different flexibility—one was the metallic tripod, by Repsold, to which refer the flexure measurements given above; the other was made by fixing the upper part of the Repsold tripod to a thick wooden plank by means of bronze bolts passing through the three holes through which the feet pass. These holes are conical, and the bolts fit exactly. I put on each bolt between the head of the support and the plank a leaden washer, so that, in tightening the bolts and compressing the washers, great stability was obtained and at the same time a horizontal position. The plank (which was 5 centimeters thick) was cut in order to make a place for the pendulum, and it was placed by force between a stone wall and a brick pillar. A slit was then cut, in which a pulley of an Atwood machine was placed to measure the flexure.

Experiments on the flexure of this support.

HOBOKEN, *May* 21, 1877.

Distance of scale before +, behind − of the center of knife in English inches.	Distance of scale to suspension plane in English inches + above, − below.	Flexure in millimeters under a weight 1 kilogramme.	Temperature C.	Observer.
Inches.	*Inches.*	*mm.*	°	
+ 1.2	− 1.3	+0.0052	18.3	E S.
+ 1.2	− 1.3	+ .0052	18.9	E S.
+ 1.2	+39.5	− .0425	20.0	C. S. P.
+13.2	+39.5	− .0367		C. S. P.

It follows that for this apparatus $S^1 = 0^{mm}.0031$, and that the difference between the values of S for the two supports is $0^{mm}.0309$. Now I find $\frac{\pi^2 l}{g} = 1.0125$ sidereal seconds and $l = 1^m$. Hence, we conclude that the difference of $\frac{\pi^2 l}{g}$ according as the pendulum oscillates on one or the other supports must be equal to

$$\frac{\pi^2 l}{g} \frac{M(S-S^1)}{l} = \frac{81}{80} \times 6.308 \times 0.0309 = 0.000197$$

I swung the pendulum three times on the less solid support and once on the most solid to verify the theory. I observed 10 consecutive passages of the pendulum across the vertical at intervals of 5 minutes, using a relay that I invented for this purpose.

A.—*Oscillations on the Repsold metallic support.*

HOBOKEN, *April* 1, 1877.

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
300	301.9652	—0.0130	301.9522	1.006507
296	297.9408	—0.0084	297.9324	528
298	299.9533	—0.0060	299.9473	535
* Mean				1.0065238

PENDULUM SUSPENDED BY THE KNIFE FARTHEST FROM THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
296	297.9094	—0.0092	297.9002	1.006420
302	303.9374	—0.0081	303.9295	389
296	297.9060	—0.0066	297.8994	417
Mean				1.0064067

Hence, we have

$$T_1^2 = 1^s.0128544$$
$$T_2^2 = 1^s.0130902$$

And since $h_1 : h_2 = 101 : 44$ we have

$$\frac{\pi^2 l}{g} = \frac{T_1^2 h_1 - T_2^2 h_2}{h_1 - h_2} = 1.013\left(1 - \frac{101 \times 0.0001456 + 44 \times 0.0000902}{57}\right) = 1.012672$$

This value is to be corrected for rate of chronometer and temperature. The chronometer lost $0^s.86$ per day, which gives a correction to T^2 of $+0^s.000020$. The temperature during the time the heaviest mass was above was 12°.7 in the mean, and 12°.9 when this mass was below. Hence, to reduce to 13° we must apply a correction of

$$\frac{0.1 \times 101 - 0.3 \times 44}{57} = 0.0000186 = -0.000001$$

Hence we conclude

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ \text{ C.} = 1.012691$$

April 7, 1877.

PENDULUM SUSPENDED BY THE KNIFE FARTHEST FROM THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
290	291.8794	—0.0103	291.8691	1.006445
296	297.9131	—0.0086	297.9045	434
298	299.9241	—0.0073	299.9168	432
298	299.9241	—0.0060	299.9181	437
358	360.3090	—0.0058	360.3032	434
Mean				1.0064357

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
288	289.9026	—0.0132	289.8894	1.006560
298	299.9648	—0.0092	299.9556	562
300	301.9760	—0.0067	301.9693	564
298	299.9564	—0.0051	299.9513	548
298	299.9591	—0.0037	299.9524	552
Mean				1.0065578

* When we have a series of equal consecutive intervals if n is the number of intervals and i is the position of one of them we should, in taking the mean, give to this interval the weight $-i\,(i-1)$.

Hence we have

$$T^2_1 = 1^s.0129128$$
$$T^2_2 = 1^s.0131586$$
$$\frac{\pi^2 l}{g} = 1^s.012723$$

Daily correction, $+0^s.44$	$+$ .000010
Temp. $15^\circ.8$ (both positions)	$-$ .000052

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ \text{ C.} = 1.012681$$

April 8, 1877.

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected time.	Period.
	s.	*s.*	*s.*	*s.*
298	299.9647	−0.0175	299.9472	1.006534
298	299.9549	−0.0111	299.9438	523
298	299.9539	−0.0080	299.9459	530
298	299.9484	−0.0055	299.9429	520
298	299.9481	−0.0039	299.9442	526
Mean				1.0065261

PENDULUM SUSPENDED BY KNIFE FARTHEST FROM CENTER OF GRAVITY.

	s.	*s.*	*s.*	*s.*
298	299.9229	−0.0066	299.9163	1.006431
298	299.9213	−0.0058	299.9155	426
297	298.9125	−0.0049	298.9076	423
299	300.9236	−0.0042	300.9194	419
298	299.9174	−0.0035	299.9136	422
Mean				1.0064246

$$T_1^2 = 1^s.0128905$$
$$T_2^2 = 1.0130948$$
$$\frac{\pi^2 l}{g} = 1.012733$$

Daily correction, $-0^s.41$	$-$ .000009
Temp. heavy end up, $13^\circ.2$ Temp. heavy end down, $13^\circ.5$	$-$ .000016

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ = 1.012708$$

Hence the three experiments on the Repsold support give for the value of $\frac{\pi^2 l}{g}$ at 13° C.

s.
April 1, 1.012691
April 7, 1.012681
April 8, 1.012708

Mean, 1.012693

B.—*Experiments made on the stiffest support.*

HOBOKEN, *May* 14, 1877.

PENDULUM SUSPENDED BY THE KNIFE FARTHEST FROM THE CENTER OF GRAVITY.

Mean instant of 10 transits.			Interval of 298 oscillations.	Reduction to infinitely small arc.	Corrected interval.	Interval of 298 oscillations.	Reduction to infinitely small arc.	Corrected interval.
h.	*m.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*
14	6	22.4307	....	....	....	....	....	....
..	7	22.8245	....	....	....	....	....	....
..	11	22.3337	299.9030	—0.0132	299.8898	....	....	....
..	12	22.7213	....	....	....	299.8968	—0.0126	299.8842
..	16	22.2313	299.8976	—0.0110	299.8866	....	....	....
..	17	22.6209	....	....	....	299.8996	—0.0106	299.8890
..	22	22.5145	....	....	....	299.8936	—0.0087	299.8849
..	23	22.9017	....	....	....	....	....	....
..	27	22.4055	....	....	....	299.8910	—0.0074	299.8836
..	28	22.7949	299.8932	—0.0072	299.8860	....	....	....
..	33	22.6896	299.8947	—0.0060	299.8887	....	....	....

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Mean instant of 10 transits.			Intervals of 298 oscillations.	Reduction to infinitely small arc.	Corrected intervals.
h.	*m.*	*s.*	*s.*	*s.*	*s.*
15	53	22.4041	....	....	....
..	58	22.3579	299.9538	—0.0198	299.9340
..	3	22.3058	299.9479	—0.0131	299.9348
..	8	22.2531	299.9473	—0.0087	299.9386
..	13	22.2119	299.9588	—0.0062	299.9526
..	18	22.1554	299.9435	—0.0044	299.9391
..	23	22.1011	299.9457	—0.0031	299.9426

Mean $T_2 = 1^s.0065104$.

Hence we find—

$$T_1^2 = 1^s.0127144$$
$$T_2^2 = 1.0130632$$
$$\frac{\pi^2 l}{g} = 1.012445$$

Daily corr. to chron. + $2^s.59$ — .000060

Temp. heavy end down, 14°.18 } — .000010
Temp. heavy end up, 15°.00 }

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ = 1.012495$$

Comparing this value with the one obtained with the other support we find a difference of 0.000198. The difference, according to the computations of the experiments on flexure, ought to have been 0.000197,* which shows a sufficient agreement.

Yours, most faithfully,

[Signed] C. S. PEIRCE,
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* In the original publication, owing to an erroneous value for the mass of the pendulum, this is erroneously calculated as 0.000191. The agreement of the experiments with theory is, therefore, much better than was supposed.