CHARLES S. PEIRCE AND THE PROBLEM OF MAP-PROJECTION*

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AMONG the official reports of the Assistants in the United States Coast and Geodetic Survey for the year 1877[1] is a letter from Charles S. Peirce to Superintendent Patterson that tells of Peirce's new experimental work on the flexure of the pendulum stand—work necessitated by his systematic attempt to compute the measure of gravity. For Peirce was a scientist in the employ of the United States government and had been charged in 1872 with the responsibilities of the investigation of gravity as well as the related one of determining the ellipticity of the earth.[2]

The letter contains, too, the following surprising remarks:

> I have something else which you might possibly like to insert in the Coast Survey Report. When I was in Paris,[3] I found that the best MS. of Ptolemy's catalogue of stars had never been properly transcribed. This I did and have set them down on a modern atlas and have the materials for new and improved identifications of them. I therefore propose to make a new edition of Ptolemy's catalogue with identifications and notes. Also with a planisphere showing the stars and the figures of the ancient constellations, which I carefully studied some years ago and laid down on my globe.[4]

Peirce's interest in map-making was further revealed in the report that he had "considered the projection of the sphere to be adopted and, setting out with the following conditions," he had invented two means of satisfying them. The conditions were:

1. The map must preserve the angles, like Mercator's and the Stereographic projection.
2. It must put the *whole* sphere on one finite map.
3. It must bring the sphere into an oblong shape, suitable for a page.

Peirce explained that both methods involved the use of "Elliptic Integrals"[5] and he suggested that the Superintendent might be interested in having a brief description of these projections of the sphere as well as others he had invented, even

* This article is an expansion of a paper read by the writer at the International Congress for the History of Science, at Cornell University, August 1962. The original paper was entitled "The Quincuncial Map-projection of Charles S. Peirce."

[1] January 24, 1877. National Archives, Washington, D. C. The writer is indebted to the personnel at the National Archives for their assistance in making the Peirce records available to her. The writer is deeply indebted, too, to the Philosophy Department at Harvard University for permission to refer to unpublished Peirce manuscripts in the Peirce Collection at Houghton Library.

[2] For further details see "Charles S. Peirce, Nineteenth Century Man of Science," by the writer in *Scripta Mathematica* **24** (1959): 305–324. Peirce presented to the National Academy of Sciences two papers on the determination of the figure of the earth by the measurement of the variation in gravity. One he read in 1880; the other in 1882.

[3] Peirce had been in Europe on a Coast Survey mission from April, 1875, until August, 1876.

[4] Appendix #7 in the Superintendent's Report for 1876 (which appeared in 1879) carried the title "A Catalogue of Stars for Observation of Latitude." The list was compiled under Peirce's direction and the data were "based on Mr. Peirce's transcript of the Paris manuscript, an account of which he has presented to the American Academy of Arts and Sciences." He had read that paper on October 11, 1876. Peirce also read a communication on the *Star Catalogue* to the National Academy of Sciences at its meetings November 14–17, 1882. Much of the material from the *Catalogue* may be found in Peirce's *Photometric Researches* published as vol. IX of the *Annals of the Harvard College Observatory*, Leipzig, 1878.

[5] See "The Rhombic Conformal Projection" by Oscar S. Adams, Mathematician, U. S. Coast and Geodetic Survey, in *Bulletin géodésique*, #5 (janvier, février, mars, 1925): 1–26. Adams claims that in the account that Peirce published in 1879 in the *American Journal of Mathematics* of a conformal projection of the sphere within a square, one finds the first application of elliptic functions to conformal mapping for geographical purposes. Adams refers to the work of H. A. Schwarz, of Halle, in 1864, and to Schwarz's proof that a circle can be conformally mapped within a regular polygon of n sides by use of the function:

$$w = \int_0^z \frac{dz}{(1 - z^n)^{2/n}}$$

The writer is indebted to Professor Victor Lenzen of the University of California for this reference.

A recent communication from Albert A. Stanley of the U. S. Coast and Geodetic Survey office has brought to the attention of the writer, on Erwin Schmid's suggestion, Special Publication #112 of the Department of Commerce (1925). It is also by Oscar S. Adams and is entitled: *Elliptic Functions Applied to Conformal World Maps.* Peirce's work is given full credit both in preface and text.

though the Catalogue of Ptolemy were not desired.

The image of Peirce concentrating his efforts on logical analysis and metaphysical speculation is now so familiar to the academic mind that the description of his role as a scientific map-maker comes as a surprise. Yet it should not be so, even from purely logical considerations. For, thinking to him was a basically diagrammatic process for which he invented in logic a diagrammatic mechanism that he called "the existential graph." He claimed that it bore "a remarkable likeness to his thoughts about any topic in philosophy,"[6] and he believed that logical truth has the same source as mathematical truth which is derived from observation of diagrams which have been set down on paper.[7] He called the *sheet* of assertion in the representation of the existential graph a photograph,[8] or rather, a *map* of such a photograph and called a map of the simpiest kind one that represents "all the points of one surface by corresponding points of another surface in such a manner as to preserve the continuity unbroken, however great may be the distortion."

The preservation of the continuity element relates Peirce's "imaging" or substitution theory in logic very closely to the conception of correspondence and Gauss's conception of the *Abbild*.[9] Peirce claimed that his early work on the projection of the spheroid in 1879 showed his familiarity even then with Gauss's logical conception. Throughout Peirce's work the concepts of correspondence, imaging, mapping are all related in his thought to the same basic logical problem of representation. And he often uses a description of geographical mapping processes to illustrate a logical point. For example he dramatically illustrated the regressive element in continuity by the map image in the following figure:

> If a map of the entire globe was made on a sufficiently large scale, and out of doors, the map itself would be shown upon the map, and upon that image would be seen the map of the map, and so on indefinitely. If the map were to cover the entire globe, it would be an image of nothing but itself, where each point would be imaged by some other point, itself imaged by a third, etc. But a map of the heavens does not show itself at all.

He explained that "a Mercator's projection shows the entire globe (except the poles) over and over again in endlessly recurring strips," and that "many maps, if they were completed, would show two or more different places on the earth at each point of the map (or at any rate on a part of it), like one map drawn upon another." He observed that "such is obviously the case with any rectilinear projection of the entire sphere, excepting only the stereographic."[10]

On the other hand, Peirce points out that

> other kinds of map such as his Quincuncial Projection . . . show the whole earth over and over again in checkers, and there is no arrangement you can think of in which the different representations of the same place might not appear on a perfectly correct map.[11]

Peirce's interest in map construction for practical purposes in the field work of the Coast and Geodetic Survey can be readily understood. A report to Patterson from Peirce dated July 17, 1879, in a section devoted to Pendulum Observation serves to illustrate this need. For Peirce

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[6] From autobiographical fragments in the Charles Peirce Collection at Houghton Library, Harvard University, Box R#8. Peirce believed, too, that he suffered from an incapacity for linguistic expression because he thought not in words but in diagrams. (Dated Aug. 24, 1909.)

[7] *Collected Papers of Charles S. Peirce* edited by Charles Hartshorne and Paul Weiss, 2.76 and 2.77.

[8] *Ibid.*, 4. 513.

[9] Fragment from R#9 at Houghton Library. Also see 8.122 and Peirce's review of Royce's *The World and the Individual*. The map metaphor is continued in 8.125.

In this autobiographical fragment Peirce refers to his 1881 paper on the "Logic of Number" in which he distinguishes "between finite and infinite collections in substantially the same way that Dedekind did six years later." He claims to have deduced the validity of the Fermatian method of reasoning about integers from the conception of correspondence. A paper entitled "Fermatian Inference and DeMorgan's Syllogism of Transposed Quantity in Peirce's Logic of Science" by the writer is being published in *Physis* (*Rivista Di Storia Della Scienza*) **5**, FASC. 2 (1963).

[10] *Collected Papers* 3.609.

[11] *Ibid.*, 4.513. Peirce writes that a Mercator's chart "represents all the surface of the earth by a strip, infinitely long, both north and south poles being at infinite distances, so that places near the poles are magnified so as to be many times larger than the real surface of the earth that they represent, while in longitude the whole equator measures only two or three feet; and you might continue the chart so as to represent the earth over and over again in as many such strips as you pleased."

A "quincunx," according to Peirce's definition in the Century Dictionary, is "an arrangement of five objects in a square, one at each corner and one in the middle (thus :·:). . . ." "Quincuncial" means that the elements are, "disposed so as to form a quincunx; also, arranged in two sets of oblique rows, at right angles to one another, so that five together form a quincunx. . . ." The illustration given for a *quincuncial arrangement* is the following:

wrote that "the theory of conform map projection has been studied with reference to its use in the study of gravity; and a new projection has been invented."[12] Again, a letter to his famous father Benjamin, who was then Consulting Geometer for the Coast Survey, carried an expression of gratitude for the father's advice on a map problem and explained that, if a map were constructed as Charles then proposed "with the great circle between New York and Queenstown as its centre, the straight line from the point of Florida to Gibraltar (the worst one on it) would diverge about 60 miles as its maximum from a great circle." Charles claimed that

> the straight lines on the projection . . . would be much better than the rhumb lines of an ordinary Mercator's projection and as these latter only differ 30 miles in length from that of a great circle in crossing the Atlantic, it follows that the deviation of my lines will be less still.[13]

It is difficult to set an exact date for the completion of the map invention in which Peirce took the greatest pride—the Quincuncial map-projection. In his 1889 *Century Dictionary* account of the history of map-making, which is included in his article on "projection," Peirce gave the date as 1876. Indeed by February 26, 1877, he had sent a paper on the map to Patterson, for he asked on that date whether such a paper had been received.[14] Correspondence in the Coast Survey files during 1877 suggests that Peirce and Charles A. Schott, Assistant in charge of the Computing Division, were exchanging views on problems in map-making in that period.[15]

Now 1877 was the year of the autumn meetings of the International Geodetic Association in Stuttgart. Recognizing the importance of the map theory that he had developed by this time, Peirce wrote to Acting Superintendent Hilgard of his desire to show his Quincuncial map at the meetings. He continued: "I think that the sketch which I sent might be traced, photographically reduced and photolithographed. The points were all laid down from computation, only that there were not enough of them." He then inquired,

> Do you not think that this could be done so as to send me a few copies by the steamer of the 22nd? I was, and am anxious that a map should be published, which should have a sufficient number of points to enable me to draw magnetical and other curves upon it.

Because of insufficient time, Peirce was willing to permit the use of a sketch in the reproduction. In a footnote he requested Hilgard to present this map for him at the next meeting of the National Academy of Sciences. That organization, however, was not to learn about Peirce's map-work until the meetings of April 15–18, 1879, when the subject of a talk given by Peirce himself was entitled "On the projections of the sphere which preserve the angles." The map was not made ready, unfortunately, for presentation by Peirce at Stuttgart.[16]

Since his map had not yet appeared in print, Peirce informed Patterson in a letter dated April 26, 1878, of the desire of Professor Sylvester of the Johns Hopkins University to publish a paper by Peirce on map-projections in a forthcoming number of the *American Journal of Mathematics* which Sylvester had just founded. Peirce sought permission to send the material to Sylvester and requested also that the Coast

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[12] Reports of Assistants, 1879. U. S. C. and G. S., National Archives. Peirce used "conform" instead of "conformal" as can be seen in his *Century Dictionary* definitions.

[13] Houghton Library. Professor Max Fisch has dated this letter April 14, 1877.

[14] Reports of Assistants, 1877. National Archives. The letter reads in part: "I sent you some time ago a paper on Map-Projections. I do not know whether it reached you or not. With it I sent a letter, in which was a remark which might be misunderstood. I recommended that the algebraic work should be duplicated by another person, or at any rate looked over. But it will be understood that it is not necessary to verify the results by other methods, which would involve much labor."

[15] On March 8, 1877, Peirce wrote to Patterson as follows: "Yours of yesterday with the sketch of a projection by Mr. Schott depending on Laplace's Coëfficients has just reached me. The projection seems to involve the novel idea of distorting the surface of the globe (while leaving it a globe) so as to counteract the subsequent distortion in projecting onto a plane. The latter projection might be of any sort whatever. This idea is well worth developing. Of course, all projections which preserve the angles are obtainable by my method; but Mr. Schott's idea is not limited to this class of projections." Reports of Assistants 1877. National Archives.

In the Civil and Naval Assistants file, Vol. #2, 1877, a note to Hilgard from Peirce, dated May 18, 1877, says in part: "On my map there was a formula given for the parallels which is only approximate. I find an exact formula is just as easy and I communicated the latter to Mr. Schott some time ago when I supposed the matter was in his hands. . . . I also enclose a note to my paper on Map Projections; it should appear as a foot-note and serves to correct a slight inaccuracy or rather looseness of statement." No such note exists in this part of the Coast Survey files today.

[16] Civil and Naval Assistants, vol. #2, 1877. C. & G. S. files, National Archives, September 10, 1877.

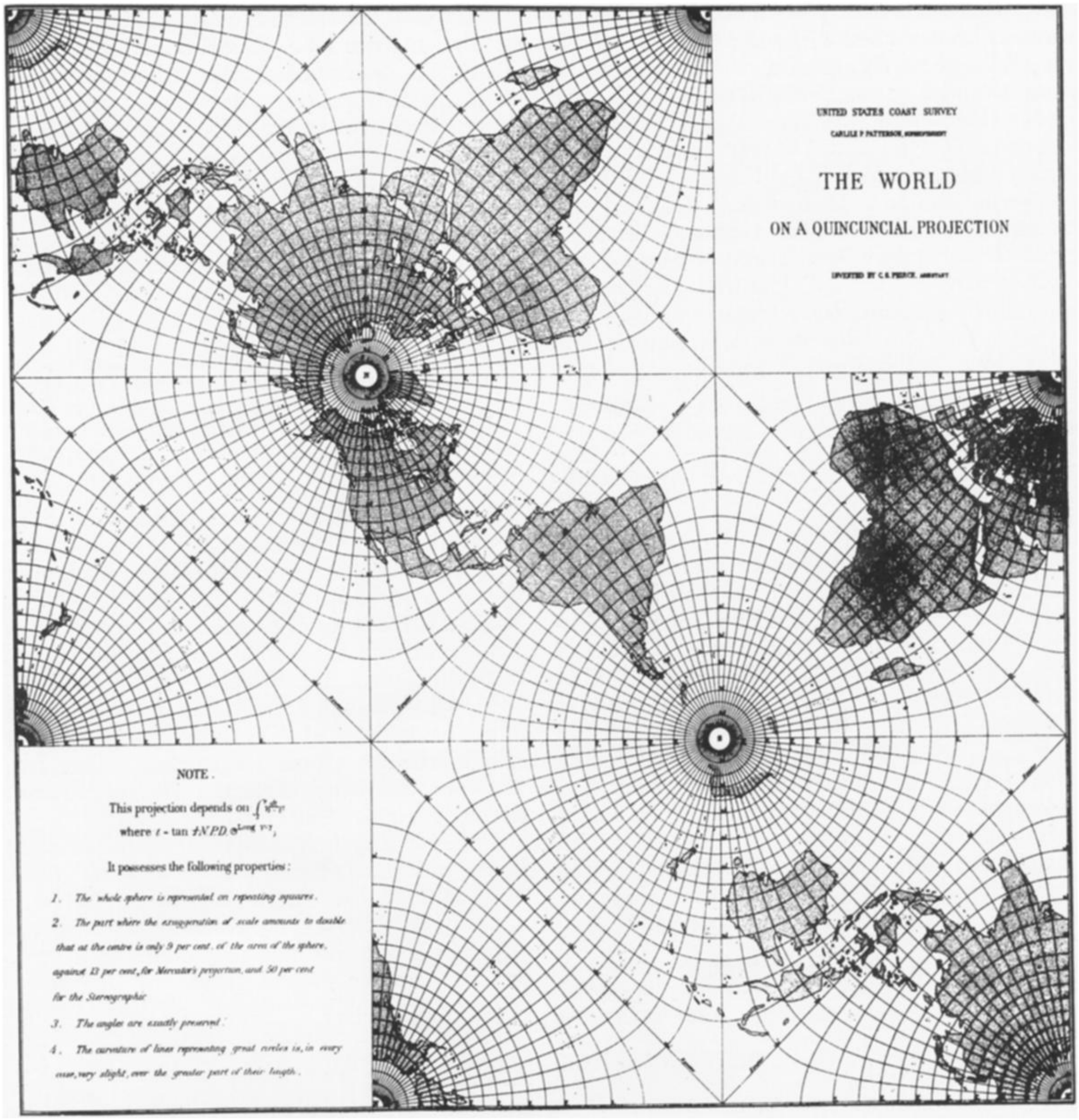

FIG. 1. Map from the *American Journal of Mathematics* **2** (1879).

Survey supply five hundred copies of the map for the publication since it would now be unnecessary to include the map in the annual Coast Survey Report.[17]

At the end of that year, on December 17, Peirce was writing again, now to Hilgard:

As I have promised months ago to give Professor Sylvester an article to be illustrated with the Quincuncial map, I would like that finished. The last I knew of it, it was about to be redrawn so as to shift the globe under the projection 5° in longitude in accordance with your suggestion.

And, as if to hurry the work, Peirce added: "Now the Superintendent also wants it for a publication of his."[18]

[17] *Ibid.* Reports of the Assistants, 1878. On April 3, 1879, Peirce increased the number requested to 750.

[18] *Ibid.* By June 20, 1879, Peirce was again writing to Patterson: "I have found a graphical method of drawing any projection of the sphere which preserves the angles as soon as the North and South poles are given. The method

In the *Report of the Superintendent* of the United States Coast and Geodetic Survey showing progress of work during the fiscal year ending June, 1879, Patterson included at last a description of Peirce's work in the following terms:

> Among several forms of projection devised by Assistant Peirce, there is one by which the whole sphere is represented upon repeating squares. This projection, as showing the connection of all parts of the surface, is convenient for meteorological, magnetological, and other purposes. The angular relation of meridians and parallels is exactly preserved; and the distortion of areas is much short of the distortion incident to any other projection for the entire sphere.

Peirce himself was to describe it later in the *Johns Hopkins University Circulars* (as he does on the published map) as possessing the following properties:

(1) The whole sphere is represented on repeating squares.

(2) The part where the exaggeration of scale amounts to double that at the centre is only 9 per cent of the area of the sphere, against 13 per cent for Mercator's projection, and 50 per cent for the Stereographic.

(3) The angles are exactly preserved.

(4) The curvature of lines representing great circles is, in every case, very slight, over the greater part of their length.

The map was published not only in the second volume of the *American Journal of Mathematics* in 1879[19] and in Appendix #15 (CS 1877) which appeared in the annual report of the Coast and Geodetic Survey for 1880, but also by Thomas Craig of the Coast Survey in 1882, in his Coast Survey publication entitled *A Treatise on Projection.* We learn from the preface of Craig's work, which is dated August 19, 1880, that Mr. C. A. Schott, Assistant in the United States Coast and Geodetic Survey, was responsible for the Appendix and that it "is a resumé in very compact form of most that is of importance in the subject of projections together with a comparison of the principal methods of projection in use at the present day."[20] Since Schott had included Peirce's invention in his summary, Craig was bound to give it an equally prominent exposition in the body of the text, in his historical review of the greatest map-makers to date. Yet Peirce's work at that time was of academic interest only and was not adopted for use then by any government or private agency.[21]

throws a fine light on the general run of mathematical functions and is chiefly useful for that. I am making a few drawings to illustrate it which I desire to have lithographed for my paper, if you will permit it. It turns out that the parallels have the same shape as the Coel surfaces about centres of attraction or repulsion inversely proportional to the distance and the meridians have the shape of the lines of force." From Reports of the Assistants, 1879, National Archives.

[19] An erratum in the above, published in *Amer. Jour. Math.* **3** (1880): v, is noted by Arthur Burks in vol. 8 of the *Collected Papers.*

[20] Craig states that the *best* book on projections was by M. D'Avezac in the "Bulletin de la Société de Géographie," Paris, 1863. Craig was a member of the Mathematics Department at the Johns Hopkins University when Peirce lectured there.

I am indebted to Professor Max Fisch for a fourth published reference to Peirce's Quincuncial map-projection. In *Science News* **1**: 225–227, a semi-annual meeting of the Metrological Society is reported as having taken place on Tuesday, May 20, 1879, at Columbia University. There one discovers that "the only complete report was that of the Committee on Standard Time—Messrs. Cleveland Abbe, E. B. Elliott, H. A. Newton, and C. S. Peirce. This was an elaborate document, fortified with numerous appendices, showing much hard work on the part of the committee in collecting historical and other data of permanent value. . . . A quincuncial projection of the world is presented in the appendix to the report. The chart was prepared by Mr. C. S. Peirce and the peculiar projection shows the meridian of 180° to great advantage."

[21] The C. & G. S. files in the National Archives reveal a bit more of Peirce's association with Craig and his work in the following sequence of letters:

(1) Peirce to Patterson, August 12, 1879: "The MS. of my paper on map projections is in Cambridge and not available for the moment. I have thought of rewriting it, but I enclose a resumé of it, from which, perhaps, Mr. Craig might construct a chapter."

(2) Peirce to Patterson, August 19, 1879: "I return herewith the interesting paper of Dr. Craig in which he has succeeded in getting out the general differential equation connecting two surfaces developable into one another. It was well worth while to obtain the equation, notwithstanding its complexity. The office is to be congratulated at having added Dr. Craig to its corps of mathematicians."

(3) Peirce to Patterson, January 10, 1880: "Tomorrow I get my accounts made up and the next day, if I have the strength, I will get my paper on the Quincuncial Projection made.

"I was always a good deal perplexed how to draw this up, for on the one hand there is very little which is absolutely new in a mathematical sense in it, nor on the other hand, do I attempt a complete rearrangement of what is already known. I was in high hopes to be able to take Craig's MS. and with mine construct from the two a chapter on the Conform Projection. But I found though his treatment showed an adequate mathematical knowledge of the subject, his method of writing is so contrary to mine that I couldn't do what I proposed without making his chapter lose its homogeneity with the rest of the book.

"Under the circumstances I think the best thing is to take my paper with the addition I have made to it and cut

However, some sixteen years after its first appearance in the *American Journal of Mathematics* the Quincuncial map study caught the attention of the American mathematician James P. Pierpont who considered Peirce's work a "very elegant representation" of the sphere on the plane. Pierpont had found a slight error in Peirce's algebraic treatment and published his own account of the matter in volume 18 of the *American Journal of Mathematics* in 1896. The final table of computed values in that article differs only trivially from that of Peirce. But the value of Pierpont's paper lies in the treatment of the algebraic detail that Peirce had failed to reproduce in his report, and it is to Pierpont that the mathematician is referred for the mathematical detail. Craig, incidentally, had become by this time Editor-in-Chief of the *American Journal of Mathematics*. He had submitted the manuscript of Pierpont's paper to Peirce before printing it in the hope that Peirce might have some comment to make on Pierpont's criticism and correction.[22] But Peirce was involved in those years in so many personal and professional difficulties that he probably had little time to follow through on any differences of opinion that he might have had with Pierpont, or even to give any thought to the ultimate fate of the Quincuncial projection.

Peirce had described his map as "an orthomorphic or conform projection formed by transforming the stereographic projection with a pole at infinity, by means of an elliptic function." Although many pages of Peirce's painstaking computations are still extant in the Peirce Collection at Harvard today, it is from the full detail of Pierpont's paper that we learn that the transformation used by Peirce had been developed originally by H. A. Schwarz in a paper in *Crelle's Journal*, vol. 70, 1869. Since Peirce was well acquainted with the mathematical publications of his contemporaries, it is to be assumed that he, too, had been familiar with Schwarz's work. Pierpont sets up Peirce's formulas[23] and in his analysis speaks of their having the very remarkable property of representing in one-to-one correspondence the interior of a square by the interior of a circle of unit radius about the

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it down so as to give it a very special character, so as to *unify* it, and make it say the particular thing I have particularly to say with clearness and force and say nothing else.

"I worked a good deal over this, and have it nearly ready but writing these very short things is often more labor than would be supposed."

(4) A letter dated May 31, 1883, from Peirce in Paris to Hilgard, then the Superintendent, contains a number of interesting items, among which is to be found the statement:

"I have also written for science a careful review of Dr. Craig's work on projection—a job upon which I have spent a great deal of time."

[22] A note from Craig to Peirce in the Peirce Collection at Harvard and dated May 1, 1896, implies that Peirce had sent some comment on Pierpont's paper to Craig. The note follows:

"I wrote you first about Pierpont's paper a year ago. You replied briefly to that note. After waiting several months I wrote you again and still later a third time but received no answer to either letter. It was not just for me to keep Pierpont waiting so long especially as I was in absolute uncertainty as to whether you were going to do anything in the matter.

"I shall forward your note to Pierpont. Any criticism that you wish to make on his paper I shall gladly publish." Pierpont's paper was entitled "Note on C. S. Peirce's Paper on 'A Quincuncial Projection of the Sphere.' "

There is a note in the Peirce Collection at Houghton Library from Pierpont to Peirce, written late in 1894 (December 10), in which Pierpont wanted to know whether Peirce had "ever published any other papers on the Quincuncial Projection of the Sphere than the one in Vol. II of the *American Journal of Mathematics*. In case you have," he continued, "I should be greatly obliged to you if you would refer me to them."

[23] The function is:

$$\zeta = cn\left(z, \frac{1}{\sqrt{2}}\right)$$

"Let θ, l, p be the longitude, latitude and north polar distance, resp. of a point P on the sphere. If $\zeta = \xi + i\eta$ be the stereographic projection of this point on the equatorial plane ("ζ-plane") we have $\zeta = \rho e^{i\theta} = \tan\frac{p}{2}\cdot e^{i\theta}$. Let now $\zeta = cn\left(z, \frac{1}{\sqrt{2}}\right)$; the ζ plane and thus the sphere itself is conformally represented on the "z-plane." Being given ζ, it is not difficult to find formulae for determining the coordinates of z and thus follow the movements of P in the z-plane.

"The formulae given by Prof. Peirce for this purpose are:

1) $x_k = \frac{1}{2}F(\phi)$ where x_k is *one* of the coordinates of

$z = x + iy$.

2) $$\cos^2\phi = \frac{\sqrt{1 - \cos^2 l \cos^2\theta} - \sin l}{1 + \sqrt{1 - \cos^2 l \cos^2\theta}}$$

and as usual:

$$F(\phi) = \int_0^\phi \frac{d\phi}{\sqrt{1 - \frac{1}{2}\sin^2\phi}}."$$

Pierpont included in his paper two interesting diagrams showing the mapping of elements in the ζ-plane on the z-plane. See illustrations.

The Journal of which Crelle was editor was the *Journal für die reine und angewandte Mathematik* and the paper by Schwarz was entitled, "Über einige Abbildungsaufgaben."

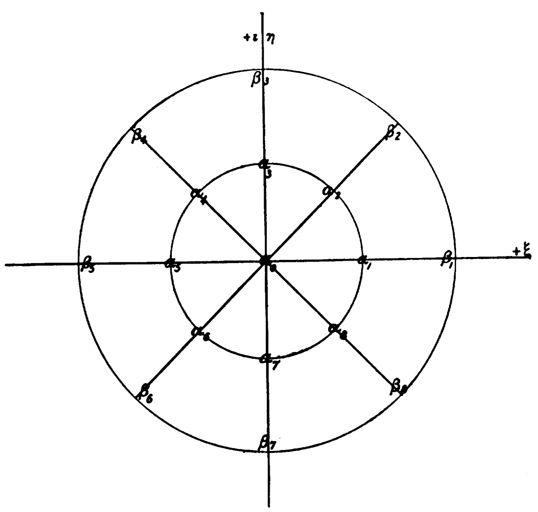

FIG. 1.—ζ-plane.

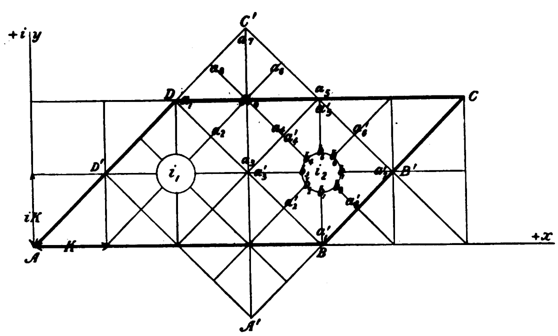

FIG. 2.—z-plane.

FIG. 2. Diagram from Pierpont's paper in *American Journal of Mathematics* **18** (1896).

origin on the plane of the stereographic projection. He observed that only at the corners does this representation cease to be conformal.[24]

Peirce's interest in map construction extended to mappings in topology and to the famous four-color problem. When Dr. Story presented a communication from A. B. Kempe of London on the "Geographical Problem of the Four Colors" at a meeting of the Scientific Association at the Johns Hopkins University in 1879,[25] it was reported that "Remarks were made upon this paper by Mr. C. S. Peirce." A month later Peirce is again reported to have "discussed a new point in respect to the Geographical Problem of the Four Colors, showing by logical argumentation that a better demonstration of the problem than the one offered by Mr. Kempe is possible."[26] Much later, Peirce read a long paper on the subject before the National Academy of Sciences and a manuscript is extant in the Peirce Collection at Harvard today[27] which the writer has reason to believe gives the substance of that paper.

Peirce later invented another important geographical chart which he called the "Skew Mercator." In a letter to his brother James Mills he described it[28] as a "Mercator with the poles of an oblique great circle at infinity. Its value is great." He wrote of starting "with a stereographic centered at the pole of that circle," and asserted that he had not, at that time, quite solved the problem of taking account of the compression *exactly*. But he noted that the solution would involve elliptic functions and that "the whole earth would be represented ∞^2 times but probably not in rectangular checkers." Nor was it quite clear to him "that the scale of the map would be uniform over the whole of the geodesic taken as central and reduced to a straight line."

In another place[29] he wrote of the accuracy of

Schwarz had presented some of this material in 1864 in a mathematics seminar at the University of Berlin.

[24] Another commentary on Peirce's work by Von I. Frischhauf appeared in Vol. 19 of the *Amer. Jour. Math.* in 1897. It was entitled "Bemerkungen zu C. S. Peirce Quincuncial Projection."

[25] I am indebted to Professor Max Fisch of the University of Illinois for this reference and the next following.

[26] From the Johns Hopkins University Circulars, Vol. I, p. 16, Circular #2, January 1880, Reports of meetings on 5 November and 3 December, 1879.

[27] Box IIIB, Cartography Folder, Peirce Collection, Houghton Library. The paper was presented to the Academy on November 15, 1899, and is entitled in the manuscript: "On the Problem of Coloring a Map." In the published material one may find brief references to Peirce's interest in this subject as a problem in topology.

[28] Dated April 5, 1894. Peirce Collection, Houghton Library. Manuscript fragments in Box IIIB, Cartography Folder in the Peirce Collection, dated March 27, 1894, and entitled "New Theory of the Skew Mercator" suggest that Peirce intended to write a long paper on the subject in which Chapter I outlined the mathematical theory. He pauses at one point to observe that a certain result "agrees with Craig's Projection, p. 65."

[29] *Ibid.* Fragments. Box IIIB The fragment opens as follows:

"C. S. Peirce's 'Skew Mercator' is a projection *of the sphere* (i.e., not corrected for the compression of the earth, since in measuring distances on a map of any considerable fraction of the earth's surface it will always be necessary to use numerical computation, either to assure oneself that the number read from the scale used sufficiently represents what one ought to mean for the special purpose in view, by the "distance" between two points on the earth's surface, or else to deduce this "distance" from the measure on the map) upon the plane of the great circle passing through latitude 60°, this projection being of the same kind as that the chart to within 30° of the equator as "being astonishing, as compared with any visual aspect of it from without." Moreover, he continued,

it happens that all parts and possessions of the United States of America, from Puerto Rico to Alaska and straight on to the Philippines lie within that distance of a Great Circle excepting Hawaii on one side and the Panama Canal on the other; and these are not very far outside of 30° from that central circle.

The fragments of the caveat he filed with the Commissioner of Patents "to protect his right" until he "matured his invention" still exist.[30] Upon its completion Peirce sent the invention to the Bureau of Navigation of the Navy Department. A letter[31] from the Hydrographic Office of that Department, dated March 31, 1894, tells of the chart as having been examined and as fulfilling the conditions Peirce had claimed for it. While its greatest advantage seemed to be "The uniform mile-scale for measuring distances," it was pointed out that there were practical disadvantages that would make the chart unpopular with the general run of navigators, and we hear no more about it.

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of the ordinary Mercator's Projection, which ought to be a conform projection *of the sphere.*

"But a map of a large fraction of the earth's surface ought not to be regarded as intended to measure upon closer than to the first significant figure of our customary numerical notation, but rather to be viewed with the unaided eye. Now if any figure is drawn upon a plane surface held square before the eyes, and this figure is then turned in its own plane through a right angle, so that what was, before the turning, its height becomes its width right and left and *vice versa*, its *apparent* shape, to a spectator not specially trained, will undergo a change ascribed to the greater effort required to turn our eyes up and down than to turn them right or left. But this change of apparent shape is not found to be a great inconvenience. Now that apparent change is just about double the exaggeration of linear scale of Mercator's chart for a small area at 23° of latitude compared with a like object on the equator. . . ."

Another set of fragmentary notes in this box is entitled "Explanations of two Map-Projections suitable for showing the Territory and Possessions of the United States of America."

[30] *Ibid.* Fragments, Box IIIB, Cartography Folder.

"To the Commissioner of Patents:

"BE IT KNOWN, that I, CHARLES SANDERS PEIRCE, a citizen of the United States, residing on my wife's farm of ARISBE, in Westfall township, Pike County, Pennsylvania, have invented a new and useful improvement in MAP PROJECTIONS, and desiring further to mature the same, file this my Caveat therefor and pray protection of my right until I shall have matured my invention.

"The following is a description of my newly invented Map Projection, which is full, clear, and exact as I am able at this time to give.

"1. The object of this my invention is to represent one surface, say a part of the surface of the terraqueous globe, upon another, such as a sheet of paper or of glass, in such a systematic manner as shall facilitate the solution of certain practical questions, say how a given ship shall steer on a given day.

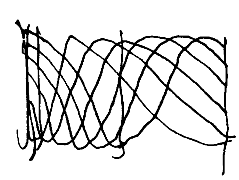
"2. The particular system of representation which constitutes my invention I term the Skew Mercator, and I would extend the same designation to any chart made according to its rules.

"3. While applicable to any two surfaces whatever, this system has been specially developed, by means of new and useful discoveries in mathematics, for the case of a part of an oblate ellipsoid of revolution having a compression of about 1/300 represented upon a plane surface.

"4. For the sake of clearness it will be well to remark that a geodetic line (which is a line drawn upon a given surface in such a manner that taking any two points upon it not too far apart, a portion of this line is the shortest that can be drawn upon that surface between those lines) does not generally return into itself at the end of any finite length of arc; and in particular on an oblate ellipsoid of revolution, excepting the geodetics which lie in plane through the axis of revolution or perpendicular to it, no geodetic returns into itself after traversing 360° of longitude; because it passes from an extreme latitude north (say) to an equal latitude south, not while it traverses 180° of longitude, but before it has quite got round so far, so that if the geodetic be indefinitely prolonged it will form a grating of its own cycles over all that part of the globe intermediate between its extreme latitudes, or more properly speaking a reticulation.

"5. In the Skew Mercator one oblique geodetic, which we may designate as A, is represented by a straight line, say a, on the plane, and is so represented that to equal distances on A correspond equal distances on a. Moreover, letting M represent any geodetic line on the ellipsoid (or other surface) which cuts A at right angles, this geodetic (and every one like it) is to be represented on the plane by a straight line, m, cutting a at right angles at the point that represents the point at which M cuts A at right angles. And the scale of the map is at every point the same in all directions."

The following diagram is found at the end of another draft of the caveat:



[31] The letter is signed by C. D. Sigsbee, Commander, U. S. Navy, Hydrographer. A disadvantage of Peirce's scheme was the use of a separate diagram for laying off courses. It was pointed out that the use of a chart in two parts is always a disadvantage at sea and, moreover, the distortion may affect the two parts differently. Then, too, "experience shows that navigators, as a class, greatly dislike curved or converging parallels or meridians on their charts and they will not use them if they can get others." Peirce collection, Houghton Library.

The Quincuncial Projection, on the other hand, has enjoyed a success far beyond Peirce's wildest dreams. Albert A. Stanley, Special Assistant to the Director, Coast and Geodetic Survey, has published an article in which he explains that

> The U. S. Coast and Geodetic Survey recently published Chart # 3092, showing major International Air Routes on a chart constructed on a Quincuncial Projection of the sphere . . . . The resulting configuration of land areas is conformal with the whole sphere being represented on repeating squares; also the major air routes, which for the most part follow approximate great circles, are in areas of least distortion, and in most cases are shown as straight lines. Angles of intersection are preserved exactly, and the maximum exaggeration of scale is less on the quincuncial projection than on either the Mercator or the stereographic projection.[32]

Stanley attributes the invention of the chart to Peirce and speaks of its revival as being due to its showing "the major air routes as approximate straight lines on a world outline preserving satisfactory shapes." Then, too, it provides "peoples residing in either the Eastern or Western Hemispheres with a world pattern in accordance with their inherent geographical conception." A citizen of the United States or of Asia "is able to observe the relationship of world land areas from his point of view as occupying a central geographical position."

And thus the needs of a complex scientific civilization are at last bringing slowly but surely to our scientific community an appreciation of Peirce's remarkable capacity for original scientific thought.[33]

[32] Albert A. Stanley, Special Assistant to the Director, C. & G. S. "Quincuncial Projection," January–March, 1946, *Surveying and Mapping*, the quarterly journal of the American Congress on Surveying and Mapping.

[33] On October 15, 1962, a new Coast and Geodetic Survey vessel was launched at Point Pleasant, West Virginia, and was christened the "USC and GSS PEIRCE." The program issued for the launching ceremonies carried the following note: "The ship is named in commemoration of Charles Sanders Peirce who contributed a wide variety of technical and scientific accomplishments to the Coast and Geodetic Survey during his 30 years service in the Bureau, extending from 1860 to 1890. He was the son of Benjamin Peirce who was Superintendent of the U. S. Coast Survey from 1867 to 1874. His research and development in pendulums, map projections and gravity made lasting contributions to the Bureau's scientific standing. Charles Sanders Peirce was the outstanding American logician of his day and is known especially as the founder of 'Pragmatism.' Important segments of the scientific community now hold that Charles Sanders Peirce was one of the greatest minds produced in America during the 19th century."

The ship was commissioned at Mobile, Alabama, on May 6, 1963.