APPENDIX No. 19.

DETERMINATIONS OF GRAVITY AT ALLEGHENY, EBENSBURGH, AND YORK, PA., IN 1879 AND 1880.

By CHARLES S. PEIRCE, Assistant.

I.—GRAVITY AT THE ALLEGHENY OBSERVATORY.

The Allegheny Observatory is situated in—

Latitude 40° 27′ 41″.6 north,

Longitude $5^h\ 20^m\ 2^s.93$ west of Greenwich.

It stands 1,140 feet (=348 meters) above the mean sea-level.* From a few yards in front of the observatory the descent is very sharp into the valley of the Ohio, and as this has been formed by erosion, it must be supposed to diminish the acceleration of gravity, perhaps by the one hundred thousandth part. Unfortunately the necessary calculation, which a topographical sketch would enable us to perform at once, remains impossible for the present.

The operations were conducted nearly as described in my "Measurements of Gravity at Initial Stations." The Repsold reversible pendulum was oscillated in vacuo on the Geneva support, in the cellar of the observatory, the feet of the support resting on iron bars laid upon other bars let into the great pier of the equatorial at one end and into a stone wall at the other.

Measures of the length of the pendulum were commenced 1879, January 2; but owing to the difficulty of maintaining a tolerably constant temperature in any part of the observatory that was otherwise suited for a comparing-room, no valuable results were obtained before January 18; and even after that date, it was found necessary to reject the work of several days, owing to bad conditions. The first series of measures of length was completed February 1. Four swingings of the pendulum were made on February 6 and 7 with heavy end up, and two swingings on February 8and 9 with heavy end down. On February 10, the position of the center of mass was determined and the knives were interchanged. Two days were then lost in trying to make the vacuum chamber stanch; after which two swingings were made with heavy end down, February 13 and 14, and four with heavy end up February 15, 16, and 17. On February 18 and 20, the flexure of the apparatus was measured, and these measures were supplemented by others on March 4. From February 22 to March 2, the pendulum was measured. The thermometers were compared from 1878, December 19 to 31, and again 1879, March 3.

The following table gives a synopsis of the results of the swingings, the period being corrected for the rate of the clock and for arc of oscillation, and being reduced to 15° C. and to a pressure of one million absolute C. G. S. units. The approximate pressure in millimeters of mercury and the approximate temperature centigrade are also shown. It is unnecessary to say that the air-pump was never brought into action during any swinging.

The agreement of the resulting periods is, as far as it goes, favorable to the plan of swinging *in vacuo*. It will be noticed that the oscillations were continued down to a small amplitude, but there seems to have been no increased error upon this account. Following the synopsis will be found a table of the errors of the partial swingings formed by intermediate transits, as shown on pages 502–503. The errors given are differences from the following periods, deduced from the final results:

T_d (knife 1)=$1^s.0064527$ T_u (knife 2)=$1^s.0066434$

T_d (knife 2)=1 .0064463 T_u (knife 1)=1 .0066370

*The latitude and longitude here given have been extracted from the American Ephemeris. The elevation is from data furnished to Professor Langley by the Allegheny City surveyor and by the engineer of the Pennsylvania Railway.

The errors are multiplied by the square roots of the number of oscillations, and the products are shown to be constant in the mean. It is also noticeable that this constant has the same value whichever end is up. Several obvious inferences might be made. In particular, it will be seen that the error of the result depends only on the total number of oscillations, no matter how they may be separated by intervals of rest.

HEAVY END UP. KNIFE No. 2.

Date.	Temperature.		Pressure.		Half arc in terms of radius.		Number of oscillations.	Corrected period.
	Maximum.	Minimum.	Beginning.	End.	Beginning.	End.		
1879.	°	°	*mm.*	*mm.*				*s.*
February 6.....	0. 3	0. 3	23	25	. 023	. 003	20, 891	1. 0066466
6.....	0. 8	0. 4	29	36	. 030	. 003	21, 406	1. 0066428
7.....	0. 5	0. 3	43	46	. 030	. 002	21, 420	1. 0066399
7.....	0. 7	0. 4	20	20	. 034	. 005	19, 742	1. 0066430
							83, 459	1. 0066431

HEAVY END DOWN. KNIFE No. 1.

February 8.....	0. 7	0. 1	13	14	. 033	. 002	74, 805	1. 0064533
9.....	0. 3	—0. 1	14	15	. 035	. 002	75, 680	1. 0064515
							150, 485	1. 0064524

HEAVY END DOWN. KNIFE No. 2.

February 13....	1. 5	—0. 5	17	40	. 033	. 002	61, 844	1. 0064471
14....	—0. 3	—1. 3	18	40	. 035	. 002	67, 626	1. 0064470
							129, 470	1. 0064470

HEAVY END UP. KNIFE No. 1.

February 15....	—0. 6	—1. 1	17	29	. 034	. 004	19, 822	1. 0066370
16....	—1. 0	—1. 2	17	35	. 034	. 004	20, 766	1. 0066337
16....	—0. 9	—1. 1	15	35	. 034	. 003	22, 588	1. 0066380
17....	—0. 7	—0. 9	21	37	. 036	. 003	20, 848	1. 0066411
							84, 024	1. 0066375

Errors of partial and total swingings.

Heavy end up.						Heavy end down.					
Knife No. 2.			Knife No. 1.			Knife No. 2.			Knife No. 1.		
Partial swingings.			Partial swingings.			Partial swingings.			Partial swingings.		
Error in 7^{h} place.	Sq. root. No. oscill.	Product in 5^{h} place.	Error in 7^{h} place.	Sq. root. No. oscill.	Product in 5^{h} place.	Error in 7^{h} place.	Sq. root. No. oscill.	Product in 5^{h} place.	Error in 7^{h} place.	Sq. root. No. oscill.	Product in 5^{h} place.
+43	57	25	+ 6	70	4	− 4	77	3	+80	78	63
+ 2	77	2	− 1	87	1	+ 48	83	40	+19	178	34
+24	83	20	− 3	86	3	+ 29	87	25	−27	94	25
+74	69	51				−180	31	56	− 9	88	8
			−66	85	56	+ 43	73	31	−35	87	30
+ 3	94	3	−13	79	10	+ 10	198	20			
−22	79	17	−18	85	15	− 61	93	57	−39	82	32
− 3	80	2							+38	88	33
			−15	82	12	+ 1	78	1	+53	83	44
−66	63	42	+28	92	26	+ 35	84	29	− 6	192	11
+30	59	18	+13	83	11	− 12	88	11	−23	95	22
−76	82	62				− 9	81	7			
−16	85	15	−28	81	23	− 11	178	20	Mean of products.		29
			+49	85	41	− 43	99	43			
+36	87	31	+99	83	82	− 19	82	16			
− 7	87	6	Mean of products		26	Mean of products.		26			
−58	73	42									
Mean of products.		24									
Whole swingings.			Whole swingings.			Whole swingings.			Whole swingings.		
+32	145	46	00	141	00	+ 6	273	16	+ 8	249	20
− 6	146	8	−33	144	48	− 12	275	33	+ 7	260	18
−35	146	51	+10	150	15	Mean of products.		24	Mean of products.		19
− 4	140	6	+41	144	59						
Mean of products.		28	Mean of products.		30						

Time was observed by Mr. F. W. Very, Professor Langley's assistant, with the instruments of the observatory, a fine 8-inch transit and the sidereal clock (Frodsham 1358). The chronometer, Negus 1589, was used for the pendulum observations; and this chronometer as well as two others (Hutton 202 and Bond 380) were compared upon the chronograph with the clock three times a day, between 3 and 4 o'clock in the afternoon and between 9 and 10 morning and evening.

The corrections to the chronometer used were obtained by assuming that between certain dates certain time-pieces moved with absolute uniformity, the changes of rate being supposed to be sudden. This is the same method of reduction used in my previous work, and appears to me most consonant with observed facts in regard to the running of timepieces. The standards used were as follows:

Date.	Sidereal time.	Timepiece assumed uniform from each time to next.
	h. m.	
February 4	6 18	Frodsham, 1358.
6	5 25	Do.
9	6 47	Do.
13	7 14	Hutton, 202.
15	8 02	Frodsham, 1358.
21	7 12	

The results of the comparisons of the length of the pendulum with the pendulum meter were as follows:

Measures of Length.

FIRST SERIES.

Date.	Pend. —standard.
1879.	μ
January 18	+26. 1
January 21	+24. 6
January 22	+26. 4
January 23	+20. 3
Mean	+24. 3

SECOND SERIES.

	μ
January 25	+22.8
January 29	+25.5
January 31	+23.2
February 1	+18.6
Mean	+22.5

THIRD SERIES.

February 22	+11.3
February 23	+10.2
February 24	+ 9.9
February 25	+ 9.1
February 26	+12.1
March 1	+15.0
March 2	+11.6
Mean	+11.3

These results have to be diminished by $200^{\mu}.4$, because they are referred to the mean of the three lines $999^{mm}.7$, $999^{mm}.8$, $999^{mm}.9$ of the standard instead of to the meter. They have then to be increased by $261^{\mu}.1$ in order to be referred to the meter adopted in my "Measurements of Gravity at Initial Stations." It follows that the length of the pendulum in terms of the meter adopted in my previous work (which is now known to be erroneous, but which is for the present adhered to, in order to avoid confusion) was

	m.
Before the interchange of knives	1.0000853
After the interchange of knives	1.0000732

The difference of the distances of the center of mass from the two knife-edges was found to be $0^{m}.39303$, to which the correction, +.00014, has to be applied.*

The experiments to determine the flexure of the support have already been published in the Coast Survey Report for 1881, pp. 375–377. The mean of the measurements of two observers shows that the flexure at the middle of the knife-edge, under a horizontal force equal to the weight of the pendulum, was $38^{\mu}.8$.

We now proceed to calculate [T^2 Rev.] and [T^2 Inv.], as in the paper above referred to. Only, it is to be remarked that, in consequence of what is said on page 72 of that paper (page 271 of the Coast Survey Report for 1876), one-seventh of the viscosity effect has to be subtracted in order to eliminate the effect of the bells; that is to say, T_d has to be diminished by 66×10^{-7} and T_u by 151×10^{-7}. The values have to be separately calculated for the experiments made before and after the interchange of the knives.

Before the interchange of knives.

	s.		s.
T_d	1.0064524	T_u	1.0066431
Bells and cylinder	—145	Bells and cylinder	—321
	1.0064379		1.0066110
T_d^2	1.0129172	T_u^2	1.0132657
Flexure	—270	Flexure	—118
Stretching			+ 10
Corrected T_d^2	1.0128902	Corrected T_u^2	1.0132549

* See *Measurements at Initial Stations*, p. 114 (Coast Survey Report for 1876, p. 313), where the correction is, however, applied with the wrong sign.

After the interchange of knives.

T_d	1^s.0064470	T_u	1.0066375
Bells and cylinder	—145	Bells and cylinder	—321
	1.0064325		1.0066054
T_d^2	1.0129064	T_u^2	1.0032545
Flexure	—270	Flexure	—118
Stretching			+ 10
Corrected T_d^2	1.0128794	Corrected T_u^2	1.0132437

	Before interchange.	After interchange.
	s.	*s.*
Corrected T_d^2	1.0128902	1.0128794
Corrected T_u^2	1.0132549	1.0132437
$\frac{1}{2}(T_d^2+T_u^2)$	1.0130725	1.0130615
$\frac{1}{2}(T_d^2-T_u^2)$	—1824	—1822
$\frac{h_d-h_u}{h_d+h_u}\frac{1}{2}(T_d^2-T_u^2)$	— 717	— 716
$\frac{h_d+h_u}{h_d-h_u}\frac{1}{2}(T_d^2-T_u^2)$	—4638	—4633
[T^2 Inv.]	1.0130009	1.0129899
[T^2 Rev.]	1.0126087	1.0125982
[T^2 Inv.]—[T^2 Rev.]	3922	3917

The two values of [T^2 Rev.] combined with the two values of the length, give for the seconds' pendulum at Allegheny:

	m.
Before the interchange of knives	0.9930479
After the interchange of knives	0.9930461
Mean	0.9930470

This is the final result from this station alone. But the correction for the erroneous length of the meter, as provisionally stated in the Coast Survey Report for 1881, page 463, is -162×10^{-7}, giving

m.
0.9930308;

and this may further be modified by the effect of measurements at other stations, and comparisons of [T^2 Inv.]. There is, however, reason to believe that such modification would be, in this case, insignificant.

Applying the correction for elevation, without continental attraction, diminished by one-tenth part, and the correction for latitude, as in my paper (C. S. Report, 1881, p. 445), we have

	m.
Seconds' pendulum at Allegheny	0.9930308
Elevation	+979
Latitude	—21903
Reduced to equator and sea-level	0.9909384

This would be increased if the effect of the valley were taken into account. A topographical sketch of this vicinity is the most pressing need of the work at this time.

The details of the work at the Allegheny Observatory are given in the tables appended to the edition of this Appendix, which has been published separately.

II.—DETERMINATION OF GRAVITY AT EBENSBURGH.

Ebensburgh is the chief (though not the principal) town of Cambria County, Pennsylvania, in the Allegheny Mountains. The observations were made in the house and grounds of Mrs. Frances S. McDonald, on Centre street. The place is shown on the county map by Beers (1867), where the house has marked under it "J. M. McDonald." It is at the southeast corner of the street next south from Highland street. The transit pier is 23½ meters south of the northern boundary and 28¼ meters east of the western boundary of the lot. The pendulum was observed in the cellar of the house.

The latitude of the station, +40° 27′, was determined by Mr. Marcus Baker by sextant observations upon the Sun, Jupiter, and Polaris. The longitude was determined by telegraphic exchanges with the Allegheny Observatory, the observers being Mr. F. W. Very and Mr. H. Farquhar with the result:

	h.	*m.*	*s.*
Ebensburgh east of Allegheny,	0	5	9.2
Ebensburgh west of Greenwich,	5	14	53.7

The elevation of the station has been ascertained from that of the railway at the station, as communicated by the engineer of the Pennsylvania Railway. The pendulum station was connected with the railway by a line of levels. The elevation so found is 2,137 feet (=651 meters).

It was intended to conduct the operations as at Allegheny; but various difficulties compelled me to support the pendulum on the Repsold tripod, as at my European stations. The brass footrests were placed directly upon the hard clay floor of the cellar. The old knives which had been used in Europe and in the stations at Hoboken and at Allegheny were replaced by new ones, made by Messrs. Darling, Brown, and Sharpe, of Providence. The amplitude of oscillation was measured on a fine arc by Messrs. Stackpole & Brothers, which is divided into thousandths of the radius. The arc and transits were observed with a reading telescope carrying an objective corrected for use at a short distance by Byrne, of New York. The same eye-piece was constantly used. The telescope was placed at a distance of two meters from the pendulum; and no screen was interposed between them.

The general order of the pendulum experiments was as follows:

1879.

August 14–21.—Measurements of length.
September 5.—Swinging, heavy end down; knife, 3–4.
Swinging, heavy end up; knife, 7–8.
September 6.—Swinging, heavy end up; knife, 7–8.
Swinging, heavy end down; knife, 3–4.
Center of mass determined.
Interchange of knives.
Center of mass determined.
September 7.—Swinging, heavy end down; knife, 7–8.
Swinging, heavy end up; knife, 3–4.
September 8.—Swinging, heavy end up; knife, 3–4.
Swinging, heavy end down; knife, 7–8.
September 10–13.—Measurements of length.
September 14.—Swinging, heavy end down; knife, 7–8.
Swinging, heavy end up; knife, 3–4.
September 15.—Swinging, heavy end up; knife, 3–4.
Swinging, heavy end down; knife, 7–8.
September 16.—Determination of center of mass.
Interchange of knives.
Determination of center of mass.
Swinging, heavy end down; knife, 3–7.
Swinging, heavy end up; knife, 7–8.
September 17.—Swinging, heavy end up; knife, 7–8.
Swinging, heavy end down; knife, 3–4.
September 18–25.—Measurements of length.

A synopsis of the periods of oscillation at Ebensburgh is given below. These periods have received not only the reductions for arc, rate, temperature, and pressure, but also peculiar *à priori* corrections for flexure of the support, difference of knives, and injury to the pendulum. These I proceed to explain:

After half the swingings had been made, the pendulum was measured. In adjusting the microscopes a plumb-line was used; and to attach this it was necessary to remove the two forward nuts which bind the head of the support to the legs of the tripod. These were afterward replaced for the rest of the swingings, but instead of being tightened by a wrench they were only tightened by hand. This negligence was only discovered after all the swingings were completed, and it was then too late to repeat them. Elaborate experiments (see Coast Survey Report for 1881, Appendix 14) were accordingly instituted to determine the flexure of the support when the nuts in question were hand-tightened and when they were wrenched. The values given on page 388 of the Report have been used in the reductions, and the periods have accordingly received the following corrections:

	Heavy end down.	Heavy end up.
First four days	−.0000832	−.0000362
Last four days	−.0000895	−.0000390

The knives used at Ebensburgh and York, which are marked 3-4 and 7-8, have, at my request, been micrometrically examined by Assistant Edwin Smith, to determine the distance of the edges from the plane of the bearings. He obtained the following results:

Knife 3—4. At end marked 3, 122 . At end marked 4, 125$^{\mu}$.

Knife 7—8. At end marked 7, 168 . At end marked 8, 170 .

On September 11 the record notes that a small spring belonging to the attachment of the knife at the *light* end of the pendulum was found to be broken. In consequence of this the pendulum must have lost mass, and the center of mass should have been removed toward the heavy end. In examining the measures of the position of the center of mass, we find that at York, the station occupied after Ebensburgh, the center of mass was distant $0^m.30333$ from the knife-edge at the heavy end. In fact, using an empirical correction for the relative position of the knives, the individual results (16 in number) show a probable error of ±.000013. At Ebensburgh, measures were made on September 6 and September 16. The four individual measures on September 16, with the correction for position of knives, give for h_u

m.
0.30330
0.30332
0.30330
0.30339

Rejecting the last observation, in which there seems to have been an erroneous reading, the others give $0^m.30331$, not differing sensibly from the value at York. The measures of the 6th give

m.
0.30324
0.30330
0.30327
0.30328

These show a value sensibly smaller than that of the 16th. The difference is such as would be produced by the loss of something less than a gramme at the *heavy* end. The distance between the knife-edges not having changed, no other changes can affect the result from the pendulum—considered as reversible—although the accident, whatever it was, must spoil the agreement of the different days. Although it does not affect the final result, I have, in the calculation, supposed that a gramme was lost at the *heavy* end, 2 centimeters beyond the knife-edge. The result of placing a small mass, m, on the pendulum at a distance of x meters and $l+x$ meters from the two knife-edges is easily found to be to increase the periods of oscillation by

$$\triangle T_d = T_d \frac{m}{M} \frac{x(l+x)}{2h_d l}$$

$$\triangle T_u = T_u \frac{m}{M} \frac{x(l+x)}{2h_u l}$$

Where M is the mass of the reversible pendulum, l the distance between the edges, h_d and h_u the distances of the center of mass from the two edges, and T_d and T_u the periods. In the present case we have $m=-1$, $M=6308$, $x=+.02$, $l=1$, $h_d=0.7$, $h_u=0.3$, $T_d=T_u=1$. We have, therefore,

$$\triangle T_d = -.0000023$$
$$\triangle T_u = -.0000054$$

and *these corrections have been applied to the first four days*, so as to reduce the pendulum to its state at the end of the work at this station.

Synopsis of periods of oscillation.

1879.	HEAVY END DOWN.	HEAVY END UP.
	Knife, 7–8.	Knife, 3–4.
	s.	*s.*
September 5	1.0064424	1.0065264
September 6	1.0064377	1.0065054
	Knife, 3–4.	Knife, 7–8.
September 7	1.0064482	1.0065122
September 8	1.0064400	1.0064296
September 14	1.0064377	1.0065024
September 15	1.0064389	1.0064789
	Knife, 7–8.	Knife, 3–4.
September 16	1.0064401	1.0065157
September 17	1.0064385	1.0064895

The period for September 8, with heavy end up, is obviously affected by an abnormal error. The Paris, Berlin, Kew, Hoboken observations show that the probable error of a period from a single swinging with heavy end up is $\pm 0^s.000006$. The period for September 8 differs from the mean of the others by $0^s.000077$, having thus an error about thirteen times the probable error, an event which would occur by chance only once in a million $\times$ million $\times$ million times. We may, therefore, safely say that on that day there was some extraordinary force tending to restore the pendulum to the vertical. The records of observations of arc show the following times of decrement on different days:

	From .0400 to .0180.	From .0180 to .0080.
	m.	*m.*
September 5	20.9	28.6
September 6	20.7	28.8
September 7	21.1	28.4
September 8	17.1	21.3
September 14	21.3	28.6
September 15	17.2	26.8
September 16	21.1	28.8
September 17	19.7	27.0
Mean 5, 6, 7, 14, 16	21.0	28.3

It thus appears that on the 8th there was some extraordinary force tending to bring the pendulum to rest. These facts suggest that a spider's line might on that day have connected the pendulum with the stand, and this supposition is somewhat strengthened by finding that on that day the operations commenced with oscillating the pendulum with heavy end up in the position in which it had been left the night before. On the 15th and 17th, also, the arc descended rapidly, the periods are very short, and the pendulum had been left over night with the heavy end up ready for the oscillations which were begun in this position in the morning. If there were spider lines on these mornings, we should expect the disturbing influence to decrease as the arc descended. Whether this is so in regard to the effect on the decrement on the 8th it is difficult to say, but it certainly is so on the 15th and 17th. Transits were observed shortly after the arcs reached .0400,

.0180, and .0030, so that there are two intervals from which periods can be deduced. These periods, corrected as in the synopsis, are

	HEAVY END UP.	
	First interval.	Second interval.
	s.	s.
September 8	1.0064130	1.0064385
September 15	1.0064423	1.0064931
September 17	1.0064683	1.0065020

These numbers certainly confirm the hypothesis of spider-lines; and I shall consequently entirely reject the work with heavy end up on September 8 and the first intervals on September 15 and 17. With these rejections the mean periods for pairs of days in which the circumstances were the same, except the time of beginning (for on alternate days the position of the pendulum at the first swinging alternated), are as follows:

	Heavy end down.	Heavy end up.
	s.	s.
	1. 0064400	1. 0065159
	1. 0064441	1. 0065122
	1. 0064383	1. 0064978
	1. 0064393	1. 0065088
Means,	1. 0064404	1. 0065087

The time observations at Ebensburg were made with transit No. 5 carrying a reticule divided on glass by Prof. W. A. Rogers. The equatorial intervals of the five middle wires are sensibly equal to $2^s.583$. The pivot inequality was determined by Mr. Marcus Baker to be $+0^s.030$ with illumination west. Both lamps were in place during the whole of the observations, which were made by Mr. Henry Farquhar. The reductions were made by least squares, using Mr. Schott's weights of 1872. Separate azimuths were assumed for the two positions. The chronograph was a fillet-reed instrument, by Breguet. The battery consisted of two sulphate of copper gravity cells.

Chronometer Negus 1589 was always used for the star and pendulum observations, as this was undoubtedly our best chronometer. Chronometers Frodsham 2490, Hutton 202, and Bond 380, were compared with Negus twice daily. The two former break every second omitting the 0; the two latter break every even second, and also at 59^s. Frodsham and Bond were wound at 8.30 a. m.; Negus and Hutton at 8.30 p. m. at first, afterward at 9 p. m. until September 23, and after that at 6 p. m. Chronometers Negus, Frodsham, and Bond were in their external cases. All four rested firmly on sand heaped on the cellar floor about 15 cm. from an inner foundation wall and 30 cm. from one another. They were placed in this order: Negus, Hutton, Frodsham, Bond. The boxes of Hutton, Frodsham, and Bond were never opened except to wind them. The daily range of temperature in the cellar averaged less than 5 °C. The chronometers were compared with the clock of the Allegheny Observatory twice daily.

The measurements of length before the first interchange of knives were as follows:

	Pend.—standard.
	μ.
August 18	+16.4
19	+16.3
19	+16.9
20	+16.9
20	+21.5
21	+17.5
Mean	+17.6

But these measures are uncorrected for the difference of temperature between the pendulum and the standard; and in point of fact the former carried no thermometer. We may assume that the result should have a correction of $+2^{\mu}.4$ on this account, because this is the mean value of the correction in the following series. With this correction the mean result is that the pendulum was longer than the standard by $20^{\mu}.0$.

After the first interchange the results were these:

	Pend.—standard.
September 10	+19. 4
11	+18. 6
12	+18. 4
13	+19. 5
Mean	+19. 0

After the second interchange the results were as follows:

	Pend. standard.
	μ
September 23	+19.5
23	+20.3
24	+21.5
24	+21.3
25	+17.0
25	+17.7
Mean	+19.5

We conclude that the pendulum preserved the same length at all times, and was $19^{\mu}.5$ longer than the standard. The latter at 15° C. is $261^{\mu}.1$ longer than the meter assumed in the "Measurements of Gravity at Initial Stations"; so that in terms of that meter the length of the pendulum at 15° C. was

$$1^{m}.0002806.$$

The difference in the distances of the center of mass from the two knife-edges was found to be in one position

$$0^{m}.39351$$

and in the other

$$0^{m}.39352.$$

To these values must be applied a small correction, $+.14^{mm}$, which in the "Measurements of Gravity at Initial Stations" is correctly given, but is applied with the wrong sign.

The following is the calculation of the length of the seconds pendulum from the first four and last four days' oscillations at Ebensburgh:

	First days.	Last days.
	s	s
T_d	1.0064420	1.0064388
T_u	1.0065140	1.0065033
T_d^2	1.0129255	1.0129191
T_u^2	1.0130704	1.0130489
Corr. stretching	1.0130714	1.0130499
$\frac{1}{2}(T_d^2+T_u^2)$	1.0129985	1.0129845
$\frac{1}{2}(T_d^2-T_u^2)$	—730	—654
$(h_d+h_u):(h_d-h_u)$	2.54045	2.54097
[T^2 Rev.]	1.0128131	1.0128187
Same in mean time	1.0072880	1.0072936
Length pend	1.0002806	1.0002806
Sec. pend	0.9930432	0.9930379

Seconds pendulum at Ebensburgh = $0^{m}.9930406$.

This is expressed in terms of the erroneous meter having the provisional correction -162×10^{-7}. Applying as for Allegheny the corrections for elevation and latitude, we have

Seconds pendulum at Ebensburgh....	0.9930244
Elevation	+1827
Latitude	–21399
Corrected to equator and sea-level....	0.9910672

In the tables appended to the edition of this Appendix which has been published separately are given the details of the work at Ebensburgh.

III.—DETERMINATION OF GRAVITY AT YORK.

York, Pa., is situated east of the Alleghanies in a comparatively plain country. The pendulum was oscillated in the cellar of the factory of Mr. A. B. Farquhar, near the railway station, on Duke street. The transit was about a hundred yards to the east of the factory, on land belonging to Messrs. Billmeyer and Small, in Gay alley. The co-ordinates of the station are:

Latitude, 39° 58′ north.

Longitude, $5^h\ 05^m\ 54^s$ west of Greenwich.

Elevation, 122 meters (373 feet).

The work at this station was conducted by Mr. Henry Farquhar, under my supervision. The pendulum observations were partly made according to a method of eye-and-ear coincidences invented by Mr. Farquhar. For the purpose of studying the effects of flexure, the Repsold reversible pendulum was oscillated on various supports, viz: 1st, on the Repsold tripod; 2d, on a solid support formed by bolting the head of the Repsold tripod to an oaken plank 2 inches thick; 3d, on the Geneva support and tripod, with the bells off and with the bells on (this to ascertain the effect of the bells); 4th, on the Repsold tripod mounted on a wooden support; 5th, on the Repsold tripod resting on pieces of India rubber.

Experiments were also made at this station upon the effect of substituting rollers for the knives as the bearings of the pendulum. The rollers were steel cylinders of 5^{mm} diameter, backed by steel planes. They were well constructed by Messrs. Darling, Brown, and Sharpe. The utmost pains were taken (here as well as in later experiments in Baltimore) to avoid the inclusion of dust between the roller and its support. Nevertheless the decrement of the amplitude was very rapid for arcs above .035 of the radius on each side of the vertical; and the periods show enormous variations.

The experiments on the effect of the bells of the Geneva support are also of interest, though they fail to give a very accurate evaluation of this constant.

The summary of the periods of oscillation at this station (except upon the Geneva support) has already been published in the *Coast Survey Report* for 1881, pages 423–424. This summary is here repeated, with the difference that the flexure corrections are now applied, that some errors of computation are corrected,* and that the experiments relating to the effect of the bells are added.

* The following table shows these corrections:

Support.	Method of observation.	Position heavy end.	Date.	Correction to last figure.	Cause of former error.
Repsold ..	Transits ...	Up	May 2.	–9	Error in subtraction had occasioned rejection of a transit.
Do.....	Coincidence.	Down.......	Mar. 19.	–9	Error of computation.
Do.....	....do	....do	Mar. 21.	–1	Do.
Stiffest ...	Transits	....do	Apr. 4, *bis*.	–3	Mr. Farquhar thinks he recorded the wrong minute, a fault to which he was liable. Changing the minute a rejected transit is brought into concordance with the others.

In drawing up the summary, besides the corrections for arc, pressure, temperature, and rate, the following have been applied:

Cause.	Authority for amount.	Amount.	
		Heavy end down.	Heavy end up.
Knife, 7–8 (for 3–4, with reversed sign)	See Ebensburgh report*	—.000006	+.000015
Flexure Repsold support	C. S. R., 1881, p. 424	—.000084	—.000036
Flexure stiffest support	C. S. R., 1881, p. 423	—.000022	—.000009
Flexure Geneva support	C. S. R., 1881, p. 399	—.000020	—.000009
Flexure wooden support	C. S. R., 1881, p. 423	—.000123	—.000054
Flexure rubber support	do	—.000300	—.000131
Geneva cylinder	C. S. R., 1876, p. 270	—.000004	—.000008
Geneva bells	C. S. R., 1876, pp. 270, 271	—.000012	—.000028

* At the time the paper on the flexure of pendulum supports was drawn up Mr. Smith had not measured the knives. It was consequently necessary to determine this correction *a posteriori* and slightly different corrections were thus used in the synopsis given in that report, viz, —.000004 and +.000012.

Periods of Oscillation at York.

REPSOLD SUPPORT.

Method of transits.

1880.	Heavy end down.	1880.	Heavy end up.
	Knife 7–8.		Knife 3–4.
	s.		s.
April 7	1.006413	April 7	1.006467
April 30	1.006405	April 30	1.006446
	Knife 3–4.		Knife 7–8.
May 2	1.006418	May 2	1.006486
May 3	1.006418	May 3	1.006483

Method of coincidences.

	Heavy end down.		Heavy end up.
	Knife 3–4.		Knife 7–8
March 19	1.006432	March 19	1.006490
March 21	1.006407	March 21	1.006440
June 4	1.006413	June 4	1.006472
June 5	1.006407	June 4	1.006450
	Knife 7–8.		Knife 3–4.
March 22	1.006422	March 22	1.006488
March 23	1.006406	March 23	1.006494
June 6	1.006421	June 6	1.006472
June 6	1.006429	June 6	1.006466

STIFFEST SUPPORT.

Method of transits.

	Heavy end down.		Heavy end up.
	Knife 3–4.		Knife 7–8.
	s.		s.
March 31	1.006415	March 31	1.006467
April 2	1.006419	April 2	1.006472
	Knife 7–8.		Knife 3–4.
April 4	1.006410	April 4	1.006471
April 4	1.006417	April 4	1.006463

Method of coincidences.

	Heavy end down.		Heavy end up.
	Knife 7–8.		Knife 3–4.
March 26	1.006419	March 26	1.006456
March 27	1.006423	March 27	1.006463
	Knife 3–4.		Knife 7–8.
March 28	1.006417	March 28	1.006461
March 29	1.006415	March 29	1.006463

WOODEN SUPPORT.

Method of coincidences.

	Knife 7–8.		Knife 3–4.
April 24	1.006420	April 24	1.006473
April 25	1.006417	April 25	1.006469
	Knife 3–4.		Knife 7–8.
April 27	1.006415	April 27	1.006470
April 28	1.006417	April 28	1.006488

RUBBER SUPPORT.

Method of coincidences.

	Knife 7–8.		Knife 3–4.
	s.		s.
April 18	1.006404	April 18	1.006484
April 20	1.006401	April 20	1.006482

GENEVA SUPPORT; BELLS OFF.

Method of transits.

	Knife 3–4.		Knife 7–8.
May 19	1.006425	May 19	1.006499
	Knife 7–8.		Knife 3–4.
May 22	1.006420	May 22	1.006488

Method of coincidences.

	Knife 3–4.		Knife 7–8.
May 18	1.006433	May 18	1.006509
	Knife 7–8.		Knife 3–4.
May 23	1.006431	May 23	1.006463

GENEVA SUPPORT; BELLS ON.

Method of coincidences.

	Knife 7–8.		Knife 3–4.
May 26	1.006432	May 26	Rejected.
May 27	1.006439	May 27	1.006485
May 29	1.006430	May 29	1.006459
	Knife 3–4.		Knife 7–8.
May 30	1.006432	May 30	1.006507
May 31	1.006437	May 31	1.006488

The means of the observed periods for the Repsold and stiffest supports are—

Method of transits.

	Heavy end down.	Heavy end up.
	s.	s.
Repsold support	1.006413 ± 1	1.006470 ± 5
Stiffest support	1.006415 ± 1	1.006468 ± 1
Weighted mean	1.006414 ± 1	1.006468 ± 1

Method of coincidences.

	Heavy end down.	Heavy end up.
Repsold support	1.006417 ± 3	1.006471 ± 5
Stiffest support	1.006419 ± 2	1.006461 ± 1
Weighted mean	1.006418 ± 2	1.006462 ± 1
General mean	1.006416 ± 1	1.006465 ± 1

It will be seen that the method of eye and ear coincidences is greatly inferior in accuracy, the eight observations taken in this way on the Repsold support being less valuable than the four by transits; and there can be little doubt that the means would be brought nearer to the truth by rejecting all the observations by these coincidences. We shall accordingly allow observations with this method only one-fourth weight. With these weights, the above periods become—

Corrected periods 1.006415 1.006468

The observations on the Geneva support, with the bells off, give

Heavy end down.	Heavy end up.
s.	*s.*
1.006424	1.006492

The differences from the corrected periods just ascertained are—

+.000009	+.000024

These numbers are in such a proportion as to indicate some force acting equally on the pendulum in its two positions. Experiments subsequently made in Baltimore, to be described in another memoir, leave no doubt that the effect is connected with the supporting planes of the Geneva receiver.

The observations with the bells on, all made by the method of coincidences, give—

Heavy end down.	Heavy end up.
s.	*s.*
1.006435	1.006485

From these numbers it would seem that the effect of the bells may be a little larger than was calculated; but the error, if any, can hardly be sensible when the receiver is pumped out.

The time observations were made with the same transit instrument used at Hoboken and at Ebensburgh. The eye-piece not being quite steady, the variations of collimation were considerable, and the instrument could not be kept free from dust. Time was kept by the four chronometers:

Negus 1589
Frodsham 2490
Hutton 202
Bond 380

They seem to have required cleaning, and show large diurnal variations. An attempt was made in the computations to take account of these, but not successfully.

The measurement of the pendulum on March 3 showed—

Pendulum − standard = +26.$^{\mu}$9

On May 7 and 8 three sets were taken with heavy end up, on which account 1. 0 has to be added to the results. (See "Measurements of Gravity at Initial Stations.") With this correction the results are as follows:

	μ.
Pendulum − standard =	+26.9
	+23.4
	+25.8
Mean	+25.3

On June 9, the knives having been interchanged, four sets gave

	μ.
Pendulum − standard =	+27.8
	+25.5
	+31.3
	+30.0
Mean	+28.6

These figures are uncorrected for the difference of thermometers on the pendulum and standard, because such correction would make the accordance of the measures much less good. We must assume the excess of length of the pendulum in the first position to have been $+26^{\mu}.1$, and for the mean of the two positions $+27^{\mu}.3$. Since the standard is $+261^{\mu}.1$ longer at 15° C. than the assumed meter, it follows that the length of the pendulum in terms of that meter (now known to be false) was

$$1^{m}.0002884$$

I prefer to retain the erroneous meter for the present, in order to avoid further confusion.

The difference of the distances of the center of mass from the two edges was found to be

Date.	Knife, 3-4 at heavy end.	Knife, 7-8 at heavy end.	First roller at heavy end.	Second roller at heavy end.
	m.	*m.*	*m.*	*m.*
March 22	0.39343	0.39353	....	....
March 28	0.39340	0.39349	....	....
April 26	0.39353	0.39351	....	....
May 10	....	....	0.39388	0.39387
May 30	0.39344	0.39353	....	....
Means	0.39345	0.39351	....	....

In the mean of the two positions of the knives we have 0.39348, to which .00014 has to be added on account of the error of the standard. (See "Measurements of Gravity at Initial Stations.")

The following is the calculation of the length of the seconds' pendulum at York:

T_d	$=1.006415$	T_u	$=1.006468$
T_d^2	1.012871	T_u^2	1.012978
$\frac{1}{2}(T_d^2+T_u^2)$	1.012925	Corr. stretching	1.012979
$\frac{1}{2}(T_d^2-T_u^2)$	-54		
$\frac{h_d+h_u}{h_d-h_u}\frac{1}{2}(T_d^2-T_u^2)$	-137		
[T^2 Rev.]	1.012788		

Whence the length of the seconds' pendulum in York referred to the meter heretofore used is:

	$0^{m}.993073$
Provisional correction to meter	-16
Elevation	$+104$
Latitude	-2146
Reduced to sea-level and equator	0.991015

These reductions have been made, like those of Allegheny, in accordance with the principles of my memoir on the ellipticity of the earth (Coast Survey Report for 1881, Appendix No. 15).

Details of the work at York are printed in tables appended to the edition of this Appendix which has been published separately.