APPENDIX No. 17.

ON THE VALUE OF GRAVITY AT PARIS.

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The very good agreement between the figures given by Borda and Biot for the value of gravity at Paris, and the quantity found by Kater at London, reduced to Paris by means of the transportation of invariable pendulums, gives us great confidence in the exactness of the result.

The three values for the length of the seconds pendulum are as follows:

	mm.
Borda	993.827
Biot	993.845
Kater	993.867

However, it might be supposed that this agreement was merely the result of chance. It is known, in fact, that none of these numbers have received the correction for the inertia of the air drawn along by the pendulum, a correction which was first made by Bessel. Now, it is not necessarily to be supposed, before having made the computations, that this correction should be the same for all three pendulums; Borda's being composed of a platinum ball and an iron wire 4 meters long; Biot's being the same platinum ball, to which was attached a copper wire $0^m.6$ long; and that of Kater being made of brass, and irregular in form. But the effect of the atmosphere upon a sphere suspended by a thin cylinder can be exactly calculated by the formulæ which Mr. Stokes has given in his important memoir on this subject. Two elements unite in producing this effect; one arises from simple atmospheric pressure, and the other from that property of the air which the English physicists call *viscosity*, and the Germans *internal friction*. To calculate this last element we must take the value of the viscosity of the air given by modern experiments, those of Maxwell, for instance. Stokes adopted a value for the viscosity much too small. This affects especially the values expressing the effects of the *viscosity* on the suspended wires, and this is why his comparisons between observation and theory do not show the true value of the latter. The atmospheric effect on the caps attaching the platinum ball, and on the sides of the chamber in which the pendulums of Borda and Biot were swung, can be approximately calculated. Of course these corrections are confirmed as well by the observations of the pendulums at different pressures as by analysis.

Biot's observations were also affected by the oscillation of the supports. In regard to the supports used by Borda, according to his description, I believe them to have been very solid, and the correction to be applied to the value of gravity, being inversely proportional to the length of the pendulum employed, must be small in this case. Biot's supports are still at the observatory; however, they have received two modifications: 1st, the sides have been strengthened by two cross-pieces; 2d, the piece which held the pendulum has been replaced by another, which is very solid. With the kind permission of Admiral Mouchez I took off the cross-pieces and measured the flexure of the supports (still provided with the new head), subjected to the effect of a force of 2 kilos. and 5 kilos., applied in a horizontal direction.

The following are the measures—

	Displacement with 2 kilos.	Displacement with 5 kilos.
	μ	μ
	13.5	34.8
	12.9	34.8
	——	35.5
Mean,	13.2	35.6
Per kilo.,	6.6	35.2
		——
	Mean,	35.2
	Per kilo.,	7.0

In order to appreciate the effect produced, not by the large support, but by the little piece which supported the pendulum in Biot's experiments, a careful experimental study will be necessary, aided with the application of a theory entirely different from that which is applicable to elastic supports. For the present I neglect this effect.

Applying the other corrections I get the following numbers:

	Borda.	Biot.
Length given	993827	993845
	μ	
Hydrodynamic effects	31.4	31.4
Viscosity (sphere)	35.0	23.1
Viscosity wire	22.6	1.8
Effect of caps	2.1	6.2
Effect of sides	0.2	0.2
Flexure (known part)	—	5.0
Corrected length	993918.0	993913.0
New measure	993934	

If we adopt seven microns as the effect of the unknown part of the flexure of Biot's support, it will be seen that far from weakening our confidence in the exactness of the observations of those illustrious physicists, our corrections only bring into agreement their results. The number expressing the result of my experiments (993.934) differs sensibly from the others. Nevertheless, a careful study of all sources of error has convinced me that it is correct within 10 microns.

The length of the seconds pendulum at Paris, calculated from the experiments of Kater, is $0^{m}.99387$; that is, shorter than my determination by $0^{mm}.07$. If we place confidence in the experiments of General Sabine, made with Kater's pendulum at different pressures, we must add to his measure a correction not less than $0^{mm}.16$, which is two times too great for the agreement of the determinations.

But General Sabine made too few experiments to establish so improbable a result. We can, therefore, assert nothing from the experiments of Kater. In any case, I think, I have sufficiently proved by what precedes, that the number heretofore given for the value of gravity at Paris must be increased by its 1 ten-millionth part.

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NOTE.

In my report upon the Measurement of Gravity at Initial Stations, the unit of measure used is derived from the German Normal Meter No. 49. But Professor Förster has communicated new data with reference to the correction of that bar, in consequence of which it appears that the assumed meter of my publication was 16.6 microns too short. In an article in the American Journal of Science, Vol. XX, October, 1880, it is stated that the United States Office of Weights and Measures makes the same meter 19.2 microns too short. But this statement assumed the committee

meter to be correct. According to Barnard and Tresca, however, this meter is 3.4 microns too long. The meter of my paper is, therefore,

By the German comparisons, 16.6
By the American comparisons, 15.8

too short. Applying the mean of these corrections, my value of the seconds' pendulum at Paris becomes $0^{m}.9939175$, which is substantially identical with the value from Borda's corrected experiments, and is probably very close to the correct conclusion from Biot's work.

Correction.—On page 432 above, the heading "Dynamical Flexure" is not in the original. This heading correctly describes the experiments, but this phrase was first used later by Professor Plantamour.—[C. S. P.]