Appendix No. 17.

ON THE EFFECT OF UNEQUAL TEMPERATURE UPON A REVERSIBLE PENDULUM.

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The upper part of a pendulum swinging in a room not very lofty is always warmer than the lower part. Let us suppose that the temperature varies uniformly along the length of the pendulum.

A reversible pendulum consists of two parts, (first) the part symmetrical about the middle of the pendulum, and (second) the load, however distributed. We may distinguish letters referring to these two parts by the subjacent indices 1 and 2. Letters without an index may refer to the whole pendulum.

Considering, first, the symmetrical part, it is plain that the inequality of temperature cannot sensibly alter the radius of gyration of this about its center of mass, which depends on the mean temperature only, since the center of mass of this part is at the center of the pendulum. The distance of the center of mass from the supporting knife-edge will, however, be increased, say, by σ. Then, if M stand for mass, l for the length of the corresponding simple pendulum, and h for the distance of the centre of mass from the point of support, the moment of inertia will be (neglecting σ^2)

$$Mlh + M_1 l\sigma,$$

that is, it will be multiplied by

$$1 + \frac{M_1}{M}\frac{\sigma}{h}$$

. But the moment of gravity will be

$$Mh + M_1\sigma,$$

so that it will also be multiplied by

$$1 + \frac{M_1}{M}\frac{\sigma}{h}$$

and the period of oscillation, which depends on the ratio of the moment of inertia to the moment of gravity, will not be affected.

Let us now consider the load, which we may suppose to be symmetrical about its middle. Denote the distance of its center from that of the pendulum by η. Let $\delta\tau$ be the excess of the temperature of the upper knife above that of the lower one. Then $\frac{\eta}{l}\delta\tau$ will be the difference of temperature of the center of the load from the mean temperature of the pendulum. It will have the *minus* sign with heavy end down, the *plus* sign with heavy end up. Let k be the coefficient of expansion. Then, every dimension of the load will, owing to the inequality of temperature, be multiplied by

$$1 \mp \frac{\eta}{l}k\delta\tau,$$

where the double sign corresponds to the two positions of the pendulum. The radius of gyration about the center of the load (which we may denote by γ_2) will be multiplied by the same amount;

and consequently the moment of inertia and the square of the period of oscillation will be multiplied by

$$1 \mp \frac{M_2}{M} \frac{\gamma_2^2}{hl} \cdot \frac{\eta}{l} k\delta\tau$$

The middle of the distance from the supporting knife-edge to the center of the load will be at a distance

$$\mp \tfrac{1}{2}\eta + \tfrac{1}{4}l$$

above the center of the pendulum in the two positions, and this distance will be

$$\pm \eta + \frac{1}{2}l$$

The inequality of temperature will therefore in both positions raise the centre of the load by

$$\tfrac{1}{2}\frac{\eta^2 - \frac{1}{4}l^2}{l} k\delta\tau$$

The moment of inertia will therefore be multiplied by

$$1 \mp \frac{M_2}{M} \frac{\eta + \frac{1}{2}l}{l} \frac{\eta^2 - \frac{1}{4}l^2}{hl} k\delta\tau$$

the moment of gravity will be multiplied by

$$1 \mp \frac{M_2}{M} \frac{1}{2} \frac{\eta^2 - \frac{1}{4}l^2}{hl} k\delta\tau$$

and the square of the period of oscillation will be multiplied by

$$1 \mp \frac{M_2}{M} \frac{\eta}{l} \frac{\eta^2 - \frac{1}{4}l^2}{hl} k\delta\tau$$

In order to make these effects as small as possible, the load should be concentrated as much as possible about one knife-edge. For this purpose, the symmetrical part might be an unloaded tube about $\sqrt{3}$ times the distance between the knife-edges in length. This construction would also have the advantage of eliminating the error due to the flexure of the pendulum staff.

In the case of the Peirce metre pendulums, we have

$\gamma_2^2 = \frac{1}{4}(3\,cm)^2 + \frac{1}{12}(16.03\,cm)^2 = 23.66\,(cm)^2$

$M_2 = 4000$ grams $\quad \eta = 65.63\,cm \quad M = 105000$ grams

$h_d = 75\,cm \quad h_u = 25\,cm \quad l = 100\,cm \quad k = .00001863 \quad T = 1^s.003$

And the correction to the time of 5000 oscillations with heavy end up, or 15000 with heavy end down, per degree centigrade of difference of temperature between the knife-edges is

$$\pm\, 0^s.00856.$$

In the case of the Peirce yard pendulum,

$\gamma_2^2 = \frac{1}{4}(3\,cm)^2 + \frac{1}{12}(15.19\,cm)^2 = 21.48\,(cm)^2$

$M_2 = 3770$ grams $\quad \eta = 60.68\,cm \quad M = 10000$ grams

$h_d = 68.58 \quad h_u = 22.86 \quad l = 91.44 \quad k = .00001863 \quad T = 0^s.9594$

and the same correction is

$$\pm\, 0^s.00863.$$