APPENDIX No. 16.

ON THE INFLUENCE OF A NODDY ON THE PERIOD OF A PENDULUM.

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Suppose a noddy, adjusted to accord with a reversible pendulum, remain on the pendulum-support throughout the experiments to determine gravity. How much can the results be affected by this circumstance?

Let us use this notation:

l and l', the lengths of the single pendulums corresponding to the pendulum and noddy, respectively; that is, in each case the square of the radius of gyration divided by the distance between the center of mass and center of rotation;

μ and μ', the ratio of any linear displacement of the support to the angular displacement of the pendulum or noddy required to produce it;

τ and τ' the natural periods of pendulum and noddy;

T the period of either harmonic constituent of the motion.

Then, the formula, easily derived from my paper on two pendulums on one support, is:

$$T^2 = \tfrac{1}{2}\left\{\left(1+\frac{\mu}{l}\right)\tau^2 + \left(1+\frac{\mu'}{l'}\right)\tau'^2\right\} \pm \sqrt{\tfrac{1}{4}\left\{\left(1+\frac{\mu}{l}\right)\tau^2 - \left(1+\frac{\mu'}{l'}\right)\tau'^2\right\} + \frac{\mu\,\mu'}{l\,l'}\tau^2\,\tau'^2}$$

Any increase of τ' always produces an increase of T; and of the two values of T^2, one is always smaller, the other greater than

$$\left(1+\frac{\mu}{l}\right)\tau^2$$

Consequently, the greatest effect is produced when one value of T^2 is as much greater as the other is less than

$$\left(1+\frac{\mu}{l}\right)\tau^2$$

that is, when

$$\left(1+\frac{\mu'}{l'}\right)\tau'^2 = \left(1+\frac{\mu}{l}\right)\tau^2$$

In this case,

$$T^2 = \left(1+\frac{\mu}{l}\right)\tau^2 \pm \sqrt{\left(\frac{\mu\,\mu'}{l\,l'}\right)\tau^2\,\tau'^2}$$

Denote by M and M′ the masses of the pendulum and noddy, respectively, and by h and h' the distance in each between the center of mass and the center of rotation. Then

$$\mu\tau^2 : \mu'\tau'^2 = \frac{Mh}{l} : \frac{M'h'}{l'}$$

and

$$\sqrt{\frac{\mu\,\mu'}{l\,l'}\tau^2\tau'^2} = \frac{\mu}{l}\tau^2\sqrt{\frac{\mu'}{\mu}\frac{\tau'^2}{\tau^2}\frac{l}{l'}} = \frac{\mu}{l}\tau^2\frac{l}{l'}\sqrt{\frac{M'h'}{M\,h}}$$

Assuming

$$\frac{M'}{M}=\frac{1}{100}, \frac{h'}{h}=\frac{1}{36}$$

for heavy end down, $\frac{1}{12}$ for heavy end up, and $\frac{l}{l'}=20$, it would follow that the effect of the noddy might be as great as $\frac{1}{3}$ of the flexure with heavy end down, and as $\frac{1}{\sqrt{3}}$ times the flexure with heavy end up. But it could not produce a sensible effect in both positions.