APPENDIX No. 15.

NOTE ON A DEVICE FOR ABBREVIATING TIME REDUCTIONS.

By C. S. PEIRCE, Assistant.

The simple suggestion that I wish to make is that by multiplying the observed times of transit (after applying a provisional correction and subtracting the right ascension) by the cosine of the declination, the field reduction is much facilitated, and the labor of the least-square reduction is decidedly lessened. This arises from two circumstances: First, that the azimuth, level, and collimation coefficients are much quicker taken out, and for northern stars quicker used; and second, that the weights to be assigned to the observations have a small range and are readily applied, while in a rough reduction they may be omitted without disadvantage.

The following are the values of the coefficients:

	Old way.	New way.
Azimuth.	$\sin\zeta \sec\delta$	$\sin\zeta$
Level.	$\cos\zeta \sec\delta$	$\cos\zeta$
Collimation.	$\pm \sec\delta$	± 1

The old coefficients have to be taken out of a table of double-entry (which is always disagreeable), and this table extends over eight pages quarto at least. The new coefficients are taken instantly and with greater accuracy from a single-page three-place table of sines and cosines. The old table is of no use for any other purpose, but the little table of trigonometric functions is invaluable for a thousand purposes, and ought to be at every observer's elbow, whether this way of reducing time-observations is used or not. The old coefficients have often three significant figures, even when only two places of decimals are used; but the new ones never exceed unity, and a two-by-two place multiplication table, such as Waldo's, is all that is required with them.

The weights of times of transit of stars of different declinations, according to the researches of Mr. Schott, agreeing with the formula of Dr. Albrecht, are given by the formulæ

$$p = \frac{1}{1+\left(\frac{36}{63}\right)^2 \tan^2\delta} \text{ for the C. S. large instruments,}$$

$$p = \frac{1}{1+\left(\frac{63}{80}\right)^2 \tan^2\delta} \text{ for the C. S. small instruments.}$$

The weights for $D = d\cos\delta$, or the residual times multiplied by the cosines of the declinations, are the above multiplied by $\sec^2\delta$, which gives

$$P = \frac{1}{0.663 + 0.337 \cos 2\delta} \text{ for the large instruments,}$$

$$P = \frac{1}{0.81 + 0.19 \cos 2\delta} \text{ for the small instruments.}$$

Thus, for the large instruments, the weight ranges only from 1 at the equator to 3.06 at the pole, and for the small instruments from 1 to 1.6. The old way of reducing observations, as if all the times had equal weight, is entirely inadmissible even in a rough field-reduction. But after multiplying by cos δ, observations with the small instruments may be treated as of equal weight; while with large instruments, in a not very refined reduction, we may use the rule that P = 1 from the equator to 41° of declination, P = 2 from 41° to 70°, and P = 3 from 70° to the pole. It will never be worth while to use more than one place of decimals for the weights, which are given in the following table:

Declination.	P	.7 P	Declination.	P	.7 P
°			°		
0	1.00	0.70	50	1.65	1.15
5	1.01	0.71	55	1.82	1.27
10	1.03	0.72	60	2.02	1.41
15	1.05	0.74	65	2.24	1.57
20	1.09	0.76	70	2.47	1.73
25	1.14	0.80	75	2.69	1.88
30	1.20	0.84	80	2.88	2.02
35	1.28	0.90	85	3.02	2.11
40	1.38	0.97	90	3.06	2.14
45	1.51	1.06			

In a field-reduction, the unknown chronometer correction being small, the azimuth will be determined in each position by a north and a south star, and the residual D calculated for each star. This is corrected for aberration, and then the simple half difference of the mean residuals in the two positions of the instrument is the collimation, while the half sum (taking only high stars) is the chronometer error multiplied by the cosine of the latitude.

In reducing by least squares, we have the same quantities to calculate as by the old way; but these are obtained in a different manner, as shown by the following formulæ. The multiplications are generally easier by the new method.

Quantities used in least square work.

$$
\begin{aligned}
p &= \mathrm{P} \cos^2 \delta \\
p\mathrm{A} &= \mathrm{P} \cos \delta \sin \zeta \\
p\mathrm{C} &= \pm\ \mathrm{P} \cos \delta \\
pd &= \mathrm{PD} \cos \delta \\
p\mathrm{A}^2 &= \mathrm{P} \sin^2 \zeta \\
p\mathrm{AC} &= \pm\ \mathrm{P} \sin \zeta \\
p\mathrm{A}d &= \mathrm{PD} \sin \zeta \\
p\mathrm{C}^2 &= \mathrm{P} \\
p\mathrm{C}d &= \mathrm{PD} \\
\sqrt{p}\,.\,d &= \sqrt{\mathrm{P}}\,.\,\mathrm{D}\,.
\end{aligned}
$$

The following is an example: On 1885, April 5, I observed time with transit No. 5 (large size), at Key West, latitude + 24° 33½′. The correction of the chronometer had been found to be

April 3, 10^h sid. t.	+ $12^s.07$
April 4, 10^h sid. t.	+ $11^s.34$.

It was, therefore, supposed to be about $+10^s.60$ on the 5th at 10^h, with a diminution of $0^s.01$ every twenty minutes. The level was observed as follows:

			Corrected for pivots.		C
	h. m.	d.	d.	s.	s.
Clamp E.	9 18	— 19.1	— 20.7	= — 0.420	} — 0.372
	9 48	— 14.3	— 15.9	= — 0.323	
Clamp W.	9 52	— 22.4	— 20.8	= — 0.422	} — 0.402
	10 48	— 20.4	— 18.8	= — 0.382	

The stars observed, with their constants, were as follows:

Star.	Approx. time.	δ		cos δ	cos ζ	sin ζ	p.
Clamp E.							
	h. m.	°	°				
ι Argus.	9 14	— 58.8	+ 83.3	.518	.117	+ .994	2.0
θ Ursæ Majoris.	25	+ 52.2	— 27.6	.613	.886	— .463	1.7
10 Leonis Minoris.	28	+ 36.9	— 12.3	.800	.977	— .213	1.3
ο Leonis.	35	+ 10.4	+ 14.2	.984	.969	+ .245	1.0
ε Leonis.	39	+ 24.3	+ 0.3	.911	1.000	+ .005	1.1
μ Leonis.	46	+ 26.5	— 1.9	.895	.999	— .033	1.2
Clamp W.							
υ² Hydræ.	59	— 12.5	+ 37.1	.976	.798	+ .601	1.0
γ¹ Leonis.	10 13	+ 20.4	+ 4.2	.937	.998	+ .073	1.1
9 H Draconis.	25	+ 76.3	— 51.7	.237	.620	— .785	2.7
41 Leonis Minoris.	37	+ 23.8	+ 0.8	.915	1.000	+ .014	1.1
β Ursæ Majoris.	55	+ 57.0	— 32.4	.545	.844	— .536	1.9

We begin the computation with two zenith stars observed in the two positions. The following gives the data and computation:

	Clamp E. ε Leonis.	Clamp W. 41 Leon. Min.
	h. m. s.	h. m. s.
Chron. time.	9 39 10.65	10 37 01.63
Assumed correction.	+ 10.61	+ 10.58
t	9 39 21.26	10 37 12.21
α	9 39 20.92	10 37 11.47
t — α	+ 0.34	+ 0.74
(*t* — α) cos δ	+ 0.31	+ 0.68
b cos ζ	— 0.37	— 0.37
	— 0.06	+ 0.31
Approx. azimuth.	.00	— 0.01
	— 0.06	+ 0.30
Corrected for aberration.	— 0.08	+ 0.28

The mean between the final figures or $\frac{1}{2}(-0^s.08+0^s.28)=+0^s.10$ is the chronometer error multiplied by the cosine of the latitude. Multiplying by the secant, or 1.1, we get $+0^s.11$ as the first approximation to the clock error. We therefore assume that instead of $+10^s.60$ at 10^h, the error was $+10^s.49$, with a diminution of $0^s.01$ every 17 minutes.

We now proceed to find the azimuth. The data and computations are as follows:

CLAMP E.

	ι Argus.	θ Ursæ Majoris.
	h. m. s.	h. m. s.
Chron. time.	9 13 53.76	9 25 0.55
Correction.	+ 10.52	+ 10.51
t	9 14 04.28	9 25 11.06
a	9 14 2.98	9 25 11.25
$t-a$	+ 1.30	− 0.19
$(t-a)\cos\delta$	+ 0.67	− 0.12
$b\cos\zeta$	− 0.05	− 0.33
	+ 0.62	− 0.45
sin ζ	− 0.994	− 0.463

The difference between $+0^s.62$ and $-0^s.45$ or $+1^s.07$ is due to the difference between the two sines or $+1.457$. Hence, $a=\frac{1.07}{1.457}=+0^s.734$. Similarly, in the other position, we have the following data and calculations:

CLAMP W.

	υ^2 Hydræ.	9 H. Draconis.	β Ursæ Majoris.
	h. m. s.	h. m. s.	h. m. s.
Chron. time.	9 59 23.91	10 25 9.92	10 54 45.70
Corr.	+10.49	+10.47	+10.46
t	9 59 34.40	10 25 20.39	10 54 56.16
a	9 59 33.51	10 25 21.17	10 54 55.93
$t-a$	+ 0.89	− 0.78	+ 0.23
$(t-a)\cos\delta$	+ 0.87	− 0.19	+ 0.13
$b\cos\zeta$	− 0.32	− 0.25	− 0.34
	+ 0.55	− 0.44	− 0.21
Sin ζ	+ 0.601	− 0.785	− 0.536

Combining with υ^2 Hydræ, first, 9 H. Draconis, we get

$$a=\frac{+^s.99}{+1.386}=0^s.714$$

and second, β Ursæ Majoris, we have

$$a=\frac{+0^s.76}{+1.137}=0^s.668$$

As the former star has a weight of 3 and the latter of 2, we conclude

$$a=+0^s.696.$$

Applying this value of the azimuth to the high stars, we get the following results:

	Chron. time.	Corr.	Seconds of a	$t-a$	$\frac{(t-a)}{\cos\delta}$	Level.	Az.	
Clamp E.	h. m. s.		s.					
10 Leonis Minoris.	9 28 01.73	+10.51	12.17	+0.07	+0.06	−0.36	+0.16	−0.14
o Leonis.	34 52.22	+10.51	02.31	+0.42	+0.41	−0.36	−0.18	−0.13
ε Leonis.	39 10.65	+10.50	20.92	+0.23	+0.21	−0.37	0.00	−0.16
μ Leonis.	46 04.68	+10.50	14.91	+0.27	+0.24	−0.37	+0.02	−0.11
							Mean.	−0.14
							Corrected for aberration.	−0.16
Clamp W.								
γ^1 Leonis.	10 13 29.71	+10.48	39.58	+0.61	+0.57	−0.40	−0.05	+0.12
41 Leonis Minoris.	37 01.63	+10.47	11.47	+0.63	+0.58	−0.40	−0.01	+0.17
							Mean.	+0.14
							Corrected for aberration.	+0.12

It therefore appears that the collimation is + 0s.14 and the additional chronometer correction + 0s.02, making the correction at 10h, + 10s.51. We now apply these corrections to all the stars, with the following results:

Star.	$\tau - a$	$\frac{(t-a)}{\cos\delta}$	Az.	Level.	Coll. and Ab.	D	P
Clamp E.							
ι Argus.	+ 1.32	+ 0.68	− 0.73	− 0.04	+ 0.12	+ 0.03	2
θ Ursæ Majoris.	− 0.17	− 0.10	+ 0.34	− 0.33	+ 0.12	+ 0.03	2
10 Leonis Minoris.	+ 0.09	+ 0.07	+ 0.16	− 0.36	+ 0.12	− 0.01	1
o Leonis.	+ 0.44	+ 0.43	− 0.18	− 0.36	+ 0.12	+ 0.01	1
ε Leonis.	+ 0.25	+ 0.23	+ 0.00	− 0.37	+ 0.12	− 0.02	1
μ Leonis.	+ 0.29	+ 0.26	+ 0.02	− 0.37	+ 0.12	+ 0.03	1
Clamp W.							
υ^2 Hydræ.	+ 0.91	+ 0.89	− 0.42	− 0.32	− 0.16	− 0.01	1
γ^1 Leonis.	+ 0.63	+ 0.59	− 0.05	− 0.40	− 0.16	− 0.02	1
9 H Draco.	− 0.76	− 0.18	+ 0.55	− 0.25	− 0.16	− 0.04	3
41 Leonis Minoris.	+ 0.65	+ 0.59	− 0.01	− 0.40	− 0.16	+ 0.02	1
β Ursæ Majoris.	+ 0.25	+ 0.14	+ 0.37	− 0.34	− 0.16	+ 0.01	2

Taking the weighted means of D on the two sides, we see that the collimation has to be reduced by 0s.01, while the chronometer correction remains unchanged. We then have for D

Clamp E.	Clamp W.
+ 0.02	0.00
+ 0.02	− 0.01
− 0.02	− 0.03
0.00	+ 0.03
− 0.03	+ 0.02
+ 0.02	

This concludes the field-reduction, and the magnitudes of the residuals illustrate its superiority over old methods. Finally, we will calculate further corrections by least squares. The quantities for the normal equations are calculated as follows:

$P \cos^2\delta$	$P \cos\delta \sin\zeta$	$\pm P \cos\delta$	$PD \cos^2\delta$	$P \sin^2\zeta$	$\pm P \sin\zeta$	$PD \cos\delta \sin\zeta$	$\pm PD \cos\delta$
Clamp E.							
0.5	+ 1.0	+ 1.0	+ 0.01	2.0	+ 2.0	+ 0.02	+ 0.02
0.6	− 0.5	+ 1.0	+ 0.01	0.4	− 0.8	− 0.01	+ 0.02
0.8	− 0.2	+ 1.0	− 0.02	0.1	− 0.3	0.00	− 0.02
1.0	+ 0.2	+ 1.0	0.00	0.1	+ 0.3	0.00	0.00
0.9	0.0	+ 1.0	− 0.03	0.0	0.0	0.00	− 0.03
0.9	0.0	+ 1.0	+ 0.02	0.0	0.0	0.00	+ 0.02
4.7	+ 0.5	+ 6.0	− 0.01	2.6	+ 1.2	+ 0.01	+ 0.01
Clamp W.							
1.0	+ 0.6	− 1.0	0.00	0.4	− 0.6	0.00	0.00
1.0	+ 0.1	− 1.0	− 0.01	0.0	− 0.1	0.00	+ 0.01
0.2	− 0.5	− 0.6	0.00	1.6	+ 2.1	− 0.02	+ 0.02
0.9	0.0	− 1.0	+ 0.03	0.0	0.0	0.00	− 0.03
0.6	− 0.6	− 1.0	+ 0.01	0.5	+ 1.0	− 0.01	− 0.02
3.7	− 0.4	− 4.6	+ 0.03	2.5	+ 2.4	− 0.03	− 0.02

The normal equations themselves are:

$$
\begin{array}{rrrrl}
8.4\,t & +\,0.5\,a & -\,0.4\,a^1 & +\,1.4\,c & = +\,0.02^{s.} \\
0.5 & +\,2.6 & & +\,1.2 & = +\,0.01 \\
-\,0.4 & & +\,2.5 & +\,2.4 & = -\,0.03 \\
1.4 & +\,1.2 & +\,2.4 & +\,16 & = -\,0.01
\end{array}
$$

The solution is

$$t = +.001 \quad a = +.003 \quad a^1 = -.012 \quad c = +.001$$

So that the field reduction cannot be improved.

I will add that it seems to me that this mode of reduction will be much more easily understood by inexperienced computers than the old one.