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A survey of the life of Hugh MacColl (1837–1909)

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The Scottish logician Hugh MacColl is well known for his innovative contributions to modal and non-classical logics. However, until now little biographical information has been available about his academic and cultural background, his personal and professional situation, and his position in the scientific community of the Victorian era. The present article reports on a number of recent findings.

1. Introduction

Contrary to a widespread assumption the modern history of modal logic did not start with C.I. Lewis' *Survey of Symbolic Logic* (Lewis 1918). His eminent work was preceded by some 20 years by H. MacColl's fifth article on 'The Calculus of Equivalent Statements'. This article was read at the London Mathematical Society on 12 November 1896. Some months later it was published in the Society's *Proceedings* (MacColl 1896–1897). During the following years MacColl presented his logic primarily in a series of articles on 'Symbolical Reasoning'. All of them were published between 1897 and 1906 in *Mind*, the journal in which C.I. Lewis first wrote on strict implication (in 1912).

In *Symbolic Logic and its Applications* (1906) MacColl set forth his final version of a logic for propositions qualified as certain, impossible, contingent, true or false. A main fragment of his logic has turned out to be equivalent to the logic T introduced by Feys (1937, 1938) and von Wright (1951) many decades later, i.e. to the smallest normal epistemic modal logic.[1] In recent years a number of investigations into MacColl's logic and its philosophical background have been carried out, and an edition of his *Collected Papers on Logic, Mathematics and Philosophy* is being prepared.[2] Until now, scarcely anything was known about MacColl's life and person alike. The present article sums up a number of recent findings.

2. Scottish origins

Hugh MacColl was born in Strontian, Argyllshire, on 11 or 12 January 1837.[3] Other sources say that he was born in the Scottish Highlands, probably in Glenfinnan on the borders of Invernessshire and Argyllshire.[4] Most likely,

1 Cf. Read (1999).
2 For a survey cf. Astroh and Read (1999, Introduction, p. iv)
3 Cf. Ville de Boulogne (1887).
4 Cf. Poggendorff *et al.* (1863–1938, vol. 3, pt. 2, p. 850).

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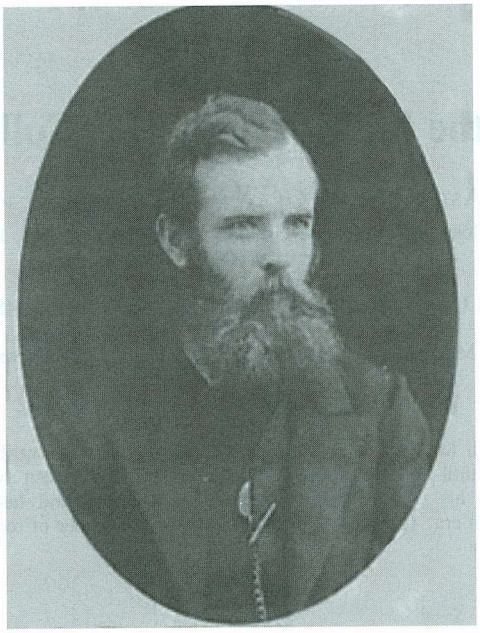

Hugh MacColl in 1886.[5]

MacColl's second marriage contract, naturally signed by himself, is the more reliable source. This document confirms that he was the son of John MacColl and Martha, née Macrae. Hugh, named after his grandfather, was their youngest child. He had three brothers and two sisters.

John MacColl was a shepherd and tenant-farmer from Glencamgarry in Kilmalie (between Glenfinnan and Fort William), and he married Martha Macrae in the parish of Kilmalie on 6 February 1823. Their first child, Alexander, was born on 6 April and baptised in the Church of Scotland on 22 April 1824. The marriage and baptism are recorded in the Parish Registers of the Church. However, John MacColl and his family belonged to the Episcopal Church of Scotland (as did many MacColls—and also Sir Walter Scott, for example), so that none of the later births is registered in the records of the established Church.

The second son, John, was born in 1826 or 1827 at Glenfinnan. Apparently, he suffered from asthma. As late as 1875 his bad health made him and his family emigrate to Australia. He died in 1892 at Clifton Hill, Victoria.

Around 1829 a first daughter, named Christine, was born. There is no further information about her or about her younger sister who, seemingly, was also older than Hugh.

The third son, Malcolm (known in the family as 'Callum'), born at Glenfinnan on 27 March 1831, was the most famous of the MacColls' children. A memoir and edition of his voluminous correspondence was published by G.W.E. Russell in 1914 (Russell 1914, esp. pp. 1–16). It contains a useful chapter on Malcolm's early life, and provides most of what little evidence we have concerning his younger brother Hugh's family background. It is also a delightful book to read; Malcolm MacColl was clearly quite a character.

5 By courtesy of Mrs Rosemarie Maconchy.

Martha Macrae, who came from Letterfearn in Kintail, was herself a Presbyterian who joined the Episcopal Church on her marriage. She spoke only Gaelic. According to Russell's memoir, John MacColl was sufficiently educated to teach his boys elementary Latin, Greek and mathematics. However, he died at the age 45 when his youngest son was only three years old.

This early loss had a far-reaching impact at least on Hugh MacColl's life. In particular, his scholarly education did not respond adequately to the needs of his talents. He was almost 40 years old and living on private teaching in Northern France when he took his BA in mathematics as an external student of the University of London. Likewise, Hugh's elder brother John was not content with his early formation as a schoolmaster. When he was nearly 50 years old he managed to receive an Oxonian BA in classics.

The importance of the father's untimely death for Hugh MacColl's later life and career is reflected in the two novels, *Mr Stranger's Sealed Packet* (1889) and *Ednor Whitlock* (1891b) he published in 1889 and 1891 respectively.[6] In both stories the sudden death of the hero's father occurs before the son's professional education is finished, and deeply affects it.

Mr Stranger's father states in his last will and testament that his son Joseph's education 'henceforward was to be on other lines. Classics were to be completely thrown aside, and I was to devote myself wholly to science, and especially to mathematics, astronomy, chemistry, electricity, and practical mechanical engineering—a sufficiently wide curriculum' (MacColl 1889, pp. 22–23). Ednor Whitlock, on the other hand, risks not receiving an academic education at all.

In both novels the loss of the father also has the consequence that the protagonist moves to another country. The sealed parcel Mr. Stranger inherits from his father contains the plan of a rocket that allows him to travel to Mars. When Ednor Whitlock and his sister lose their parents, economic reasons make them accept inferior teaching jobs. Both children seek engagements as teachers. They find work in boarding schools at Blouville, a fictitious place, somewhere on the coast of northern France.

Although information on Hugh MacColl's life is scarce, the following quotations from his second novel undoubtedly allude to the author's personal condition:

> If Ednor Whitlock had gone up to Cambridge to try for a scholarship it is not unlikely that he would have succeeded, for his mathematical abilities and knowledge were considerably above the average; but a heavy and unexpected calamity destroyed the project of a university career. His father, while ministering consolation at the sick bed of a poor old woman who had been struck down by typhus fever, caught the fell disease, and died from it after a few days' illness. His mother's death followed soon after. Worn out by anxiety and constant attendance upon her husband, she caught the infection, and in spite of the most devoted nursing, both by her son and her daughter, passed away before their eyes. (MacColl 1891b, p. 9)

The two novels confirm the few specific autobiographical details actually known about their author. In view of the various correspondences between factual and

6 MacColl's literary writings and their cultural background in Victorian Britain are discussed in Astroh (1999) and Olsen (1999).

literary information, these novels help us grasp the problems and perspectives that have shaped his personal form of life.

After the death of John MacColl his widow moved with her children to Letterfearn and then to Ballachulish, where the children started to learn English. In 1841, Hugh's elder brother Malcolm, then aged nine, was living alone with his aunt and grandmother, both called Anne Macrae, in Letterfearn, on the shore of Loch Duich.[7] Malcolm did so well at school that a wealthy lady paid for him to attend a seminary in Dalkeith near Edinburgh where schoolteachers were trained. He taught at Callander, Stonehaven and Perth. In 1854, at the age of 23, Malcolm was admitted to the relatively newly opened Theological College at Glenalmond near Perth. In 1856 he was ordained Deacon of the new Mission of the Episcopal Church at Castle Douglas in Kirkcudbrightshire. He was ordained priest in St Mary's, Glasgow by the Bishop of Glasgow in August 1857.[8]

At this time Malcolm was supporting his younger brother Hugh in his studies, probably preparatory to his becoming a schoolmaster. He had hopes to be able to support Hugh to study at Oxford. However, Malcolm was already the outspoken controversialist for which he achieved fame in later life. He became involved in the dispute over the proper interpretation of the Eucharist which divided the Episcopal Church in 1857–1858, refusing to sign an address of loyalty to the Bishop, and in fact was party to a 'Remonstrance' against the Bishop's position. Consequently, the Bishop dismissed him from his post at Castle Douglas. He was therefore incapable of providing for Hugh as he had hoped, and too proud to accept offers of help from others. Eventually, he took up the cause of Gladstone, as a pamphleteer ('the best in England' in Gladstone's words), once he had overcome the barrier to ordination in the Church of England which was enshrined in law in the 18th century (at the time of the Jacobite rebellion) and not repealed until 1864.

In the meantime, Hugh had taken up various posts as schoolmaster in England from 1858,[9] until for unknown reasons he left the country. In 1865 MacColl moved to France. For the rest of his life he settled at Boulogne–sur–Mer. Little is known about the kind of life he was leading there; but even less is known about his intellectual development or his personal circumstances during the first three decades of his life.

3. MacColl's Boulogne

Initially, MacColl's decision to earn his living in northern France might have been a provisional one. But it seems to have been well thought out. Especially in Britain, Boulogne was known for its prosperity and the refined way of life it could offer to well off or, perhaps, to educated persons. No other town on the French side of the Channel adapted technological progress to its geographical condition in so comprehensive a manner.[10] Here, it was almost natural to learn from Britain's industrial superiority. Since 1848 Boulogne had been connected to the French railway network. Five years earlier the South Eastern Railway had been extended

7 Census for 1841, West Register House, Edinburgh.
8 These details and the following paragraph are drawn from Russell's *Memoir*.
9 Poggendorff *et al* (1863–1938).
10 For a detailed report on Boulogne's economic, political and social history before 1914 cf. Oustric (1998a), Oustric (1998b) and Hilaire (1998a).

to Folkestone. When MacColl lived at Boulogne it took five and a half hours to reach London; in 1880 Paris was at a distance of three hours.

For decades Boulogne rivalled Dieppe and Calais. At the turn of the century after continuous improvements of its docks the seaport regained its former superiority. During these years its importance for France's fishing industry certainly excelled its role as a traffic port. Throughout the second half of the nineteenth century—despite a number of recessions—Boulogne's commerce and diversified industry were growing vigorously. MacColl was living in a prospering town with close economic and cultural links with Britain:

> Comme le progrès venait alors souvent d'Angleterre qui avait effectuée sa révolution industrielle avant les autres pays, il pénétrait en France par Boulogne dans les domaines les plus divers, qu'il s'agisse des techniques textiles, de l'industrie des plumes métalliques, de la vaccination de Jenner, de l'éclairage au gaz, des améliorations nautiques, des techniques de l'hôtellerie, de la doctrine artistique néogothique de Pugin, de nombreux sports: courses de chevaux, régates, golf, football, tennis ... (Hilaire 1998b)

On both sides of the Channel Boulogne was also renowned as one of the most elegant seaside resorts and spas of the *Belle Epoque*, much frequented by the elites of both nations. At the end of the July Monarchy about 3 000 of its almost 40 000 inhabitants were British subjects.[11] But the completion of the French railway network as well as the Franco–Prussian war of 1870 led to a steady decline of their number; by the turn of the century only a third of the British had kept their French domicile.

MacColl's Boulogne was a vivid French port with a truly British flavour. At this time, English shops and pubs, various Protestant churches, English surgeons and undertakers, local newspapers and regular theatre productions in English were natural facets of a balanced and liberal form of urban life. At the peak of European nationalism, living at Boulogne could have meant that there is not only 'nothing sacred about signs', as MacColl used to hold,[12] but likewise about cultural identity.

However, the British inhabitants of Boulogne kept up their genuine forms of interaction and communication. Merridew's English Library and its club-like Reading and Conversation Rooms fully equipped with Britain's major newspapers and reviews, like the *Westminster* or the *Edinburgh Review*, was a well known institution of Boulogne's cultural life. Apparently, MacColl frequented the place: a letter to Bertrand Russell on the Reading Room's writing paper has been preserved.

Merridew's (1866) *Guide to Boulogne* was published in French and English, and sold on both sides of the Channel. It provided not just general tourist information but also specific references to various colleges or schools at Boulogne offering a French education under Protestant conditions. Here, already in Europe but still close to the UK, a young person of some standing could receive a schooling sufficiently French to acquire or to preserve a higher-ranking social position in British society. Even in the 1930s Boulogne's *Collège Communal*, then called *Collège Mariette*, counted 35 per cent of British pupils among its boarders (Vasseur n.d.).

11 If not otherwise specified our information about Boulogne's British community stems from Hilaire (1998b).

12 Cf. MacColl (1906, §1, pp. 1–2).

MacColl came to Boulogne in order to earn his living as a teacher. Professional and intellectual aspiration let him become a part of the local, continental enclave of British society. Gradually, time turned his decision into a definite form of life. But still, and perhaps under the encouragement of his brother Malcolm's growing political fame, Hugh hoped to return to his cultural mainland, and to receive some kind of public recognition for his pioneering contributions to logic. Even as late as 1901, MacColl when 64 years old recommended himself to Bertrand Russell as a lecturer in logic (MacColl 1901a). Due to a University of London Parliamentary Act of 1898 some changes to the university's teaching arrangements were made. Apparently, MacColl got to know about these changes.

4. Marriage and family

While still in Britain MacColl married Mary Elisabeth Johnson of Loughborough in Leicestershire. Two years younger than Hugh, she accompanied him to Boulogne where their family soon started to grow. All five children, four girls and a boy were born there. In April 1866 MacColl reported to W.J.C. Miller, then mathematical editor of the *Educational Times*, on the birth of his first daughter, Mary Janet (MacColl 1866b). Martha Christina followed in October 1867, Flora in May 1869, Hugh Ernest in May 1871 and Annie Louise in November 1873 (Ville de Boulogne 1866–1873). Hugh Ernest was to study at Christ Church College Cambridge and pass a legal career in Burma.

Throughout these years the MacColls were living in the centre of Boulogne-sur-Mer at *4 rue du Temple*.[13] In this area where mostly craftsmen and employees were living the family occupied a decent flat. They lived in the close vicinity of the original seat of the *Collège Communal* where MacColl was teaching, at least initially. Nowadays it is the *Université du Littoral–Côte d'Opale*.

In May 1883, C.S. Peirce intended to visit MacColl at Boulogne. It is not known whether they actually met; but in a letter written shortly before the planned visit, MacColl informed Peirce about the circumstances of his personal life, and writes: 'We have been in Boulogne now eighteen years. We have five of a family—four girls besides the little boy already mentioned. We live a very quiet life.' (MacColl 1883).

Meanwhile the MacColls had moved to an apartment at *73 rue Siblequin*[14] even closer to the town's commercial centre. At least during these months and during the family's immediate future this quiet life was not a happy one. The letter to Peirce reported on a severe and long-lasting illness of MacColl's wife:

> ... You will be glad to hear that our little boy is now quite out of danger & appears to be quite well again. I wish I could say the same about my wife; still, she too is much better than she was when I wrote to you last. She went out the day before yesterday in a bath-chair—her first going out for more than eight months. I hope the coming warm weather will do much to set her up again. The beginning of her illness was a severe cold which the[15] doctor who then attended has culpably neglected as of no importance. (MacColl 1883)

13 The street is now called *rue Ernest Hamy*, situated between the former *rue Siblequin* and the former *rue des Vieillards*, presently *rue Faidherbe* and *rue Félix Adam* respectively. For general information on the streets of Boulogne cf. Charles (1992).

14 Presently this address is *119 rue Faidherbe*.

15 MacColl scratched out 'our'.

Mary Elisabeth MacColl died on 2 February 1884. We do not know the cause of death; nor is there any personal information on his reaction to her death and the loss of his children's mother. Mary Janet, the eldest daughter, was 22 years old, while her youngest sister Annie Louise was still a girl of 11 years.

MacColl's published novels both tell of the death or severe illness of a major female character. In *Mr Stranger's Sealed Packet* the hero's Martian wife dies from a lack of resistance against 'food and air' on earth. In *Ednor Whitlock* Mrs Nora Kent, the mother of the main female character, repeatedly suffers from inflammation of the lungs.[16] Within just one year her life is twice seriously threatened. When the second attack is imminent Mr. Kent, the worried husband, says to her:

> 'You are rather feverish, dear, ... It's probably nothing but the beginning of an ordinary cold. Still as small things may become serious if neglected, I will send at once for Dr. Karcher.' (MacColl 1891b, p. 268 and likewise pp. 178–9)

In view of the quoted letter to C.S. Peirce it is highly probable that MacColl is drawing here on personal experience.

Three years after the death of his first wife, on 17 August 1887, MacColl married Mlle Hortense Lina Marchal, at Boulogne–sur–Mer. An official announcement of the event was sent to Peirce (MacColl 1887). MacColl was now 50 years old, 12 years older than Mlle Marchal who came from Thann, Alsace. In several respects this union was a mutually thoughtful choice; the couple built up a harmonious family life never lacking a solid economical basis. Both were Protestants, and both had a keen interest in teaching (Anonymous 1918). The marriage certificate indicates that the parents of Hortense Marchal were living on a private income, while her brother Jules Marchal, and her brother-in-law Gustave Busch, were shopkeepers at Boulogne. The fact that a marriage contract was put up might point to some prosperity to be dealt with; however, there is no evidence that MacColl ever owned property at Boulogne (Ville de Boulogne 1887).

Most likely, the families Marchal and Busch had moved to Boulogne from Alsace, which since 1871 had been possessed by Germany. In *Ednor Whitlock* MacColl refers extensively to this political context: Ethel Whitlock, the hero's sister, has become an assistant teacher in northern France while her brother is still living with the family of their father's successor:

> Ethel's life at mademoiselle Lacour's *pensionnat* was pretty much what she had anticipated. Her main work was to teach English to the French pupils, of whom there were altogether fifteen, including the eight externes. With the English girls she had little to do, at least during school hours. She gave them a short English dictation every day, and three lessons a week in arithmetic—that was all. Her very imperfect knowledge of French, coupled with her youth and inexperience, exposed her to some trials at first, but as she and her pupils got to understand each other better these gradually diminished. Human nature is pretty much the same everywhere, and French girls, like English girls, have their good as well as their bad points. (MacColl 1891b, p. 38)

16 Mr. Stranger's father dies of the same disease.

One day, the furious reaction of a pupil to a letter from her cousin leads to an extensive argument between this girl and the school's German teacher, Frl. Hartmann:

> Among the boarders was a little, dark-eyed, fiery Alsatian, of about sixteen, named Suzanne Müller, whose parents had sold their property in Alsace for a fraction of its value, and had emigrated into the interior of France in order that their sons might not be obliged to serve in the German army. One evening, after class, this girl received an unusually bulky letter with the German stamp on it: it was from a cousin in Alsace—her father's brother's daughter. Mademoiselle Lacour, Ethel, the German governess, and all the boarders were present. (p. 39)

The argument that follows is not worth reporting. Finally, Mlle. Lacour calms down the women's excitement and restores the emotional balance on which a pedagogical institution naturally depends. Her arguments coincide well with the narrator's, and presumably MacColl's own benevolent, but, of course, reserved excuse for the girls' treatment of young and unexperienced Ethel:

> '... Let us ... hear no more upon the subject. I don't like these discussions on our different nationalities. If we read out histories, be we French, German or English we shall all find things to blush at as well as things to be proud of.' ... Thus ended the discussion which Suzanne Müller's letter had started. It had gone much further than the prudent *maîtresse de pension* had foreseen, and she made a mental resolve that nothing of the kind should happen again, at any rate in her presence. Fräulein was a necessity in her establishment. (p. 46)

Together with her sister, Mme Busch–Marchal, Hortense Marchal was running a well-known 'pensionnat de jeunes filles'. Possibly, MacColl took his second wife as a model for Mlle Lacour. At any rate, MacColl's novel depicts the social and personal context of his later life at Boulogne. He himself was teaching at the sisters' school (Anonymous 1909).

When MacColl married Hortense Marchal the *pensionnat* was apparently situated in 37 *rue Basse des Tintelleries* opposite to a park.[17] Though the name of the *quartier des Tintelleries* refers back to an ancient cloister this area belonged to the most modern and elegant parts of the Boulogne of the late 19th century; it even had its own railway station. The building was large enough to house even all members of the new family. As the domicile's quality indicates, MacColl's marriage with Hortense clearly was an economic and social improvement of his condition.[18]

5. Private teaching

During his first years at Boulogne MacColl earned his living as a schoolmaster for mathematics and English at the *Collège Communal*.[19] An advertisement from 1866 in which MacColl offers private tuition confirms this employment (Merridew 1866). But in later advertisements he never refers to himself as a 'Professor at the Communal

17 This address is now 45 *rue Basse des Tintelleries*.

18 A detailed description of the *quartier des Tintelleries* is given in Tiébaut (1998). Even a picture of the house of the boarding school is included (p. 333).

College'. In June 1870 the publishing company *Longman* advertised MacColl's *Algebraical Exercises and Problems* in its *Notes on Books* in the following terms:

> By Hugh MacColl, late Mathematical Master at the Collège Communal, Boulogne–sur–Mer. (Anonymous 1870)

In view of this reference to MacColl's professional situation the letter to Peirce makes it rather likely that from about 1870 onwards MacColl's income was essentially derived from private lessons:

> My income, a very fluctuating & precarious one, is derived entirely from my private teaching. (MacColl 1883)

In contrast with this description of his professional situation, an obituary published in *La France du Nord* on 30 December 1909 depicts MacColl as a successful teacher of the *Collège Communal*. It has not been possible to identify MacColl as one of its professors. The college's advertisements and brochures do not mention him as being responsible for a course on any subject. However, both the English and the French edition of *Merridew's Guide to Boulogne* regularly publish his private advertisements.[20] The example points to the major purpose of MacColl's instruction at Boulogne. When the hero of his second novel moves to France, he starts teaching mathematics at the boarding school of 'Mr C.H. Kent, MA, of Trinity College Cambridge'. His professional condition is that of MacColl himself:

> This Mr. Kent ... was what is commonly called a *crammer*. He prepared young men for the English competitive examinations, especially the examinations for the Army and Indian Civil Service. He had selected Blouville for his scholastic establishment in order to give his pupils greater facilities for learning the French language, for which a good many marks were allotted in the said examinations. (MacColl 1891b, p. 52)

19 Cf. Anonymous (1910). In *Merridew's Guide to Boulogne* the college's pedagogic profile is described as follows:

> This important school is supported by municipal grants, and the revenue arising from payments made by pupils; it is placed under the immediate control of a principal, a chaplain, and a large staff of professors, who are all appointed by the Minister of Public Instruction, under whom the pupils receive an educational course, comprising French, English, German, Latin, Greek, History, Drawing, Mathematics, Natural History, Natural Philosophy, Chemistry, and Rhetoric; in short, such an instruction as will enable a pupil to take a Bachelor's degree, to enter the naval or military schools, to fill public situations, or to follow any of the liberal professions. If the parents or guardians desire a commercial education only, the rules of the college admit of that particular branch being exclusively taught, ...
>
> Boarders, half-boarders, and day-scholars are received. Religion is inculcated, and those belonging to the Protestant faith receive instruction in their own religion from proper tutors, and attend divine service at the Protestant Church in Rue du Temple every Sunday. Corporal punishments are never resorted to. (Merridew 1882, pp. 56–57)

20 One of them reads as follows:

> HUGH M'COLL, BA
> (LONDON UNIVERSITY)
> Gives Lessons in MATHEMATICS, CLASSICS, ENGLISH, LOGIC, with all the other subjects of the University Course, and prepares Young Gentlemen for the Naval and military Examinations.
> 73 RUE SIBLEQUIN 73. (Merridew 1882)

This self-description was not correct. MacColl's degree was awarded by the University of London (UoL); "London University" was the name of the private company which had launched itself in the mid 1820s, and which was renamed "University College London" in 1836 when the UoL was established as a degree-examining and -awarding body.

In most of his writings MacColl presented himself as a serious academic with high moral standards and outspoken religious beliefs. Nevertheless, and foremost in this very context, he was able to reflect upon his professional condition from an ironical, if not depressing point of view:

> Ignorance of Nature's laws and errors of judgement bring their punishment no less surely than deliberate crime. This is Nature's way of teaching; and Nature, as already stated, is but another name for God—God as manifested in the working of his laws. God's teaching through the operations of his laws is slow, sure, and thorough—the exact opposite of that of the modern crammer. He allows men to commit errors, and he allows them to commit crimes (which are the most grievous of errors) in order that the discomforts and sufferings which those errors and crimes sooner or later entail, here or hereafter, may in the long-run purify their souls and accelerate their progress upwards. (MacColl 1909, pp. 156–7)

6. Academic misfortunes

Almost nothing is known about MacColl's early life in Britain. It is thus difficult to know that the specific reasons for his emigration or to understand the various conditions that obstructed the intended academic career. At the very least, economic difficulties prevented him from studying as a young man.

In *Ednor Whitlock* the name of Mrs Kent's doctor, Mr Karcher, points to the possibility that MacColl's move to France resulted from his personal contacts with well-established academics. In a letter from 1866 to W.J.C. Miller he writes:

> The words of Prof. Sylvester ... must have had reference to the little pamphlet on Ratios which I published in 1861. I left one of these with my friend Mr Karcher about a year ago, but I had no idea that he had shown it to Prof. S. (MacColl 1866a)

By then James Joseph Sylvester (1814–1897), one of the most influential mathematicians of the Victorian era, was teaching at the *Royal Military Academy* at Woolwich, London. He shared a keen interest in poetry with his colleague, Prof. Théodore Karcher. At least from 1867 onwards MacColl's friend edited a number of readers of French literature as well as courses of translation from English into French—books most valuable for a scholarly establishment like the fictitious one of Mr Kent at Blouville.[21]

The 'pamphlet on ratios' from 1861 mentioned earlier is MacColl's earliest known publication, when he was 24 years old. It has not so far been possible to locate a copy of this text.[22]

At least from 1865 onwards MacColl contributed *Questions*, *Problems* and *Solutions* to the *Educational Times*. He participated in these discussions until 1872. Then, for about five years, he abandoned this public forum of mathematical investigation, most likely in order to prepare for the BA in mathematics he took in 1876 at the University of London. He gained his degree, with a 'second division'

21 Cf. the following examples: Brette *et al* (1867), Cassal and Karcher (1876, 1885).
22 The quoted letter to W.J.C. Miller suggests that this paper was an early version of (MacColl 1866c).

grade, by private study in Boulogne between 1873 (matriculation) to the degree in 1876, with an 'Intermediate Arts' qualification at first division en route in 1874; he did not proceed to take the Honours Examination.[23] In later terms he was an external student of the University, not fulfilling residence qualifications; he may even have taken the examinations in France. (University of London 1877, p. 60.)

Three years earlier than Hugh his older brother John MacColl could afford to study classics at Oxford University:

> John McColl, second son of John McColl, land steward, of Kilmarly, Invernessshire, matriculated on 15 October 1870, aged 44; he was an 'unattached' or non-collegiate student. The pass list published in the *Oxford University Gazette* in December 1873 records that he satisfied the examiners in the School of Literae Humaniores in Michaelmas Term 1873 but was not awarded honours. The degree of BA was conferred on him in 1874. (Bailey 2001)

On 22 May 1875 John MacColl, his second wife, Theresa Whitgreave, and the seven children at their responsibility emigrated to Australia where he became headmaster of St James Grammar School in Melbourne.

Like Hugh MacColl himself, his character Ednor Whitlock makes a considerable effort in order to obtain an academic degree:

> Ednor's probation lasted a little over four years, during which he was no sluggard. He worked hard as a teacher in the school, and still harder out of school hours as a student for the London University. He passed all his examinations creditably, taking mathematical honours in the final examination. His place on the honours list was not quite so high as he had hoped; but considering what a small margin his school duties allowed him for study, it was no small achievement to take honours at all. He owed much of his success to the help which Mr. Kent occasionally gave him, and especially to the judicious course of reading which the latter had recommended. Left entirely to himself, Ednor would no doubt have wasted much time and energy in mastering things which seldom turn up in examinations, while he might be neglecting things which Mr. Kent knew from experience to be more paying.
>
> As soon as Ednor had taken his degree, Mr. and Mrs. Kent allowed his probationary engagement with their daughter to terminate in the way which all had hoped, namely, in the intimate, lifelong union of marriage (MacColl 1891b, pp. 339–340).

In contrast with his hero, MacColl took his degree not as a younger, unmarried man, but in the middle of his life, and with economic responsibility for a growing family. Hence, it was for good reason that he interrupted his work for the mathematical section of the *Educational Times* where his academic teachers were competing with one another. In a situation like MacColl's even a highly talented person risked failure. As the *Calendar of London University* reports, 17 of the 25 candidates

23 The degree is listed. Cf. University of London (n.d., p. 389). The details are logged in the University Archives (Senate House, London) in these ledgers: 'Matriculation I' (ledger D3), 1873, entry no. 230 (supported by 'his brother', presumably Malcolm, if not John MacColl); 'First BA Examination' (D32), 1874, entry no. 129 (supported by one 'G.C. Hope Esqr. D.D.'); and 'Second BA Examination' (D32), 1876, entry no. 44 (examination in November).

above the age of 30 failed in the BA exams of July 1876. In October of the same year 25 of the 33 elder candidates were not successful. After his graduation MacColl joined the university's congregation. (University of London 1877). Sporadically, its reports mention him participating in decisions.

We do not know MacColl's earliest writings. But, apparently, he could not impress influential academics like Sylvester sufficiently. The letter to Miller from 1866 quoted earlier reported the professor's reaction to the young man's pamphlet:

> The *whole* of his criticism is as follows:
>
> 'The interpretation affixed by the author to trilinear or quadrilinear coordinates is not, I think, new: I scarcely remember the time when I was not acquainted with it.'
>
> 'The notation of angles does not seem to be very important although probably ingenious. YX and XY are always understood to be complementary in respect to 360°: Mr. McColl seems to manage to express with precision the inclinations of two lines complementary in respect to 180°. The rule for fixing the sign absolutely of perpendiculars to lines and planes appears to me at first blush sound.'
>
> Next time I wrote to Mr. Karcher I desired him to thank Prof. S. in my name for his criticism, and to tell him that I had brought the title & language of my paper down to my own lowered opinion of its importance ... I have at present in my possession Professor Sylvester's tract on the proof of Newton's rule for discovering imaginary roots, & from the way he speaks of Dr. Young I cannot help thinking that the worthy professor has a somewhat peppery disposition. (MacColl 1866a)

It seems, however, that such experiences did not prevent MacColl from pursuing his, perhaps, original ideas, and defending them publicly—even if context and occasion were by no means suitable. On the other hand MacColl seems to have been supported by the Rev. Robert Harley (1828–1910).[24] From 1864 the latter was professor for mathematics and logic at Airedale College, Bradford (Poggendorff *et al.* 1863–1938, p. 588) He was a specialist in Boolean Algebra, and was a good friend of Boole. Through his work on the *Stanhope Demonstrator* he influenced the history of logical machines (Harley, 1879).

Sylvester was neither the only nor the most important critical reader of MacColl's work. In 1880, four years after his graduation, MacColl wrote to W.S. Jevons:

> It seems to me that Mr. Venn's method of criticism, if consistently & logically carried out, would deprive every inventor that ever lived (Boole not excluded) of all claim to originality; for *all* inventions are built out of and may be resolved into simple truisms which all then would know. This kind of criticism does not seem to me either just or generous. I am not surprised that with Mr. Venn for examination I was plucked in logic in 1875. (MacColl 1880)

24 In an Appendix to 'The Calculus of Equivalent Statements (III).' MacColl writes:

> I am also indebted to the kindness of Rev. Robert Harley for the loan of Boole's 'Laws of Thought', in which, as in Jevons's 'Pure Logic' in spite of a very different notation and mode of treatment, I find many points of resemblance to my method. My method, however, differs both from Prof. Boole's and Prof. Jevons's in three cardinal points, which a perusal of this paper will show to be so important as to necessitate an essentially different treatment of the whole subject. (MacColl 1878–1879, p. 27)

Actually, MacColl took his BA in 1876.[25] On 28 October 1876 Rev. J. Venn, MA set the BA pass examination for *Logic and Moral Philosophy*. Either MacColl was wrong about the year of his BA examination, or he did not refer to it. In these years not only Venn, but likewise Karcher and Sylvester were examiners of the University of London.

In the appendix to his third paper on *The Calculus of Equivalent Statements* MacColl reports how his first period without scientific research incidentally came to an end:

> ... my thoughts did not again revert to the subject till two or three months before the appearance of my article on 'Symbolical Language' in the *Educational Times* for July 1877. (MacColl 1878–1879, p. 27)

For the next seven years MacColl published regularly on logic and mathematics. Once again, he participated in the discussions published in the *Educational Times*. In a series of papers, read at the *London Mathematical Society*, MacColl designed a first, apparently non-classical propositional calculus. A letter of 1905 to Bertrand Russell describes this period of his development as follows:

> When ... I discovered by Calculus of Limits, or as I then called it, my 'Calculus of Equivalent Statements and Integration Limits', I regarded it at first as a purely mathematical system restricted to purely mathematical questions ... When I found that my method could be applied to purely logical questions unconnected with the integral calculus or with probability, I sent a second and a third paper to the *Mathematical Society*, which were both accepted, and also a paper to *Mind* (published January 1880). These involved me in a controversy with Venn & Jevons, of which I soon got tired, as I saw it would lead to no result. I sent a fourth paper (in 1884) to the *Math. Soc.*, on the 'Limits of Multiple Integrals', which was also accepted. This I thought would be my final contribution to logic or mathematics, ... (MacColl 1905b)

The long illness of MacColl's first wife and her premature death in February 1884 may have contributed to his long-lasting abstention from research. During this second period without research MacColl wrote a number of literary works: probably five novels, two of which were published,[26] a short story,[27] and, anonymously, a first essay on the religious impact of evolution theory.[28] To a large extent these works are a first articulation of his metaphysical and religious beliefs. In the last few years before his death he elaborated them in a series of essays and in a book on *Man's Origin, Destiny and Duty*.

No earlier than 1896 MacColl returned to his former investigations. At least he says so in the already quoted letter to Bertrand Russell,

> ... for ... twelve or thirteen years, I devoted my leisure hours to general literature. Then a friend sent me Mr. Dodgson's ('Lewis Carroll's') *Symbolic Logic*, a perusal of which rekindled the old fire which I thought extinct. My

25 Cf. University of London (1877, p. 389).
26 Cf. Anonymous (1896).
27 Cf. MacColl (1891a).
28 Cf. MacColl (1891b, p. 343). So far it has not been possible to identify this paper.

> articles since then I believe to be far more important from the point of view of general logic than my earlier ones; but unfortunately the views which they express are far more subversive of the orthodox or usually accepted principles in symbolic logic. (MacColl 1905b)

This self-assessment certainly is correct, but it also mirrors the reservation of those who opposed the publication of some of his later articles. As in previous years MacColl submitted his papers to the London Mathematical Society. However, after his sixth and seventh papers on 'The Calculus of Equivalent Statements' and an 'Explanatory Note and Correction' the Society refused to publish further contributions. An eighth paper with the familiar title was rejected in October 1898. Alfred Bray Kempe (1849–1922), a close cooperator of Sylvester remembered for his work on the four colour problem and multisets, and Alfred North Whitehead had been appointed referees. MacColl reacted to the decision with a revised and extended version of the paper. After a report on the matter by Kempe the Society's council decided to 'decline further communication on the subject, from Mr M, at any rate for the present ... ' Between then and 1903 MacColl submitted two further articles to the London Mathematical Society. Both were declined.[29] However, the two places in which MacColl's final views on his logical system are found are an address to the Paris conference of 1900 (MacColl 1901b) and his book *Symbolic Logic* of 1906. Sadly, there is evidence that even here in his last statement of his ideas, unfortunate circumstances made MacColl reduce the text by half. On 22 July 1905 he writes to Bertrand Russell:

> I am working hard re-casting and re-writing my book on Symbolic Logic. The ex-publisher Mr. Grant Richards, having seen my articles in the Athenaeum, asked me to write a treatise on the subject, which I did. But Mr. G.R. unfortunately failed before the work was finished, and other publishers don't feel inclined to take the risk. I cannot afford to pay for the publication myself unless I reduce the book by at least one-half; and even so, I must expect to lose. Still, I have made up my mind to face the loss by the publication of a small edition of about 500 copies. (MacColl 1905a).

During the last 13 years of his life MacColl wrote several articles in which he steadily altered, and, perhaps, improved his systems of modal logic. Almost all of them appeared in *The Athenaeum* and in *Mind*—the journal that published his controversy with Bertrand Russell on "'If" and "Imply"', and in which from 1912 onwards Lewis presented his initial account of strict implication (Lewis 1912).

7. Local recognition

Living permanently in Boulogne, Hugh MacColl acquired French citizenship. An obituary published in the *Revue de Métaphysique et de Morale* suggests that he did so on the occasion of his second marriage (Anonymous 1910). However, there is no firm evidence as to when MacColl was naturalized in France.

Boulogne's social niche allowed him to participate unreservedly in a French form of life without endangering his own cultural identity. At the end of his life Hugh

29 For this passage cf. The London Mathematical Society (1894–1914).

Hugh MacColl and his family in 1907.[30]

MacColl as well as Hortense Marchal were rather well-known citizens highly respected for their pedagogical and academic achievements:

> Quand il quitta son cours du collège, après y avoir fait un nombre incalculable d'élèves qui lui doivent leur connaissance raisonnée et pratique de la langue anglaise, M. Mac Coll [sic] continua cet enseignement ainsi que celui des mathématiques dans le pensionnat de jeunes filles si réputé que tenaient en notre ville Mmes Mac Coll [sic] et Busch-Marshal.
>
> Cette collaboration ... à l'œuvre pédagogique entreprise chez nous par sa non moins érudite épouse et Mme Busch fut pour beaucoup dans le succès toujours croissant de cete excellente pension d'éducation dont les pensionnaires se recrutaient non seulement en Angleterre, mais encore dans toutes les parties de l'Amérique. Elle comptait en outre comme externes bon nombre de nos concitoyennes qui en ont conservé le meilleur souvenir.
>
> En dehors du professorat proprement dit M. Mac Coll [sic] était un érudit et un linguiste du plus sérieux mérite, comme en témoignent ses divers ouvrages de critique, de philosophie et de mathématique, qui jouissent encore chez nos voisins, d'outre-Manche, d'une vogue des mieux justifiées (Anonymous 1909).

The family's domicile between 1906 and 1908 at 28 *rue Beaurepaire* truly was a 'maison de maître'.[31] Presumably, the boarding school had been transferred to this

30 By courtesy of Mrs Rosemarie Maconchy.

31 Presently this address is 53 *rue Beaurepaire*. The house does not exist any longer.

place. About one year before his death MacColl and his wife retired and moved to 67 *rue Porte Gayole*—a house close to Boulogne's ancient noble centre.[32]

Here MacColl died on 27 December 1909 (Ville de Boulogne 1909). He was, however, working actively until a few days before his death, as is evidenced by a letter to Bertrand Russell of 18 December. His elder brother, Malcolm, predeceased him, on 5 April 1907, and his son Hugh died in London in 1924 shortly after retiring to Boulogne after his legal career in Burma. Hortense died on 13 October 1918 (Ville de Boulogne 1918).

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32 This address presently is 77 *rue Porte Gayole*.

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