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[FROM THE UNITED STATES COAST SURVEY REPORT FOR THE YEAR 1876.]

APPENDIX No. 15.

MEASUREMENTS OF GRAVITY AT INITIAL STATIONS IN AMERICA AND EUROPE.

UNITED STATES COAST AND GEODETIC SURVEY,
ALLEGHENY, PA., *December* 13, 1878.

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Superintendent United States Coast and Geodetic Survey, Washington, D. C.

DEAR SIR: I present herewith the first part of my report on the measurement of gravity at initial stations of Europe and America. I here describe the methods employed and communicate the main results of the research. The discussion of the amount and nature of the errors, of the comparison of the present results with those deducible from the experiments of other men, and of the resulting figure of the earth, together with some other matters, are postponed for a subsequent report.

The acceleration of gravity is one of those quantities which it is the business of a geodetic survey to measure. So it has always been considered; and it is usage which fixes the meaning of the word "survey" in its geodetical sense. The geodesist is expected to do more than make a map of the country. He not only determines, for instance, the declination of the magnetic needle, which may be laid down on the chart, but also the other magnetic constants which cannot be so laid down. Were he to omit to determine the total force of magnetism, he would be held by all scientific men to have neglected a part of his duty. Now, in the same relation in which this constant stands to magnetical declination and inclination, just so stands the acceleration of gravity to the latitude and longitude; and by as much more as the latitude and longitude are essential to a survey relatively to the direction of magnetism, by so much more is the measurement of gravity indispensable in comparison with that of the magnetic force. The very first duty of the geodesist—paramount even to the drawing of a map—is the study of the figure of the earth; and an operation of surveying in which this problem was left out of view would neither merit nor receive the name of geodetical work. But it was the variation of gravity with the latitude which first proved the earth's ellipticity; and it may very well yet turn out that this method is the best way of determining it. At all events, the study of local variations of vertical attraction will find an application in the measurement of the level surface of the earth by triangulation. It is, also, quite certain that the solution of some high problems of geology must be facilitated by the integrated soundings which the pendulum virtually makes of the earth's interior.

While the absolute amount of the acceleration of gravity is, no doubt, a geodetical constant necessary to be determined, a very precise knowledge of it can, in the present state of science, find no practical nor theoretical application. What is chiefly of importance is the relative gravity at different places and times. This is also a quantity far easier to measure. To determine the acceleration absolutely, we must accurately measure both an interval of time and a length; to determine it relatively, we have only to carry the same rigid piece of metal from place to place and determine the duration of some phenomenon in which gravity is chiefly concerned. Moreover, we can fix in some measure the probable error of relative determinations. Most of the conditions of the experiments other than the amount of gravity itself are alike at the different stations. If they were precisely so, no constant errors could affect the relative result except in the second order of magnitudes. Now the accidental errors of observations, the only ones which would remain, can readily be determined by the method of least squares. It is not quite true that no conditions other than gravity vary from station to station. The temperature, for example, varies; and in such a manner that an erroneous coefficient of expansion will produce errors in the relative gravity of stations near the poles and near the equator in a constant direction and similar amount; and so will slightly affect the deduced compression of the earth. So an error in the coefficient of atmospheric effect will produce a constantly similar error in the relative gravity of an elevated and a depressed station, and may thus lead to an extremely erroneous value of the absolute modulus of gravitation and of the mean density

1878. *Experiments in April and May. Illustration No.* 34.—Before heating up the room in which the pendulum was swung the chronometers were moved outside, to a place which was not so favorable in uniformity of temperature and in other respects. Their rates were therefore not so good as they had previously been. Chronometer 1589 was taken as the standard, and supposed to run uniformly until the 2d, and from the 2d to the 5th of May. Between the 5th and the 9th there were evidently two changes of rate; the times when these occurred were determined from the comparisons with the other chronometers. The rate was uniform again from the 9th to the 13th.

		Rate of 1589.
		s.
April	18.43 to 32.35	+1.036
May	2.35 to 6.18	+1.29
	6.18 to 7.20	−2.52
	7.20 to 13.50	+1.667

The rates of No. 380 during the nights of pendulum-work were then found to be—

		Rate in seconds.	In decimals of a day.
April	24	+0.07	+.0000008
	26	+0.07	+ 008
	30	−0.08	− 009
May	2	−0.39	− 045
	4	−0.39	− 045
	5	−0.39	− 045
	6	+0.47	+ 054
	8	−0.51	− 059
	10	+0.24	+ 028
	11	+0.24	+ 028

CORRECTION FOR ARC.

The factor for reducing the time of oscillation of the pendulum to an infinitesimal arc is best developed according to powers of the arc itself. Such a development is far more convergent than those found in the books. The factor is

$$1 - \frac{1}{64} A^2 + \frac{1}{49152} A^4 - \frac{5}{1179648} A^6 + \text{etc.}, \quad = 2\cos\frac{\Phi}{4} - 1$$

or you correct by subtracting $4T\sin^2\frac{\Phi}{8}$

where A represents the whole amplitude of the oscillation expressed in parts of the radius.

The Repsold pendulum tripod is provided with a metallic arc for reading the amplitude of oscillation. This is divided into spaces of 10′ each. In the experiments on the Geneva support an arc divided into thousandths of the radius was made use of.

At Geneva, the amplitude was read by bringing the vertical wire of the telescope so as to bisect the point of the pendulum at the extremity of an oscillation, the wire having been turned in a direction radial from the line of the knife-edge. The time was noted and the position of the wire between two lines of the graduated arc was estimated at leisure. At the other stations a far better method was used. The wire was placed in exact coincidence with a line of the graduated arc and the time was noted at which the pendulum was bisected by it at the extremity of its oscillation. The arc was so placed that its zero was 1′ or 2′ away from the vertical, so as to permit the observation to be made both to the right and the left.

Table showing the time of decrement of the arc from 1° 10′—Continued.

HEAVY END DOWN—Continued.

BERLIN.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
Apr. 19	−35 37	−32 24	....	....	− 7 24	+22 41	....
20	....	−32 50	−25 05	....	− 7 23	+22 15	+37 44
24	−35 42	....	−24 51	−20 04	− 8 7	+22 24	+36 51
25	....	....	−24 52	....	− 7 51	+21 42	+38 06
26	−35 39	....	....	−20 02	− 7 34	+21 38	....
28	....	....	−25 02	−20 15	− 7 57	+21 53	+38 40
29	−35 55	−32 43	−24 57	−20 05	− 7 49	+22 35	+37 52
30	−35 56	−32 44	−24 52	−20 19	− 7 59	+21 56	+37 38
May 2	−35 39	−32 59	−24 49	−20 06	....	....	+37 23
Means...	−35 45	−32 44	−24 58	−20 9	− 7 46	+22 8	+37 45

KEW.

	2° 20′.	2° 10′.	1° 50′.	1° 40′.	1° 20′.	0° 50′.	0° 40′.
	m. s.	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
July 2	....	−33 9	−25 07	−20 07	− 7 58	+22 16	+37 15
3	....	−33 6	−25 21	....	− 8 06	+22 19	+37 54
4	....	−33 17	....	−20 31	− 8 10	+21 20	+37 27
4	....	−33 14	−25 33	−20 29	− 7 58	+21 35	+38 40
7	....	−33 59	....	....	− 8 15	....	+38 41
7	....	−34 4	−26 4	....	− 8 26	+22 33	+39 14
8	....	−33 55	−25 50	−20 53	− 8 24	+22 21	+38 8
9	....	−33 44	−25 41	−20 51	*	+22 29	+38 53
10	....	−33 25	−25 30	....	....	+22 13	+38 30
Means...	....	−33 31	−25 35	−20 36	− 8 09	+22 07	+38 16

The foregoing tables show the amount of discrepancy between the observations of different days. The point of the pendulum is distant 113½ cm. from the knife-edge, so that one minute of arc measures ⅓ of a millimeter. The reading-telescope was placed at a distance of about 3 meters. It may, therefore, be supposed that a single observation of the half-amplitude would be in error by something like ¼ of a minute. The following table shows how much error this would produce in the noted time of attaining the different amplitudes:

Half amplitude.	Time of decrement of ¼′. Heavy end up.	Time of decrement of ¼′. Heavy end down.
° ′	*s.*	*s.*
2 10	2	5
1 50	3	7
1 40	3	8
1 20	5	11
1 10	6	13
50	9	21
40	12	27

It will be seen that the observed discrepancies are several times as large as these values, and cannot therefore well be attributed to errors of observation. The daily discrepancies are, however, less than 7 times the numbers just given, that is less than 1¾ minutes. Such errors would produce end down have been added together; and the means have then been taken for all three stations (Paris, Berlin, and Kew). We thus obtain

Half amplitude.	Sum of times.
′	s.
130	−2880
110	−2187
100	−1779
80	− 706
70	0
50	+1927
40	+3304

The time for 140′ has been neglected as not having been generally observed with heavy end down.

To satisfy these values a form of equation has been assumed which has been copied from Professor Benjamin Peirce's *Analytic Mechanics*, and which is Coulomb's equation with a constant term added. It is—

$$\mathrm{D}_t\,\Phi = -\,a - b\,\dot{\Phi} - c\,\Phi^2 .$$

The integral of this equation is

$$\Phi = \sqrt{\frac{a}{c} - \tfrac{1}{4}\frac{b^2}{c^2}}\,\cot\left\{c\sqrt{\frac{a}{c} - \tfrac{1}{4}\frac{b^2}{c^2}}\,(t - t_0)\right\} - \tfrac{1}{2}\,\frac{b}{c} \,.$$

The values of Φ for the different values of t, as given in the table above, are sufficiently satisfied by putting (for t in seconds of time and Φ in minutes of arc)

$$a = 1547 \times 10^{-6}$$
$$b = 6418 \times 10^{-8}$$
$$c = 1421 \times 10^{-9}$$

The errors are shown in the following table:

Sum of times.	Obs. Φ.	Calc. Φ.	(O—C) Φ.
s.	′	′	′
−3191	140	138.82	+1.18
−2880	130	130.03	−0.03
−2187	110	110.33	−0.33
−1779	100	100.00	−0.00
− 706	80	79.97	+0.03
0	70	70.03	−0.03
+1927	50	49.98	+0.02
+3304	40	39.99	+0.01

By least squares, better values of the constants could be obtained; but these are evidently sufficient for our purpose.

The law of decrement of the amplitude having been made out, it was requisite to apply it to the observations. The constant t_0, being different for each experiment, had first to be determined. In doing this, it was desirable to use observations in which the arc had only been noted on the right or on the left. For this purpose it was necessary to calculate the inclination of the zero of the metallic arc to the vertical. This was readily determined from the difference of the time of reaching a given division to the right and to the left. The following tables show the results so obtained. The figures in parentheses at the top of the column show the estimated inclination from observations of the pendulum at rest.

The following tables give the observations of arc in detail, followed by the calculation of inclination, made according to the methods before given.

UNITED STATES COAST SURVEY—HOBOKEN, JUNE, 1877.—DECREMENT OF PENDULUM ARC.—FULL PRESSURE.—SOLID SUPPORT.

TIMES OF REACHING DIFFERENT AMPLITUDES.

	HEAVY END UP.										HEAVY END DOWN.									
June	11	14	15	16	17	19	20	22	29	Mean.	11	14	15	16	17	19	20	22	29	Mean.
′	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*
380	......	......	......	......	....	......	......	......	......	....	......	54. 5	......	......	......	.....	......	......	......	.....
370	......	......	......	......	......	......	......	.....	......	....	......	57. 4	....	26. 0	52. 1	13. 3	2. 0	29. 7	4. 7	......
360	......	......	......	......	......	......	......	......	......	....	......	....	......	......	......	......	......	......	......	.....
350	......	......	......	......	......	......	......	......	......	....	......	....	......	......	......	......	......	......	......	....
340	......	......	......	......	......	......	......	......	......	....	......	......	......	......	......	......	......	......	......	......
330	......	......	......	......	......	......	......	......	......	....	......	62. 5	......	31. 3	57. 2	......	7. 0	34. 8	9. 7	34. 40
320	......	......	......	......	......	......	......	......	......	....	38. 8	64. 4	......	......	59. 0	20. 0	8. 4	36. 2	11. 6	36. 07
310	......	......	......	......	......	......	......	......	......	......	......	....	......	34. 5	60. 6	21. 7	10. 2	38. 1	13. 4	37. 75
300	13. 2	......	......	24. 9	49. 4	13. 9	......	......	7. 9	20. 44	41. 8	67. 5	......	36. 0	62. 7	23. 4	......	......	15. 0	39. 41
290	......	......	......	26. 1	......	14. 7	......	......	8. 6	21. 30	......	68. 9	......	38. 0	....	24. 9	......	41. 2	16. 5	40. 99
280	14. 6	17. 6	17 8	......	50. 4	15. 4	5. 1	......	......	21. 98	44. 8	70. 7	...	39. 5	66. 0	26. 7	15. 3	42. 8	18. 4	42. 66
270	......	......	......	......	......	......	......	......	......	(22. 98)	......	....	......	......	68. 2	......	17. 8	......	......	45. 06
260	......	......	......	......	......	......	......	25. 6	......	23. 89	....	....	......	......	....	......	......	47. 3	23. 1	47. 61
250	......	......	......	......	......	......	......	......	......	(24. 88)	......	77. 0	39. 2	47. 0	72. 8	33. 8	22. 2	50. 0	25. 6	49. 64
240	......	22. 0	......	......	......	......	......	......	......	....	54. 4	81. 1	41. 7	......	....	......	25. 0	52. 3	28. 5	52. 50
230	......	23. 1	23. 1	31. 6	55. 8	20. 6	......	......	14. 4	27. 07	57. 3	....	......	......	78. 3	39. 0	27. 1	54. 9	......	54. 91
220	......	24. 5	24. 5	......	56. 8	......	11. 0	30. 0	15. 3 ·	28. 24	......	86. 2	47. 4	54. 9	80. 9	41. 5	......	58. 1	......	57. 81
210	21. 6	25. 7	26. 0	......	58. 3	22. 9	12. 4	31. 2	16. 8	29. 56	62. 5	88. 5	......	58. 1	....	44. 7	.....	61. 2	36. 4	60. 74
200	22. 7	27. 0	27. 2	35. 4	......	24. 2	13. 8	32. 6	17. 9	30. 84	67. 3	....	53. 6	60. 6	87. 0	48. 2	......	64. 1	39. 4	64. 04
190	24. 6	......	......	36. 6	61. 1	25. 6	15. 1	34. 1	19. 3	31. 09	70. 1	95. 9	56. 2	64. 8	90. 7	50. 7	....	66. 9	43. 1	67. 30
180	......	......	......	......	62. 5	27. 1	......	35. 6	....	33. 91	73. 7	....	......	68. 5	94. 2	....	....	....	......	70. 96
170	......	32. 2	......	......	....	......	....	.....	......	35. 42	....	103. 5	......	......	....	....	......	74. 9	49. 5	74. 74
160	......	34. 6	33. 8	41. 7	......	......	....	......	24. 0	37. 34	......	......	.....	......	......	......	....	81. 3:	......	(79. 80)
150	31. 2	36. 3	35. 3	43. 6	67. 8	32. 2	21. 6	40. 5	26. 0	39. 11	....	....	......	81. 5	107. 6	......	......	83. 5	59. 5	84. 36
140	33. 0	......	37. 3	......	70. 1	......	......	....	27. 8	41. 10	91. 6	117. 7'	.....	86. 2	112. 0	81. 6:	......	87. 7	....	88. 66
130	34. 8	40. 6	39. 5	47. 5	72. 1	36. 0	......	44. 4	29. 5	43. 05	96. 7	....	......	90. 7	116. 4	....	....	....	68. 0	93. 40
120	37. 1	43. 5	42. 4	49. 8	74. 1	......	28. 1	46. 4	31. 2	45. 34	101. 5	127. 2	84. 1	95. 8	121. 2	......	....	97. 4	73. 2	98. 36
110	......	46. 3	44. 9	52. 4	......	40. 4	30. 1	49. 2	33. 9	47. 91	110. 6	134. 7	90. 1	102. 6	128. 2	95. 4	....	104. 4	80. 9	105. 86
100	42. 5	49. 6	......	55. 5	79. 8	43. 5	32. 7	......	37. 2	51. 01	116. 6	142. 8	96. 2	110. 2	136. 8	103. 5	....	111. 2	88. 0	113. 16
90	45. 7	53. 9	51. 0	59. 0	83. 2	46. 6	36. 2	55. 5	......	54. 39	125. 9	152. 2	103. 4	117. 4	144. 2	......	......	119. 2	.. ..	121. 42
80	50. 2	58. 9	56. 0	......	87. 5	......	......	......	......	....	......	....	......	......	....	......	......	.. ..	......	......

1.67
1.68
1.66
1.58
1.67
2.40
2.55
2.03
2.66
2.41
2.90
2.93

The December observations were corrected for inclination by the proper additions to the time of arcs observed on the right side only. For the observations at high temperatures, the observed times on the right side were taken, and the arcs corrected,

In heavy-end-up observations, all days except the first, by	−0.0004
In heavy-end-down observations, May 4, 5, and 6, by	−0.001
In heavy-end-down observations, May 8, by	−0.00035

The differential formula connecting the arc and the time was next found. Only three constants were employed, as has been already stated, the observations not being sufficiently numerous or sufficiently exact to admit of four. The equation, then, is

$$D_{t}\phi = -b\varphi - c\varphi^2.$$

Hence, $\varphi = \frac{b}{c}\left(e^{bt} - 1\right)^{-1}$,

$$\frac{1}{16}\int \varphi^2 \, dt = \frac{b}{16c^2}\left\{ bt - \text{Nat. log.}\left(e^{bt}-1\right) - \left(e^{bt}-1\right)^{-1} \right\} + C.$$

$$= \frac{b}{16c^2}\left\{ \text{Nat. log.}\left(1 + \frac{c}{b}\varphi\right) - \frac{c}{b}\varphi \right\} + C,$$

and $t = \frac{1}{b}\,\text{Nat. log.}\left(1 + \frac{b}{c\varphi}\right)$.

The constants b and c, for heavy end up, calculated from the observed times of decrement, but not corrected by least squares, are given in the subjoined table.

From the observations of decrement were deduced, first, an equation of the form

$$b + c\varphi_m = n,$$

φ_m denoting the mean value of φ; and, second, a value of c. As c has been supposed proportional to the density of the air, we find

$$c = 0.03125 \frac{p}{\tau}$$

By substituting these values in the equations $b + c\varphi_m = n$, we obtain new values of b, called "second reduction" in the table; the "first reduction" having been obtained with the original values of c. These second values answer to the formula

$$b = 0.0013\,\tau^{3/4} + 0.00435\,p^{1/2}\,\tau^{-1/2}.$$

For the observations in which the bell-glasses were on the receiver, these alone being comparable with one another, the values of b, thus calculated, are given in the table, along with the values from the reduction of observations of decrement. Also in illustration No. 36, before alluded to.

$$t = \frac{1}{b}\log\left(1 + \frac{c}{b}\varphi\right) + \frac{1}{b}\log\frac{b}{c} - \frac{1}{b}\log\varphi$$

$$t_1 - t_2 = \frac{1}{b}\log\frac{1 + \frac{c}{b}\varphi_1}{1 + \frac{c}{b}\varphi_2} - \frac{1}{b}\log\frac{\varphi_1}{\varphi_2}$$

when c is very small $\log\left(1 + \frac{c}{b}\varphi\right) = \frac{c}{b}\varphi - \frac{1}{2}\frac{c^2}{b^2}$ [illegible]

$$t_1 - t_2 = \frac{c}{b^2}(\varphi_1 - \varphi_2) + \frac{1}{b}\log\frac{\varphi_2}{\varphi_1}$$

grammes per centimeter (second)². This may be converted into the pressure of a height of mercury by a calculation like the following:

	Names of quantities.	Numbers.	Logarithms.
	Density mercury at 0° C. compared with water at 4°	13.5959	1.133405
	Absolute density of water at 4°	0.99999 $\frac{gr}{(cm)^3}$	—4
∴ Multiplying,	Absolute density of mercury at 0°	13.5957 $\frac{gr}{(cm)^3}$	1.133401
	Gravity at Paris (60ᵐ elevation)	980.88 $\frac{cm}{s^2}$	2.991616
∴ Multiplying,	Absolute specific gravity of mercury at Paris	13335.7 $\frac{gr}{cm(s)^2}$	4.125017
	One absolute atmosphere	1000000 $\frac{gr}{cm(s)^2}$	6.000000
∴ Dividing,	One absolute atmosphere expressed in centimeters, pressure of mercury at 0° at Paris at elevation of 60ᵐ	74.986	1.874983

This is equal to 29.63 inches pressure at Hoboken at 15° C. At London it is less by 0.03 inch, a quantity which produces hardly a perceptible effect in the time of oscillation of the pendulum.

To find the density of the air under this pressure, we have, according to the experiments of Regnault (Wüllner's Experimentalphysik, 3ter Band, 3te Auflage, S. 133), the absolute density of dry air free from CO_2 at 0° C. and pressure 76 cm at Paris (6ᵐ elevation), 0.0012932. This supposes a slightly different absolute density of water from that above assumed; but that is a matter of no consequence. Then we have

		Log.
Density dry pure air at Paris, standard atmosphere	.0012932	7.11167
Centimeters, pressure in Paris, standard	76	1.88081
Centimeters in absolute atmosphere	74.986	1.87498
∴ Density pure dry air at 0° C. under one absolute atmosphere	.0012760	7.10584
Absolute temperature of 0° C. = $\frac{273}{288}$		9.97677
∴ Density pure dry air at 15° C. under one absolute atmosphere	.0012095	7.08261
Correction for usual amount CO_2 $(1 + .529 \times 4 \times 10^{-4})$		0.00009
∴ Density common dry air at 15° C. under one absolute atmosphere	.0012097	7.08270

The moisture was not observed during the pendulum-experiments but is believed to have been as much as would be contained in air at a little less than ½ saturation at 15°; or say that the density was diminished by 3 thousandths. The density of the air taken may then be taken at .001206 at 15° C. under a pressure of one absolute atmosphere.

Our pendulum never having been weighed in water, it is necessary to estimate its density. The brass of which it is composed may be supposed to be of the same density as that of the Prussian pendulum-meter, which density is given by Bruhns as 8.5. But the pendulum contains a certain amount of steel. The knives have as their dimensions $9.55 \times 1.8 \times 1.4$, the product being 24 (cm)³. A part is beveled off, but other steel about the pendulum will make up nearly the amount. The entire volume of steel would be therefore 48 (cm)³. We may take it at 45 (cm)³. Assigning to steel the density 7.82 the mass of steel would be 352 gr. The entire mass of the pendulum has been ascertained by weighing to be 6308 grammes. Subtracting the mass of the steel, we have for that of the brass 5956 grammes. Dividing this by the assumed density of the brass we have 701 (cm)³ as the volume of brass, and 746 (cm)³ as the total volume of the metal in the pendulum. Then for the density of the metal we have $\frac{6308}{746}$ or 8.46 $\frac{gr}{(cm)^3}$.

Mass of Knives 317.8 gm

position of the center of mass of this air may be estimated as 3 cm outside the knife-edges and its radius of gyration about its center as 2 cm. The hydrodynamic effect should be taken as equivalent to carrying the displaced air, as a part of this air enlarges the cylindrical stem and the rest is so shaped as to offer very great resistance. Hence the effects are

$$.0000105\ T_d \text{ and } .0000241\ T_u.$$

If we, now, add together the various parts of the effects of buoyancy, of inclosed air, and of air carried outside, we have the total calculable effect proportional to the atmospheric density, as follows:

	Heavy end down.	Heavy end up.
Buoyancy	682×10^{-7}	1565×10^{-7}
Air within stem	795	1826
Air within frames	24	70
Air without stem	946	2169
Air without bobs	141	310
Air without knives, etc	105	241
Sums	2693	6181

When the pendulum swings upon the Geneva support without the bell-glasses we have to increase these effects by

Effect of cylinders	36	71
Sums	2729	6252

When the bell-glasses are in place we have in addition

Effect of bells on stem	22	53
Effect of bells on bobs	21	46
Sums	2772	6351

The result of this calculation is probably a little too small, owing to neglected terms, and can hardly be too large.

The calculation of the effect of viscosity on the time of oscillation depends on the variation of the decrement of the arc with the pressure. Experiments were made upon the Geneva support at Hoboken at various pressures. The observations of arc made during these observations, as has been explained above, were reduced according to the formula

$$\dot{\Phi} = -b\,\Phi - c\,\Phi^2,$$

a being supposed zero in order to diminish the number of unknown quantities. The coefficient c was supposed proportional to the density and one factor was taken for all the experiments, while b was left to be determined independently for each. The result is that all the abnormal variations of the decrement which are considerable are thrown upon b, so that the latter presents an appearance of greater irregularity than properly belongs to it. The results of these experiments are shown in illustration No. 37 *c*. Those with heavy end down have been brought to heavy end up by multiplying them by $\frac{h_d}{h_u}$. The time is expressed in minutes; the pressure in inches pressure at 15° C. at Hoboken. It will be seen that the observations satisfy sufficiently well the formula

$$b = .0013\ \tau^{\frac{1}{3}} + .00435\ p^{\frac{1}{2}}\ \tau^{-\frac{1}{3}}.$$

When the bell-glasses were removed, the time of decrement was noticeably increased; but this is partly due to the change in the value of c. Upon the Repsold support there is scarcely any sensible difference between the time of decrement from that in experiments of the Geneva support with the bells removed. This is shown on illustration No. 36. To compare the observations of arc at

Paris, Berlin, and Kew, with those at Hoboken, they were recalculated with only three constants. These as corrected by least squares are

$$b = .0001082 \quad \text{(units: one second of time and one minute of arc.)}$$
$$c = .000001125$$

t_0, reckoning from $1^\circ\ 10' = 8001^s$. The agreement of these values with observation is shown below:

φ (Obs.)	φ (Calc.)	C − O
130′	129′.96	− 0′.04
110	109 .85	− .15
100	100 .17	+ .17
80	80 .05	+ .05
70	69 .89	− .11
50	49 .91	− .09
40	40 .12	+ .12

Reduced to decimal parts of radius, minutes of time and heavy end up, these values become

$$b = 0.0214, \qquad c = 0.76.$$

Observations of June, 1877, at Hoboken, give

$$b = 0.0242, \qquad c = 0.58.$$

Allowing the European observations a weight of 3 and combining the values of c, we have

$$c = 0.72.$$

Substituting this value, we find for b at Paris, Berlin, and Kew.......................... 0.0224
And at Hoboken.. 0.0212
b (weighted mean).. = 0.0221

The curve drawn in illustration No. 36 is calculated from the European coefficients, and the agreement of the observations taken at Hoboken before the bell-glasses were put on is shown by the near coincidence with it of the points distinguished by crosses. The points distinguished by circles are obtained from a combination of all the observations taken in 1877 with the bells on, when the pressure was about 30 inches. The t_0 taken in each case was the one that made the mean excess over the curve equal to zero. The influence of the bell-glasses in arresting the motion of the pendulum is thus very strikingly shown. The value of b, with the bells on, under a pressure of 30 inches, appears by the above formula to be 0.0251 for a minute of time. The mean of the experiments with bells off, as just shown, gives $b = .0221$; so that we may assume that the viscosity effect is one-seventh larger with the bells on than off.

To calculate the effect on the period of oscillation, we take the coefficient .00435, we multiply it by $\sqrt{29.63}$ to bring it to one absolute atmosphere, we divide it by 60 to bring it to seconds, and finally we divide by $\odot$, and we get as the effect, with heavy end up, .0001256 T_u. To find the effect with heavy end down, we simply multiply by $\frac{h_u}{h_d}$, which gives .0000548 T_d. When the bells are off, $\frac{7}{8}$ of these values are to be taken.

At excessively low pressures the whole theory of atmospheric viscosity fails, because the fundamental hypotheses are then violated; and, therefore, the real effect of viscosity at $\frac{1}{4}$ inch pressure will probably be somewhat smaller than calculation would make it.

Experiments have been made at Hoboken on the Geneva support, in order to determine the effect of atmospheric pressure *à posteriori*. A series of experiments were made in September, 1877, with heavy end down, and another in December, 1877, with heavy end up. The duration of each experiment was generally long, and the agreement of the results is all that could be expected. These observations were made by Mr. Farquhar. The temperature, during the September experiments, was about 20° C.; that during the December experiments was about 10° C. They have been corrected so as to bring them exactly to these temperatures. The results of these experiments are exhibited on illustrations Nos. 37*a* and 37*b*. It will be seen that the sidereal time of oscillation, with heavy end down, satisfies the formula

$$T_d \overset{s.}{=} 1.006072 + .00000985\ p + .0000081\ \sqrt{p};$$

and those with heavy end up, the formula

$$T_u = 1.005740 + .00002264\ p + .0000234\ \sqrt{p},$$

where p is the pressure in inches at 15° C.

Taking one absolute atmosphere, or 29.63 inches, as the unit of pressure, and reducing the coefficients to 15° C., we have the general formulæ,

$$T_d \overset{s.}{=} 1.006027 + .0002969\ \frac{p}{\tau} + .0000442\ \frac{\sqrt{p}}{\sqrt[3]{\tau}},$$

$$T_u = 1.005785 + .0006598\ \frac{p}{\tau} + .0001271\ \frac{\sqrt{p}}{\sqrt[3]{\tau}}.$$

The values which we have obtained *à priori* are

$$T_d = x \qquad + .0002789\ \frac{p}{\tau} + .0000551\ \frac{\sqrt{p}}{\sqrt[3]{\tau}},$$

$$T_u = y \qquad + .0006388\ \frac{p}{\tau} + .0001263\ \frac{\sqrt{p}}{\sqrt[3]{\tau}}.$$

The difference between observation and *à priori* calculation is perhaps not greater than ought to be expected. The values which have been used in the reductions are

$$T_d = x \qquad + .0002917\ \frac{p}{\tau} + .0000512\ \frac{\sqrt{p}}{\sqrt[3]{\tau}},$$

$$T_u = y \qquad + .0006694\ \frac{p}{\tau} + .0001175\ \frac{\sqrt{p}}{\sqrt[3]{\tau}}.$$

These values were used before the last calculations of the *à priori* values were completed, and it was not thought worth while to change them; but the *à priori* values are preferred.

COEFFICIENT OF EXPANSION.

The coefficient of expansion of the pendulum has been determined, by comparing it directly with a meter obtained from the German Imperial Eichungsamt and there designated as Normal Meter No. 49, and also by assuming it to have the same coefficient as the pendulum-meter, the pendulum-meter having also been compared at different temperatures with No. 49. The coefficient of expansion of No. 49 has been absolutely determined by comparison with meter made for the purpose and marked "U. S. C. S.—C. S. P.—1878.—A." The comparisons have been made

Between No. 49 at 13° and A at 3°
Between No. 49 at 3° and A at 13°
Between No. 49 at 4° and A at 4°
and between No. 49 at 18° and A at 18°

The two meters were compared by means of the vertical comparator belonging to the reversible pendulum. They stood in two vertical brass tubes, 4½ cm. in diameter and 1¼ m. long, polished on the exterior and closed at the bottom by a foot terminating in a conical point. One of them rested in the step designed for the pendulum-meter, and was held at the top in a stirrup, movable upon a screw in such a way as to vary its distance from the microscope. The meter rested at the bottom of the tube in a species of trap, in which it was compressed just sufficiently to hold it in place. At about ⅘ m. from the bottom it was lightly held, by springs on its four sides, into a frame which was capable of being moved in any direction, by means of four horizontal screws penetrating the walls of the brass tube. India-rubber washers and nuts kept the screw-holes water-tight. Opposite the lines at the top and bottom of the meter, two windows were inserted in the brass tube, setting into little sashes formed of brass casting. These windows were made of plate-glass, about 3 mm. thick, which was carefully selected with a view to the parallelism of its sides, and placed in the sash in such a way that the slope of the wedge should be horizontal. These glass windows were kept water-tight by rubber washers, having a brass washer over them screwed down by four thumb-

Careful reformation table p 76

H. end down

Temp.	T_d
19°.57	1.0063895
35.05	1.0065315
15.48	1420
Per degree	92

H. end up

Temp	T_u
19.59	1.0065633
35.60	1.0067039
16.01	1406
	88

These give as the coefficient of expansion—

From oscillations with heavy end down.......................... $18^{\mu}.5$
From oscillations with heavy end up.............................. 18 .3

The value 18 .3 has been used in the reductions.

CORRECTIONS FOR THE WEARING DOWN AND ROUNDING OFF OF THE KNIFE-EDGES.

If 5 kilogrammes' weight be put upon an absolutely sharp knife-edge of steel hardened in oil and having a bearing length of 2 centimeters, the steel edge will be crushed until the breadth of the bearing surface is 1 micron. Accordingly, from the very beginning, a knife-edge will wear down and round off. The wearing down and the blunting will have very different effects upon the period of oscillation.

The removal of the point of support of a pendulum from its center of mass, will have an effect which is readily calculated, thus:

$$\mathrm{d}\,T^2 = \mathrm{d}\,.\,\frac{\odot^2}{g}\left(\frac{r^2}{h} + h\right) = \frac{\odot^2}{g}\left(-\frac{r^2}{h} + h\right)\frac{\mathrm{d}\,h}{h}.$$

For a reversible pendulum,

$$r^2 = h_d\ h_u.$$

Hence, we have,

$$\mathrm{d}\,T_d^2 = T_d^2\,\frac{h_d - h_u}{h_d + h_u}\cdot\frac{\mathrm{d}\,h_d}{h_d}$$

$$\mathrm{d}\,T_u^2 = -\,T_u^2\,\frac{h_d - h_u}{h_d + h_u}\ \frac{\mathrm{d}\,h_u}{h_u}$$

If a pendulum rolls upon a cylindrical surface of radius ρ, the instantaneous axis of rotation is the instantaneous line of contact; and a velocity of rotation about this axis is equivalent to the same velocity of rotation about the line of contact in the equilibrium position of the pendulum combined with such a translation velocity along the length of the pendulum as is necessary to fix the instantaneous axis; this is $2\rho\sin\frac{1}{2}\varphi\,.\,\frac{\mathrm{d}\varphi}{\mathrm{d}\,t}$. It follows that the amount by which the *vis viva* of the pendulum is affected by a cylindricity of the knife-edge is of the order of $\rho^2\,\varphi^2$ and may consequently be neglected. The moment of gravity is, however, obviously the same as if the axis of the cylinder were the axis of rotation, that is, it is multiplied by $\left(1 + \frac{\rho}{h}\right)$. Hence we have

$$\delta\,T^2 = -\,T^2\,\frac{\rho}{h}.$$

If the section is not circular, then obviously some sort of a mean radius of curvature must replace ρ. If the section is flatter than a circle, that is, if the lower parts in repose have the greater radii of curvature, then the mean radius, and consequently the effect on the period of oscillation, will be greater for small arcs than for large ones; while if the section is somewhat pointed downwards the reverse will be the case.

We know too little of the laws of crushing and grinding to be able to calculate the radius of curvature from the amount worn off. In fact, the ratio would probably depend on the hardness of the material. Neither can the radius be measured directly with any accuracy. But it may obviously be very large. When the pendulum is first brought down to rest on the edge, why may not the blunted surface be nearly flat? If it were so, the small oscillation through 4° or 5° could not round the edges enough to make the ratio of the radius to the wearing down at all small. Under these circumstances it is a question deserving consideration and experimental examination whether

Temperature.	Pl — 49
°	μ μ
0	+ 24.4 ± 0.20
5	− 26.0 ± 0.15
10	− 76.5 ± 0.12
15	− 126.9 ± 0.13
20	− 177.3 ± 0.17
25	− 227.8 ± 0.23

The uncertainty pertaining to these values may be much larger than the probable errors indicate because of the uncertainty of the relative expansion and of the imperfect methods of determining the temperatures, and because the observations are not sufficiently numerous.

This says Förster under date of 1880 Nov 15 is confirmed by new indirect comparisons.

If we take the result of the comparison made in Paris between our platinum meter and the mètre des Archives, namely: Mètre des A. − Pl = − 3 .01, we obtain the equation:

$$\text{M. d. A.} - \text{No. } 49 = \{ + 21^{\mu}.4 - 10^{\mu}.086\ t \}\cdot$$

This equation, however, is subject to doubt on account of the imperfection of the comparisons at Paris, previously mentioned, a doubt which can only be removed at some future time with the assistance of the more accurate equation established between 1605 and 49. The direct comparison between our platinum meter and the mètre des Archives (A) appears much less complete than the indirect comparison through the medium of the mètre du Conservatoire (C). On the assumption made at Paris that the coefficient of expansion of A, C, and Pl are nearly equal, the following results were obtained for the indirect comparisons. Direct observations between C and Pl gave:

$$\text{C} - \text{Pl} = - 16\ .2$$

But according to subsequent determinations (Procès-Verbaux de la Section Francaise, 1870), $\text{A} - \text{C} = - 3^{\mu}\cdot 2$, it would follow that $\text{A} - \text{Pl} = - 19^{\mu}.2$, and using this last equation, we get:

$$\text{A} - 49 = + 5^{\mu}.2 - 10.086\ t,$$

and if for 1° Centigrade we take the expansion $= 8^{\mu}.60$, we obtain:

Förster says in above letter that this is wrong

$$\text{No. } 49 = \overset{mm.}{999.9948} + \overset{mm.}{0.01869}\ t,$$

a value subject to a much greater uncertainty than is indicated by the probable errors given above on account of the imperfections of the Paris results.

The results of the scale values of the cylinders are given in conclusion (the lines designated by 1 are those nearest the contact ends)·

Space.	Cylinder I.		Cylinder II.	
	μ		μ	
Line 1 to 2	54.2		49.4	
1 to 3	104.3	50.1	100.7	51.3
1 to 4	154.5	50.2	148.7	48.0
1 to 5	207.8	53.3	199.4	50.7
1 to 6	252.8	45.0	249.7	50.3
1 to 7	305.5	52.7	303.4	53.7
1 to 8	355.8	50.3	352.7	49.3
1 to 9	407.8	52.0	401.3	48.6
1 to 10	454.0	46.2	452.9	51.6
1 to 11	504.4	50.4	503.6	50.7

For the Imperial Commission on Weights and Measures.

[Signed] FOERSTER.

The comparisons made in 1875 between the U. S. and German pendulum-meters give:

$$\text{U. S. meter} - \text{German meter} = + 131^{\mu}.9,$$

and those made in 1877 give:

$$\text{U. S. meter} - \text{German meter} = + 131\ .3;$$

The mean of these values, or + 131 .6, is adopted. The comparisons in detail are given in the following tables. The unit is one-thousandth of the revolution of the micrometer-screws.

CONCLUDED LENGTH OF THE PENDULUM.

The indirect comparisons of the U. S. pendulum-meter with the German meter No. 49 give the following result, when reduced to 15° C.:

	μ
U. S. meter — German pendulum-meter	+131.6
German pendulum-meter — No. 1605	−103.1
No. 1605 — No. 49	− 40.7
U. S. meter — No. 49	− 12.2
Direct comparison, U. S. meter — No. 49	− 20.0

The mean of these values, or $-16^{\mu}.1$, is taken, the likelihood of error of the two methods being estimated as equal. The bad temperature conditions at New York have prevented a more accurate determination of this quantity; but a new determination will be undertaken at the first opportunity.

But we have, for 15° C.:

$$\text{No. } 49 - \text{M} = +\,277^{\mu}.2, \text{ hence}$$
$$\text{U. S. pendulum-meter} - \text{M} = +\,261.1$$

The length of the pendulum at the different stations is, therefore, as follows:

	cm.
Geneva	100.00624
Paris	100.00799
Berlin	100.00840
Kew	100.00871
Hoboken	100.00960

CENTER OF MASS.

The quantity $h_d - h_u$, or twice the distance of the center of mass of the pendulum from the center of figure, was observed at the beginning and end of each series of experiments, and also before and after each transposition of knife-edges. The apparatus, method of observing, etc., have been elsewhere described.* Comparisons at the U. S. Office of Weights and Measures show that the 39 centimeters on the staff of the balance, from 17 to 56, are 0.14 mm too long. This correction applied, we obtain for $h_d - h_u$,

This seems to be applied with wrong sign.

	cm.
At Geneva	39.284
At other stations	39.292

These are the values used. The separate observations are shown in the following table:

* The idea of determining the center of mass of a reversible pendulum, instead of moving a weight upon it, belongs exclusively to Bessel.

PERIODS OF OSCILLATION AND VALUES OF GRAVITY.

The pendulum was swung in Hoboken in various ways, to wit:

1. The regular set was made on the Geneva support with the bells off. This set cannot be compared with others on the Repsold support, if the reductions be made on the principle of the reversible pendulum, owing to the different ways in which the knives rest on the two supports. The comparison may, however, be made on the principle of the invariable pendulum, so as to eliminate this effect.

2. Sets of experiments were made at various pressures on the Geneva support with the bells on. The knife-edges not having been interchanged, these are strictly only comparable among themselves.

3. Half a regular set was made on the Geneva support with the bells on at about 35° C. There were a few additional experiments at this temperature with heavy end up at different pressures.

4. The pendulum was swung on the Repsold support and also on a very stiff support having the head of the Repsold support as a part of it (so as to have the same bearing on the knives). The object of these experiments was to determine the effect of flexure of the support.

A conspectus of all these experiments is given in the following table:

Periods of oscillation of the pendulum at Hoboken; reduced to one absolute atmosphere and to 15° C., and to values on rigid support without bells or cylinder.

Heavy end down.					Heavy end up.					
Pressure.	Temp. C.	T_d	No. days.	No. thousand oscillations.	Pressure.	Temp. C.	T_u	No. days.	No. thousand oscillations.	
ON GENEVA SUPPORT.										
With bells on.—Regular set to determine gravity.—Knives interchanged.										
in.	°	s.			in.	°	s.			
30	20	1.006344	8	42	30	20	1.006537	8	18	
With bells on.—At high temperatures.										
in.	°	s.			in.	°	s.			
30	35	1.006346	4	22	30	35	1.006540	4	19	Knives interchanged.
					30	34	1.00652[illegible]	2	6	Knife No. 1.
					$2\frac{1}{4}$	38	1.00653[illegible]	1	5	Knife No. 1.
					$1\frac{1}{4}$	34	1.00652[illegible]	1	2	Knife No. 1.
With bells on.—At various pressures.—Knife No. 1 at heavy end.										
in.	°	s.			in.	°	s.			
30	18	1.006349	3	16	30	10	1.006539	3	13	
					29	11	1.006560	2	7	
					27	10	1.006537	1	4	
					$22\frac{1}{2}$	11	1.006549	1	2	
15	18	1.006337	2	20	15	10	1.006545	1	3	
					$7\frac{1}{2}$	10	1.006533	2	10	
5	20	1.006336	1	15						
$1\frac{1}{2}$	20	1.006342	1	21						
					1	10	1.006524	1	11	
					$\frac{3}{4}$	9	1.006530	4	53	
					$\frac{4}{10}$	9	1.006532	4	73	
ON REPSOLD SUPPORT.										
One day, knife No. 1 at heavy end; two days at light end.										
in.	°	s.			in.	°	s.			
30	14	1.006355	3	4	30	14	1.006516	3	4	
ON STIFFEST SUPPORT.										
Knife No. 1 at light end.										
in.	°	s.			in.	°	s.			
30	14	1.006366	1	2	30	15	1.006544	1	2	

The reductions in the above table have been made with the *à priori* constants of atmospheric effect, and with the coefficient of expansion, $18^{\mu}.38$ per degree Centigrade. A correction of $+ 73 \times 10$ for inequality of knives has been applied to the three last results with heavy end up at high temperatures.

The agreement of the several experiments of the regular set is shown by the following table of the observed periods (uncorrected for the effect of the cylinders and of flexure):

Hoboken.—Regular set.

	Obs. T_d	Obs. T_u
	s.	*s.*
	1.006352	1.006559
	352	560
	361	546
	360	558
	358	544
	350	539
	363	551
	356	534
Mean	1.006357	1.006548

The agreement of the several experiments of the half set at high temperatures is shown in the following table:

Hoboken.—Half set at high temperatures.

Obs. T_d	Obs. T_u
s.	*s.*
1.006533	1.006709
541	713
536	708
534	708
	706
	716
	708
	709

It will be seen that the mean results of the experiments of the regular set agree as well as could be expected with those made at high temperatures; which shows both that the coefficient of expansion is correct, and also that the correction for bell-glasses is happily not in error. Further to compare these two sets of experiments we may calculate the mean T^2 which is to be used when the reduction is made on the principle of the reversible pendulum, and also the mean T^2 which is to be used when the reduction is made on the principle of the invariable pendulum. Denoting the former by [T^2 Rev.] and the latter by [T^2 Inv.] we have algebraically

$$[T^2 \text{ Rev.}] = \frac{T_d^2 h_d - T_u^2 h_u}{h_d - h_u} \qquad [T^2 \text{ Inv.}] = \frac{T_d^2 h_d + T_u^2 h_u}{h_d + h_u}$$

The values will be

	[T^2 Inv.]	[T^2 Rev.]
	s^2	s^2
From regular set	1.012846	1.012410
From ½ set at high temperatures	1.0128[illegible]	1.012[illegible]
Difference, in seconds, per day	0^s.[illegible]	0^s.[illegible]

We may next examine the experiments at various pressures. Their concordance with one another is very good; but the reader will hardly desire to see the single experiments set forth here; particularly as all the means are given in the tables at the end of this paper. We may exhibit [T^2 Inv.] and [T^2 Rev.] for pairs of experiments under nearly the same pressure but in reversed positions. But these results can have no value in the determination of gravity; nor can they be expected to accord with those just obtained, for, not to speak of the non-interchange of knife-edges, the observations in the two positions were taken at intervals of months, under conditions very different in many respects, and were never intended to be used for obtaining gravity, but only to show the variation of the period and decrement of arc with the pressure.

	[T^2 Inv.]	[T^2 Rev.]
From experiments at 30 inches down and 30 inches up.....	1.012855$(s)^2$	1.012425$(s)^2$
From experiments at 15 inches down and 15 inches up.....	1.012842	1.012376
From experiments at 5 inches down and $7\frac{1}{2}$ inches up.....	1.012832	1.012389
From experiments at $1\frac{1}{2}$ inches down and 1 inch up.......	1.012836	1.012424
From mean of experiments at $\frac{1}{2}$ and $\frac{1}{4}$ inch down and experiments at $\frac{4}{10}$ inch up..................................	1.012841	1.012415

The agreement is sufficient to show that the coefficient of atmospheric pressure is well determined.

We pass now to the experiments on the Repsold support and on the stiffest support. These were not very carefully made, being only intended to show that the effect of flexure was really what calculation had predicted. Upon these supports the knife-edge rested on steel instead of glass, and consequently the reductions on the principle of the reversible pendulum are not comparable with results of experiments on the Geneva support until the slip shall have been measured on both stands. The reductions on the principle of the invariable pendulum are, however, comparable.

	[T^2 Inv.]	[T^2 Rev.]	
From experiments on stiffest support	1.012879$(s)^2$	1.012512$(s)^2$	367
From experiments on Repsold support	1.012855	1.012514	341
From regular set	1.012846 (30)		
Δ Repsold and stiffest support, in seconds, per day.	$1^s.0$	$0^s.1$	
Δ Repsold support and regular set	$0^s.6$		

Corr. for effect of Bach!

We may now proceed to compare the results at the European stations. First, the results of the single experiments at each station will be compared, with only such corrections as vary from day to day. Next, the values of [T^2 Inv.] and [T^2 Rev.] will be given for each station after correcting them for the wear of the edges so as to reduce them to what they would have been for Paris, just after the knives had been ground. Lastly, we shall use the determinations by the principle of the reversible pendulum of the absolute length of the seconds' pendulum (still uncorrected for slip) at each station, in combination with the determinations of relative gravity on the principle of the invariable pendulum, in order to find four independent values of the length of the seconds' pendulum at each station. These, being corrected for elevation, will be comparable with the results of other experiments. The Hoboken experiments on the Geneva support cannot be used to determine [T^2 Rev.] until the slip has been ascertained; those made at Hoboken, on the Repsold support, must, therefore, be used in place of them for the present.

Paris.—Periods of oscillation.

	Heavy end down.	Heavy end up.
	s.	s.
	1.006051	1.006192
	048	210
	047	190
	048	195
	048	185
	052	185
	062	208
	053	213
Mean..........	1.006051	1.006197

The results on the last two days at Paris were affected by excessive damp.

Berlin.—Periods of oscillation.

	Heavy end down.	Heavy end up.
	s.	*s.*
	1.005899	1.006052
	901	037
	896	026
	890	036
	901	037
	896	034
	899	046
	895	034
Mean..........	1.005898	1.006038

Kew.—Periods of oscillation.

	Heavy end down.	Heavy end up.
	s.	*s.*
	1.005935	1.006077
	30	68
	29	73
	27	64
	25	70
	31	71
	28	64
	31	66
	29	70
	30	66
		59
		63
		72
		73
		66
		74
		77
		67
Mean..........	1.005930	1.006069

The following table shows the results of Paris, Berlin, and Kew in comparison:

	[T^2 Inv.]	[T^2 Rev.]	Diff.
Paris	1.0121042$(s)^2$	1.0116956$(s)^2$	4086
Berlin	1.0117925	1.0113934	3991
Kew	1.0118560	1.0114548	4012

These differences involve double the square roots of the sums of the squares of the errors of the periods of oscillation with heavy end down and with heavy end up. Therefore, the Berlin and Kew differences are remarkably close together, while that at Paris is rather divergent. The experiments at Hoboken show a wide discrepancy in this difference, owing to the use of the Geneva support. Consequently the [T Rev.] cannot be used.

Now, reducing the values of [T^2 Rev.] to mean solar time and dividing into the length of our pendulum at Paris we obtain the length of the seconds' pendulum at the several stations. The ratios of the [T^2 Rev.] at the different stations, being inversely as the length of the seconds' pendulum, may be used to obtain the length of the seconds' pendulum at any station from the value deduced from the [T^2 Rev.] at any other station. So that at each station we shall have not only a value of the seconds' pendulum deduced from the [T^2 Rev.] of that station but also two other values deduced from the [T^2 Rev.] of the two other stations. These three values will have nearly equal weight. They are as follows:

For Paris.

	m.
From Paris observations	.9939390
From Berlin observations	02
From Kew observations	20
Mean	.9939337
Reduction to sea-level	+ 163
Seconds' pendulum at Paris reduced to sea-level	.9939500

For Berlin.

	m.
From Paris observations	.9942362
From Berlin observations	.9942452
From Kew observations	.9942382
Mean	.9942399
Reduction to sea-level	+ 83
Seconds' pendulum at Berlin reduced to sea-level	.9942482

For Kew.

	m.
From Paris observations	.9941757
From Berlin observations	.9941830
From Kew observations	.9941740
Mean	.9941776
Reduction to sea-level	+ 14
Seconds' pendulum at Kew reduced to sea-level	.9941790

No comparison can be attempted between these results and those of previous experimenters until the former have been corrected for the slip of the knives and the latter have been reduced anew according to modern methods. These matters will be treated in the second part of this report with results which will be found satisfactory.

The pendulum at Geneva was virtually a different pendulum from that used at the other stations, because of the accident that befel it after the Geneva experiments. These experiments can, therefore, only be reduced on the principle of the reversible pendulum. The concordance of the single experiments is shown in the following table:

GENEVA.

T_d^2	T_u^2
1.012599	1.012814
581	775
589	797
607	793
593	789
580	767
582	763
598	783

The resulting value of the length of the seconds' pendulum after correcting for flexure in the manner explained under that heading is

Length of seconds' pendulum at Geneva.

	m.
Experiments of Coast Survey	0.993556
Professor Plantamour's result	0.993550

The appended tables show the details of the experiments at the different stations.

Respectfully submitted.

C. S. PEIRCE,
Assistant.

UNITED STATES COAST SURVEY.—PENDULUM AT HOBOKEN.

HEAVY END DOWN.

True temp	Date.	Chr.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
				h. m. s.	s.								
18°.90	June 11	380	D	15 33 13.345									
			A	49 3.937	950.592	.056	950.536	944½	1.0063913	− 57	−60	−363	1.0063433
			B	16 16 18.874	1634.937	.048	1634.889	1624½	3952	− 57	−60	−363	472
			B	17 8 24.798	3125.924	.031	3125.893	3106	4047	− 57	−60	−363	567
								(5675)					(1.0063518)
19.[illegible]1	June 14	387	A	13 59 53.591									
			D	14 14 44.815	891.224	.051	891.178	885½	1.0064069	−288	−63	−377	1.0063341
			B	38 14.865	1410.050	.043	1410.007	1401	4289	−288	−63	−377	561
			D	15 30 14.824	3119.959	.035	3119.924	3100	4273	−288	−63	−377	545
								(5386½)					(1.0063516)
19.82	June 15	380	D	15 22 38.937									
			B	35 12.769	753.832	.044	753.788	749	1.0063923	− 68	−51	−436	1.0063368
			D	59 17.029	1444.260	.046	1444.215	1435	4214	− 68	−51	−436	659
			D	16 45 50.878	2793.849	.027	2793.822	2776	4200	− 68	−51	−436	645
								(4960)					(1.0063607)
20.25	June 16	380	B	14 28 36.999									
			B	43 4.571	867.572	.050	867.522	862	1.0064056	− 68	−32	−472	1.0063484
			B	15 10 53.272	1668.701	.049	1668.651	1658	4240	− 68	−32	−472	668
			D	59 55.059	2941.787	.029	2941.758	2923	4174	− 68	−32	−472	602
								(5443)					(1.0063604)
							Change of knife-edges.						
19.78	June 17	202	B	15 54 32.711									
			D	16 10 51.967	979.256	.055	979.200	973	1.0063720	+142	−54	−432	1.0063376
			B	16 36 40.868	1548.901	.044	1548.857	1539	4048	+142	−54	−432	704
			D	17 26 28.880	2988.012	.029	2987.982	2969	3934	+142	−54	−432	590
								(5481)					(1.0063584)
20.07	June 19	202	B	14 15 25.293									
			B	29 40.743	855.450	.050	855.399	850	1.0063525	+240	−39	−456	1.0063270
			B	55 18.561	1537.818	.047	1537.771	1528	3946	+240	−39	−456	691
			D	15 45 54.822	3036.261	.038	3036.223	3017	3717	+240	−39	−456	462
								(5395)					(1.0063497)
(19.68)	June 20	202	D	15 4 14.856									
			D	20 0.891	946.035	.053	945.981	940	1.0063631	+240	−59	−416	1.0063396
19.21	June 22	202	C	14 31 41.930									
			A	44 53.972	792.042	.047	791.995	787	1.0063471	+336	−46	−386	1.0063375
			B	15 12 33.042	1659.070	.050	1659.020	1648½	3814	+336	−46	−386	718
			C	16 1 24.137	2931.095	.029	2931.066	2912½	3745	+336	−46	−386	649
								(5348)					(1.0063630)
19.41	June 29	202	D	10 7 24.685									
			B	21 10.950	826.265	.048	826.217	821	1.0063547	+426	−53	−404	1.0063516
			A	46 1.924	1490.974	.046	1490.927	1481½	3636	+426	−53	−404	605
			B	11 30 27.294	2665.370	.030	2665.340	2648½	3582	+426	−53	−404	551
								(4951)					(1 0063561)

Ther. I seems to have received no correction so that T_d should be increased by 42×10^{-7} and T_u by 85×10^{-7}

Correct. Temp

18.91
19.70
19.84
20.24
19.85
20.11
(19.52)
19.17
19.38

UNITED STATES GEODETIC SURVEY.—PENDULUM AT HOBOKEN, 1877.

HEAVY END UP.

Date.	Chr.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp.	Period, corrected.
			h. m. s.	s.								
June 11	202	C	14 11 38.249									
		D	18 15.367	397.118	.020	397.097	394½	1.0065830	+ 94	−139	−306	1.0065479
		A	29 24.260	668.893	.016	668.877	664½	5872	+ 94	−139	−306	5521
		A	48 31.797	1147.537	.010	1147.526	1140	6019	+ 94	−139	−306	5668
							(2199)					(1.0065590)
June 14	387	B	16 13 38.972									
		C	19 53.944	374.972	.021	374.951	372½	1.0065820	−288	−141	−323	1.0065068
		C	30 5.987	612.033	.018	612.015	608	6038	−288	−141	−323	5286
		D	56 20.856	1574.869	.016	1574.853	1564½	6594	−288	−141	−323	5842
							(2545)					(1.0065596)
June 15	380	A	14 15 1.901									
		A	20 22.026	320.125	.017	320.108	318	1.0066299	− 68	−114	−378	1.0065739
		B	30 52.683	630.657	.018	630.639	626½	6069	− 68	−114	−371	5516
		C	53 24.044	1351.361	.013	1351.348	1342½	5909	− 68	−114	−364	5363
							(2287)					(1.0065457)
June 16	380	B	13 22 44.800									
		B	28 45.183	360.383	.019	360.374	358	1.0066304	− 68	− 73	−399	1.0065764
		C	39 27.931	642.748	.018	642.729	638½	6233	− 68	− 73	−394	5698
		D	14 1 19.039	1351.108	.012	1311.096	1302½	5998	− 68	− 73	−391	5466
							(2299)					(1.0065577)
						Change of knife-edges.						
June 17	202	B	14 47 8.665									
		B	53 0.975	352.310	.018	352.292	350	1.0065497	+142	−119	−368	1.0065152
		D	15 5 0.698	719.723	.019	719.703	715	5780	+142	−119	−368	5435
		D	25 36.797	1236.099	.011	1236.088	1228	5861	+142	−119	−368	5516
							(2293)					(1.0065435)
June 19	202	"XOXO"	16 11 16.343									
		D	17 40.855	384.512	.021	384.491	382	1.0065222	+240	− 97	−386	1.0064979
		A	29 19.952	699.097	.019	699.078	694½	5927	+240	− 97	−386	5684
		D	48 43.042	1163.090	.010	1163.079	1155½	5591	+240	− 97	−386	5348
							(2232)					(1.0065389)
June 20	202	A	14 7 40.020									
		A	17 48.012	607.992	.017	607.975	604	1.0065815	+240	−136	−345	1.0065574
		B	38 36.656	1248.644	.011	1248.633	1240½	5564	+240	−136	−345	5323
							(1844½)					
June 22	202	D	16 22 58.721									
		C	27 32.028	273.307	.012	273.295	271½	1.0066110	+336	−112	−321	1.0066013
		C	37 27.926	595.898	.015	595.883	592	5586	+336	−112	−321	5489
		B	57 35.297	1207.371	.011	1207.360	1199½	5531	+336	−112	−321	5434
							(2063)					(1.0065512)
June 29	202	A	9 6 0.137									
		B	10 58.584	298.447	.016	298.431	296½	1.0065133	+426	−121	−335	1.0065103
		B	21 44.824	646.240	.019	646.220	642	5734	+426	−121	−335	5704
		D	40 39.187	1134.363	.012	1134.351	1127	5231	+426	−121	−335	5201
							(2065½)					(1.0065343)

UNITED STATES GEODETIC SURVEY.—PENDULUM AT HOBOKEN, 1877.—SWINGS AT LOW PRESSURES.

HEAVY END DOWN.

[All temperatures in low-pressure and high-temperature experiments are to be corrected by $+0^{\circ}.3$ C.]

On the contrary apply $-0^{\circ}.1$ C

Date.	Pressure.	Temperature.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Period, corrected.
	In.	°		*h. m. s.*	*s.*						
Sept. 25	30.25	67.6	A	10 48 26.194							
			A	11 7 45.625	1159.431	.044	1159.387	1152	1.0064123	+ 13	1.0064136
			A	11 27 25.156	1179.531	.022	1179.509	1172	4258	+ 13	271
			A	11 57 54.835	1829.679	.014	1829.665	1818	4164	+ 13	177
								(4142)			(1.0064173)
Sept. 26	15.19	66.1	A	10 48 22.909							
			A	11 44 26.903	963.994	.042	963.952	958	1.0062129	+ 13	1.0062142
			A	11 41 32.780	2225.877	.053	2225.824	2212	2495	+ 13	508
			A	12 23 24.403	2511.623	.019	2511.604	2496	2516	+ 13	529
								(5666)			(1.0062457)
Sept. 27	30.25	68.5	A	8 50 47.365							
			A	9 1 15.420	628.055	.032	628.023	624	1.0064471	+ 13	1.0064484
			A	9 19 34.448	1099.028	.030	1098.998	1092	4084	+ 13	097
			A	9 58 14.881	2320.433	.026	2260.407	2246	4145	+ 13	158
								(3962)			(1.0064194)
Sept. 29	0.50	68.3	D	10 7 12.887							
			B	10 38 21.285	1868.398	.146	1868.252	1857	1.0060592	+128	1.0060720
			A	11 8 15.753	1794.468	.127	1794.341	1783½	0785	+128	0913
			B	11 37 59.106	1783.353	.112	1783.241	1772½	0598	+128	0726
			B	12 5 17.109	1638.003	.098	1637.905	1628	0842	+128	0970
			D	12 35 59.323	1842.214	.091	1842.123	1831	0748	+128	0876
			D	13 7 48.930	1909.607	.085	1909.522	1898	0705	+128	0833
			B	13 42 28.560	2079.630	.082	2079.548	2067	0706	+128	0834
			D	14 12 50.654	1822.094	.061	1822.033	1811	0922	+128	1050
			D	14 57 16.812	2666.158	.076	2666.082	2650	0687	+128	0815
			D	15 24 6.555	1609.745	.040	1609.705	1600	0656	+128	0784
			B	15 55 2.794	1856.237	.042	1856.195	1845	0677	+128	0805
			D	16 25 57.012	1854.218	.037	1854.181	1843	0677	+128	0805
			D	17 9 32.831	2615.819	.042	2615.777	2600	0681	+128	0809
			B	17 29 4.907	1172.076	.017	1172.059	1165	0592	+128	0720
			D	18 26 24.687	3439.780	.044	3439.736	3419	0649	+128	0777
			B	18 57 57.142	1892.455	.020	1892.435	1881	0792	+128	0920
			A	19 34 33.931	2196.789	.020	2196.769	2183½	0767	+128	0895
								(33834½)			(1.0060879)
Oct. 1	4.99	68.3	A	8 18 49.961							
			D	8 49 38.866	1848.905	.121	1848.784	1837½	1.0061410	+ 9	1.0061419
			B	10 19 42.950	5404.084	.166	5403.918	5371	1288	+ 9	297
			B	10 51 26.588	1903.638	.028	1903.610	1892	1364	+ 9	373
			D	11 23 1.196	1894.611	.021	1894.590	1883	1535	+ 9	544
			D	12 11 51.086	2929.890	.012	2929.878	2912	1394	+ 9	403
			B	12 31 21.201	1170.115	.009	1170.106	1163	1101	+ 9	110 rej.
								(13895½)			(1.0061379)
Oct. 1	1.50	68.0	B	12 48 29.189							
			B	13 24 56.944	2247.755	.161	2247.594	2234	1.0060850	+ 9	1.0060859
			B	13 53 7.292	1680.348	.100	1680.248	1670	1365	+ 9	1374
			D	14 28 10.769	2103.477	.096	2043.381	2050	0760	+ 9	0769
			D	15 1 39.367	2068.598	.074	2068.524	2056	0914	+ 9	0923
			D	15 33 22.956	1903.589	.055	1903.534	1892	0962	+ 9	0971
			D	16 1 31.528	1748.572	.040	1748.532	1738	0596	+ 9	0605
			B	16 41 46.912	2355.384	.045	2355.339	2341	1252	+ 9	1261
			D	17 6 24.907	1477.995	.020	1477.975	1469	1095	+ 9	1104
			B	18 16 19.357	4194.450	.046	4194.404	4169	0935	+ 9	0944
			A	18 41 44.122	1524.765	.012	1524.753	1515½	0990	+ 9	0999
								(21115½)			(1.0060999)

UNITED STATES GEODETIC SURVEY.—PENDULUM AT HOBOKEN.—SWINGS AT HIGH TEMPERATURE.

HEAVY END UP.

Date.	Pressure.	Temp. F.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscilla-tions.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp. to 35° C.	Period, corrected.
1878.	*In.*	°		*h. m. s.*	*s.*								
Apr. 24	1.24	....	D	11 52 17.358									
		92.5	B	12 5 25.088	787.730	.042	787.688	783	1.0059872	+ 8	......	+100	1.0059980
		92.6	D	12 18 20.755	775.667	.035	775.632	771	60078	+ 8	......	+ 91	60177
		92.0	D	12 30 31.137	730.382	.028	730.354	726	59972	+ 8	......	+128	60108
								(2280)					(1.0060088)
Apr. 26	2.25	....	B	11 3 13.343									
		99.3	B	11 24 0.896	1247.553	.038	1247.515	1240	1.0060605	+ 8	......	−244	1.0060369
		100.8	D	11 53 54.768	1793.872	.034	1793.838	1783	60785	+ 8	......	−316	60477
		101.5	B	12 24 6.767	1811.999	.022	1811.977	1801	60949	+ 8	......	−352	60605
								(4824)					(1.0060497)
Apr. 24	29.888	....	B	10 8 11.240									
		89.9	D	10 19 46.835	695.595	.030	695.565	691	1.0066064	+ 8	−59	+178	1.0066191
		90.5	B	12 32 16.830	749.995	.012	749.983	745	66886	+ 8	−59	+157	66992
		91.0	D	10 50 8.937	1072.107	.006	1072.101	1065	66696	+ 8	−59	+135	66780
								(2501)					(1.0066670)
Apr. 26	29.886	....	D	8 49 46.929									
		96.5	B	8 57 4.888	437.959	.020	437.939	435	1.0067563	+ 8	−60	− 78	1.0067459
		97.0	D	9 5 31.263	506.375	.011	506.314	503	66885	+ 8	−60	−100	66733
		97.8	D	9 25 41.330	1210.067	.010	1210.057	1202	67030	+ 8	−60	−135	66843
								(2140)					(1.0066936)
Apr. 26	29.890	....	D	9 28 27.344									
		97.8	D	9 33 9.253	281.909	.018	281.891	280	1.0067536	+ 8	−60	−135	1.0067349
		97.9	D	9 42 12.885	543.632	.016	543.616	540	66963	+ 8	−60	−135	66776
		98.6	B	10 5 51.382	1418.497	.015	1418.482	1409	67296	+ 8	−60	−164	67080
								(2229)					(1.0067039)
Apr. 30	30.000	....	D	9 4 59.956									
		102.8	B	9 12 1.115	421.859	.019	421.840	419	1.0067780	− 9	−85	−328	1.0067358
		102.8	B	9 21 49.060	587.945	.014	587.931	584	67312	− 9	−85	−328	66890
		102.7	D	9 44 53.336	1384.276	.010	1384.266	1375	67389	− 9	−85	−328	66967
								(2378)					(1.0067017)
Apr. 30	30.007	....	B	9 47 45.072									
		102.7	D	9 55 33.220	468.448	.027	468.121	465	1.0067263	− 9	−87	−328	1.0066839
		102.5	D	10 5 27.224	594.004	.016	593.988	590	67593	− 9	−87	−320	67177
		102.3	B	10 26 40.780	1273.556	.011	1273.545	1265	67549	− 9	−87	−313	67140
								(2320)					(1.0067060)
May 2	29.897	....	D	11 05 33.347									
		95.2	D	11 13 58.729	505.382	.025	505.357	502	1.0066873	−45	−61	− 28	1.0066739
		95.4	D	11 28 44.682	885.953	.016	885.937	880	67466	−45	−61	− 36	67324
		95.6	B	11 46 26.759	1062.077	.006	1062.071	1055	67024	−45	−61	− 43	66875
								(2437)					(1.0067007)
May 2	29.892	....	D	11 49 35.047									
		98.0	D	11 55 31.417	356.370	.019	356.351	354	1.0066412	−45	−60	− 64	1.0066243
		96.3	D	12 03 26.619	475.202	.013	475.189	472	67563	−45	−60	− 71	67387
		96.7	D	12 22 53.429	1166.810	.012	1166.798	1159	67282	−45	−60	− 85	67092
								(1985)					(1.0067011)
May 10	29.899	....	D	10 11 18.725									
		91.1	D	10 17 37.254	378.529	.017	378.512	376	1.0066809	+28	−62	+135	1.0066910
		91.5	D	10 27 11.098	573.844	.013	573.831	570	67211	+28	−62	+114	67291
		92.3	D	10 51 59.023	1487.925	.011	1487.914	1478	67077	+28	−62	+ 85	67128
								(2424)					(1.0067132)
May 10	29.892	....	B	10 55 38.701									
		92.5	D	11 3 12.735	454.034	.022	454.012	451	1.0066785	+28	−60	+ 78	1.0066831
		92.5	D	11 13 22.820	610.085	.014	610.071	606	67178	+28	−60	+ 78	67224
		92.8	D	11 33 30.913	1208.093	.010	1208.083	1200	67358	+28	−60	+ 64	67390
								(2257)					(1.0067[illegible]4)
May 11	29.968	....	B	9 42 29.411									
		93.7	D	9 48 56.995	387.584	.016	387.568	385	1.0066701	+28	−78	+ 28	1.0066679
		94.0	D	9 58 28.845	631.850	.012	571.838	568	67570	+28	−78	+ 21	67541
		94.4	B	10 21 35.105	1386.260	.010	1386.250	1377	67175	+28	−78	0	67125
								(2330)					(1.0067153)
May 11	29.980	....	D	10 25 16.895									
		94.4	B	10 32 48.935	452.040	.024	452.016	449	1.0067171	+28	−81	0	1.0067118
		94.3	B	10 42 40.894	591.959	.012	591.947	588	67126	+28	−81	+ 7	67080
		94.0	D	11 10 49.181	1688.287	.012	1688.575	1677	67233	+28	−81	+ 21	67201
								(2714)					(1.0067161)

PENDULUM AT HOBOKEN.—SWINGS AT HIGH TEMPERATURE—Continued.

HEAVY END DOWN.

True temp C	Date.	Pressure.	Temp. F.	Method.	Time of transit.	Intervals.	Cor. for arc.	Corr. time.	Number oscillations.	Time, one oscillation.	Rate.	Cor. for pressure.	Cor. for temp. to 35° C.	Period, corrected.
	1878.	*In.*	°		*h. m. s.*	*s.*								
36.01	May 4	29.794	....	D	9 19 15.340									
			96.0	B	9 31 37.201	741.861	.041	741.820	737	1.0065400	−45	−16	− 75	1.0065264
			96.8	D	9 54 29.156	1371.955	.039	1371.916	1363	65414	−45	−16	−108	65245
			97.3	D	10 59 34.625	3905.469	.031	3905.438	3880	65563	−45	−16	−133	65369
									(5980)					(1.0065327)
34.66	May 5	29.868	....	B	9 13 26.830									
			94.0	D	9 26 12.843	766.013	.038	765.975	761	1.0065375	−45	−24	+ 25	1.0065329
			94.2	B	9 49 22.897	1390.054	.036	1390.018	1381	65301	−45	−24	+ 8	65240
			94.8	D	10 54 11.255	3888.358	.028	3888.330	3863	65571	−45	−24	− 17	65485
									(6005)					(1.0065409)
34.96	May 6	30.027	....	D	11 05 53.067									
			95.1	D	11 15 40.900	587.833	.026	587.807	584	1.0065188	+54	−40	− 33	1.0065169
			95.1	B	11 33 32.896	1071.996	.028	1071.968	1065	65427	+54	−40	− 33	65408
			95.1	B	12 30 31.132	3418.236	..030	3418.206	3396	65389	+54	−40	− 33	65370
									(5045)					(1.0065356)
34.75	May 8	29.843	....	B	9 57 19.379									
			96.5	D	10 9 35.213	735.834	.042	735.792	731	1.0065554	−59	−21	− 91	1.0065383
			96.1	D	10 30 23.375	1248.162	.036	1248.126	1240	65533	−59	−21	− 75	65378
			94.0	B	11 23 35.137	3191.762	.034	3191.728	3171	65367	−59	−21	+ 25	65312
									(5142)					(1.0065338)

Reduction of the regular set at Hoboken.

HEAVY END DOWN.

		s.
Mean T_d corrected for rate, temperature, and pressure	=	1.0063565
Correction for cylinder	= −	36
Correction for flexure	= −	89
		1.0063440
Correction for improved constants	−	2
	T_d =	1.0063438

HEAVY END UP.

		s.
Mean T_u corrected for rate, temperature, and pressure	=	1.0065487
Corrected for cylinder	= −	71
Corrected for flexure	= −	39
		1.0065377
Correction for improved constants	= −	3
	T_u =	1.0065374

Reduction of experiments at high temperatures.

HEAVY END DOWN.

The half set.

		s.
Mean T_d corrected for rate and pressure and reduced to 35° C.	=	1.0065357
Reduction to 15° C. Correction for expansion	= −	1850
Correction for atmospheric effect—*First* part	= +	181
Second part	= +	5
Correction for flexure, bells, and cylinder	= −	168
	T_d =	1.0063525
Correction for erroneous temperature	= −	65
		1.0063460

HEAVY END UP.

The half set.

		s.
Mean T_u corrected for rate and pressure, and reduced to 35° C.	=	1.0067097
Reduction to 15° C. Correction for expansion	= −	1850
Correction for atmospheric effect—*First* part	= +	414
Second part	= +	10
Correction for flexure, bells, and cylinder	= −	~~168~~ 209
	T_u =	1.0065503
Correction for erroneous temperature	= −	66
		1.0065~~437~~ 396

Reduction of experiments at high temperatures—Continued.

HEAVY END UP—Continued.

At 30 inches (not included in half set).

		s.
Mean T_u corrected for rate and pressure, and reduced to 35° C.	=	1.0066882
Correction for inequality of knives..................	= +	74
Reduction to 15° C. Correction for expansion	= −	1850
Correction for atmospheric effect—*First* part........	= +	414
Second part	= +	10
Correction for flexure, bells, and cylinder...........	= −	~~168~~ 209
	T_u =	1.0065362
Correction for erroneous temperature................	= −	66
		1.0065~~296~~ 55

At 2.25 inches.

			s.
1878, April 26.	T_u corrected for rate and brought to 35° C ...	=	1.0060497
	Correction for inequality of knives	= +	74
	Correction for expansion from 15° C......	= −	1850
	Correction for atmosp. effect—*First* part..	= +	5935
	Second part.	= +	919
	Correction for flexure, bells, and cylinder.	= −	~~168~~ 209
		T_u =	1.0065407
	Correction for erroneous temperature.....	= −	78
			1.0065~~329~~ 288

At 1.25 inches.

			s.
1878, April 24.	T_u corrected for rate and brought to 35° C...	=	1.0060088
	Correction for inequality of knives.......	= +	74
	Correction for expansion from 15° C......	= −	1850
	Correction for atmosp. effect—*First* part..	= +	6138
	Second part.	= +	1007
	Correction for flexure, bells, and cylinder .	= −	~~168~~ 209
		T_u =	1.0065289
	Correction for erroneous temperature.....	= −	79
			1.0065~~210~~ 169