Nova (1843) designated by any integral magnitude, and comparing these integral estimations (made by the naked eye) with the photometric measures of the same stars, found the light ratio between stars of the 2nd and 3rd magnitudes to be 2.94, between stars of the 3rd and 4th magnitude, 2.44, and between stars of the 4th and 5th magnitude to be 1.75. *(Memoirs R.A.S.*, **47** (1883), 363.)

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Charles S. Peirce and Die Europäische Gradmessung

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Gravity surveys became applicable to geodesy when Richer found that his pendulum clock lost time after transport from Paris to the Equator. Initially, gravity was determined with a ball pendulum, and its intensity was expressed by the length of the simple pendulum that beat seconds. Bohnenberger proposed, but Kater first constructed, a convertible pendulum, a body with two knife-edges, about each of which it was oscillated with identical periods, by adjusting weights on it. By a theorem of Huygens, the length of the seconds pendulum was the distance between the knife-edges divided by the period squared. Bessel used a ball pendulum, but designed a reversible pendulum, geometrically symmetrical and hung by interchangeable knives, thereby eliminating effects of surrounding air and different curvatures of knife-edges. The length of the seconds pendulum was determinable from the periods of oscillation and distances of the centre of gravity from the knife-edges. Messrs. Repsold created a gravity apparatus with a Bessel pendulum hung on a plate supported by a tripod, and about 1864 constructed the first exemplars for the Imperial Academy of Sciences, St. Petersburg and the Swiss Geodetic Commission.

Meanwhile, the European Geodetic Association, founded by Lieutenant General Baeyer, held its First General Conference in Berlin, 1864. A resolution called for gravity surveys and recommended the reversible pendulum. The Second Conference, also in Berlin, 1867, upon a report by M. Hirsch, Director of the Neuchâtel Observatory, of Swiss observations, specifically recommended Repsolds' apparatus. The Third Conference in Vienna, 1871, called for comparisons of instruments and observations with different pendulums at a principal station.

Charles Sanders Peirce was put in charge of pendulum operations of the United States Coast, Survey, 30th November 1872, and ordered a Repsold apparatus which he received in Hamburg, in May, 1875. Peirce than conferred in Berlin with General Baeyer, founder and President of the Royal Prussian Geodetic Institute, who questioned the stability of the Repsold stand. Peirce next went to Geneva and by arrangement with Professor Plantamour, Directeur of the Observatory, swung his pendulum there. He also measured the flexure of the Repsold stand and calculated its effect upon the length of the seconds pendulum.

A special commission on the Pendulum, with Baeyer as Chairman, had been appointed by the Fourth Conference in Dresden, 1874 and met in Paris, 1875, concurrently with the Permanent Commission of the Association. Baeyer declared that Repsolds' apparatus was an unsatisfactory field instrument, and recommended relative determinations with invariable pendulums at temporary stations. Bruhns and Peters, Directors of the Leipzig and Kiel Observatories, respectively, agreed with Baeyer, but Hirsch, and Professor von Oppolzer of Vienna, defended Repsolds' apparatus. Peirce, in attendance by invitation, reported his measurements of flexure at Geneva. The Special Commission resolved that Repsolds' apparatus, with slight modifications, met requirements for gravity surveys. It recommended that different pendulums be swung at the Imperial Standards Office in Berlin, near Bessel's historic station, and expressed support for Peirce's plan to swing his pendulum at historic stations in Paris, Berlin and London. The Permanent Commission, with Peirce present, approved these decisions.

Peirce also measured flexure in Paris and Berlin, and swung his pendulum in Paris, Berlin and at Kew Observatory. After his return to America, he experimented at the Stevens Institute, on flexure, and on pendulum oscillations on a "Geneva Support" in a vacuum chamber which he had designed.

The Permanent Commission met in Brussels, 1876 and again discussed pendulum apparatus. Baeyer, assisted by Bruhns, reported Peirce's measurements of flexure of the Repsold stand in Berlin. Hirsch and von Oppolzer replied that the observed flexure applied to particular tripods which lacked strength.

The Fifth Conference met in Stuttgart, 1877. On invitation, Peirce submitted a memoir on the influence of the flexibility of the support on the oscillation of a pendulum, and with credentials from Superintendent Patterson, participated in the Conference. Peirce's results were accepted by two notes of von Oppolzer; by a memoir of Cellérier, who after conversation with Peirce had developed the theory by a different method; and by Plantamour's memoir which reported experimental confirmation of the Peirce-Cellérier theory of flexure. Plantamour distinguished between statical and dynamical measures, in consequence of which Peirce made further investigations and modified his original view that the difference was negligible. Peirce sent to the Permanent Commission in Hamburg, 1878, a report on work subsequent to the Stuttgart Conference. His memoir, "Measurements of gravity at initial stations in America and Europe". dated 13th December 1878, was published as an appendix to the Superintendent's Report for 1875-76. It received special notice at the Munich Conference in 1880, and is listed as a basic monograph on the pendulum in *Encyklopädie der mathematischen Wissenschaften*, Band IV, 1904.

M. Faye, President of the Bureau of Longitudes, Paris, proposed at Stuttgart that flexure be eliminated by swinging simultaneously two similar pendulums from one support, with equal amplitudes and in opposite phases. Hirsch criticized this procedure, but Peirce subsequently analyzed the method in a paper, "On a method of swinging pendulums for the determination of gravity, proposed by M. Faye." Peirce declared that the suggestion was as sound as it was brilliant, and depicted a support for two pendulums. Copies of his paper in Silliman's Journal were sent to Europe, and Faye reported on it to the Academy of Siences, Paris, and Baeyer had copies distributed to the Permanent Commission in Geneva, 1879. M. Hirsch questioned the mathematical basis of Peirce's analysis, and Plantamour and Cellérier were appointed to report on Faye's proposal to the Sixth Conference in Munich, 1880. Cellérier there submitted a general analysis based upon Peirce's equations, but proposed that flexure be corrected from observations of an auxiliary pendulum, and this procedure was recommended by Plantamour and Cellérier. In Paris, Peirce had measured the flexure of supports used by Borda and Biot in their determinations of gravity, and on 14th June 1880 reported to the Academy of Sciences corrections to those values for hydrodynamical, viscous and flexures effects. He was called home by the fatal illness of his father, before the Conference, but reported to it by a letter to Faye in which he described an invariable, reversible pendulum of cylindrical form.

At Pennsylvanian stations, Peirce experimented on the effect of enclosures on oscillations of a pendulum, and demonstrated that oscillations are quickly damped if pendulums are swung on rollers. He determined the ellipticity of the earth from corrected observations with Kater invariable pendulums. He swung two pendulums simultaneously from the same support, and at a conference on gravity in Washington, 1882, advanced this method of eliminating flexure. The Coast and Geodetic Survey constructed four Peirce invariable, reversible pendulums of cylindrical form, and with them Peirce established an initial base at the Smithsonian Institution in 1883. That year he measured the flexure of a pier for pendulum observations at Kew Observatory, with a noddy, an inverted pendulum with adjustable weight, and gave a mathematical theory of this method. He reported the effect of the flexure of a pendulum on its period of oscillation, but this received attention only after Kühnen rediscovered it in 1894 at the Geodetic Institute, Potsdam, and Lorenzoni, Helmert and Almansi offered new theories for it. Peirce also derived the effect of unequal temperatures upon the pendulum. His comprehensive report, which contained comparisons between experiment and Stokes' theory of viscous resistance to the motion of a cylinder, submitted 20th November 1889, remained unpublished and is missing.

Professor von Oppolzer, by direction of the Permanent Commission at The Hague, 1882, reported on gravity apparatus to the Seventh Conference in Rome, 1883. He fully reported Peirce's contributions, but described Faye's method of eliminating flexure as impracticable, and recommended Cellérier's procedure with an auxiliary pendulum.

Meanwhile, Major von Sterneck of Austria-Hungary developed an apparatus for relative determinations with a short pendulum. In view of difficulties of absolute determinations in this field, von Sterneck's apparatus was generally adopted. In order to test pendulums for invariability, two or four of them were hung on a stand. At Potsdam, corrections for flexure were determined from the effect of an oscillating pendulum upon another initially at rest. Schumann gave a mathematical theory for the method and cited Peirce's paper for its mathematical methods. In a later mathematical paper, Ph. Furtwängler remarked that the disturbances predicted for "Faye's double pendulum" had been found not to occur.

During a gravity survey of Holland, 1913-21, Vening Meinesz swung two pendulums of a Stückrath four-pendulum apparatus in antiphase and thereby nullified the effect of mobility of the soil. The Faye-Peirce method of swinging mendulums was then widely adopted, thus confirming a prediction made by Peirce to Superintendent Patterson, in July, 1879, "The method proves to be perfectly sound in theory, and as it would greatly facilitate the work it is probably destined eventually to prevail."

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