APPENDIX No. 14.

ON THE FLEXURE OF PENDULUM SUPPORTS.

By C. S. PEIRCE, Assistant.

HISTORICAL.

The fact that the rate of a pendulum might be largely influenced by the elastic yielding of its support was first pointed out by Dr. Thomas Young in his article on Tides in the Encyclopædia Britannica, where he gave a correct mathematical analysis of the problem. Kater made use of the *noddy*, or inverted pendulum of Hardy, to assure himself that its support was sufficiently steady.

Hardy's noddy is a pendulum turning with a reed spring and provided with an adjustable bob. It differs from an ordinary pendulum, first, in being upside down, that is, having its center of mass above its point of support; and second, in having a spring so strong as to act a little more strongly than gravity. The force tending to bring the pendulum to the vertical is then the excess of the force of the spring over the moment of gravity. In this way the noddy is easily adjusted so as to have the same period of oscillation as the pendulum used to determine gravity, while its moment of inertia is very small. In a note at the end of this paper I give the mathematical analysis of this state of things, from which it will be seen that Kater might have constructed his noddy in such a manner as to detect any amount of flexure sufficient to have a serious effect upon the period of his pendulum.

Bessel, at the end of §3 of his great memoir on the length of the seconds' pendulum at Königsberg, states that he also used Hardy's noddy, and that he swung his pendulum again after stiffening the support. He adds that the effect on the period would probably be the same for his long pendulum as for his short one—a very just remark—which made it less necessary for him to attend to the rigidity of the stand.

The construction of English pendulum supports, that of Bassevi, for example, shows that in that country this source of error was never overlooked. It is noticed even in brief accounts in English of the process of measuring gravity. Thus, a writer in the Encyclopædia Britannica proposed to make use of two different reversible pendulums of the same form but of different weights, in order to take account of the flexure, an idea lately borrowed by M. Cellerier.

When the reversible pendulum came into use the study of the writings of the older observers seems to have been neglected,* and the grave errors due to flexure were never suspected until Albrecht found a value of gravity at Berlin differing by nearly 2 millimeters from that of Bessel. So little was the true cause of this discrepancy at first suspected that it was paradoxically attributed to the neglect of a buoyancy correction.

In 1875, however, General Baeyer gravely suspected that the period of a pendulum swinging upon a Repsold tripod was affected by the oscillation of the latter, and in a circular addressed to the members of the committee on the pendulum of the International Geodetic Congress, he wrote: "The necessity of suspending the pendulum from a stand is a source of error, since a pendulum swinging on a stand sets the latter into oscillation and so influences the rate of the former. The effect could be diminished by the use of a shorter pendulum and smaller stand; but whether it would be rendered entirely insensible is open to question."

*Thus, Bessel's idea of directly measuring the position of the center of mass was supposed by the Swiss *savans* to belong to M. Cellerier.

It was at this time that I first received the Repsold apparatus from the makers, of whom it had been ordered two years before, on the occasion of my first being charged with the pendulum operations of the Coast Survey. Becoming acquainted with General Baeyer's doubts, I determined to settle the question by measuring the flexibility of the Repsold tripod at the earliest opportunity. This I did at Geneva, where, though I only made a rough measurement, I found that the flexure was fully sufficient to account for the discrepancy between the determinations of Bessel and of Albrecht.

On September 25 of the same year I communicated my result to the standing committee of the Geodetical Congress. At the same sitting the reports of the different members of the pendulum committee were read. Dr. Bruhns said: "The question whether the stand is set into oscillation, and whether the rate of the pendulum is influenced thereby is, in my opinion, well worth investigation. But I should suppose that the stand could be made so stiff as to eliminate this source of error for a pendulum used only as a relative instrument." The views of M. Hirsch, who is so much occupied with the going of time-keepers, are interesting. He said: "The fear that the tripod of suspension may also enter into oscillation, unless it be a fact established by direct observations, seems to me unfounded. Indeed, it cannot be supposed that there are any true oscillations of a body of such a form resting on three points. Besides, the movement of the pendulum whose mechanical moment (*moment mécanique*) is slight on account of its small velocity, could only be communicated to the tripod by the friction of the knife on the supporting plane. Now, this friction is insignificant, as the slowness of the decrement of the amplitude shows, this being almost entirely due to the resistance of the air." It may be observed that the rolling friction of the knife edge is, in truth, very slight, but the amount of the sliding friction is sufficient to hold the knife in place on the supporting plane. Dr. von Oppolzer, the designer of the Repsold tripod in its definitive form, said that the construction of the stand rendered any serious flexure *a priori* improbable; but he did not support this opinion by any calculations.

During the spring of 1876, having already measured the flexibility of the tripod in Paris, I remeasured it in Berlin, where my experiments were witnessed by General Baeyer and a party of gentlemen attached to the Prussian Survey.

In October, 1876, at the meeting of the standing committee of the International Geodetical Union at Brussels, the result of my experiments was announced by General Baeyer. M. Hirsch described certain experimental researches undertaken by him to ascertain whether there was any such flexure in the case of the Swiss tripod. He had, in the first place, employed an extremely sensitive level, which had not entered into oscillation while the pendulum was swinging upon it. It is not clear why M. Hirsch employed a very sensitive level, the natural time of oscillation of which would differ much more from the period of the pendulum than that of a less sensitive level would do. He also used an artificial horizon in the same way. M. Hirsch's conclusion is that "there remains no doubt that the Swiss stand is free from every trace of such oscillations." Dr. von Oppolzer entirely agreed with the views of M. Hirsch.

In the following summer I addressed to M. Plantamour a paper upon the subject, to be submitted to the next meeting of the Geodetical Congress. In this note, which is reprinted at the end of the present report, I first give a mathematical analysis of the problem. I next show experimentally that the motion of the knife-edge support is not a translation, but is a rotation, so that different parts of the head of the tripod, only a few centimeters distant from one another, move through very different distances. Consequently, measures of the flexure made anywhere except at the center of the knife-edge plane require an important correction before they can be used to correct the periods. This is confirmed by experiments with a mirror while the pendulum is in motion. I next give a brief *résumé* of my statical measures of the flexure. I then give measures of the actual flexure under the oscillation of the pendulum, and show that the statical and dynamical flexibilities are approximately equal. Finally, I swing the same pendulum upon the Repsold support and upon another having seven times the rigidity of that one, and I show that the difference of the periods of oscillation agrees with the theory.

Immediately upon the reception of my manuscript, MM. Hirsch and Plantamour commenced new researches, designed to form an "étude approfondie de ce phénomène." These were embodied in a paper by M. Plantamour, which was read to the Geodetical Congress, and which has since been expanded into a memoir entitled "Recherches expérimentales sur le mouvement simultané d'un pendule et de ses supports." M. Plantamour finds fault with me, first, for having measured the flexure with a force five or ten times that of the deflecting force of the pendulum; and second, for measuring the elasticity statically instead of dynamically. The reply to the first objection is that the properties of metals are known to a great extent, that elasticity is not "une force capricieuse," and that no fact is better established than that an elastic strain is proportional to the stress up to near the limit of elasticity, which limit was not approached in the author's experiments. As to the second objection, I had shown by experiment that the statical and dynamical flexures are nearly equal; and I am willing to leave it to time to show whether this will not be assumed in future measures of the flexure of future pendulum supports. M. Plantamour caused a fine point fixed into the head of the tripod to press against a little mirror, mounted on an axis; and then observed the reflection of a scale in a telescope. The length of the path of light from the scale to the telescope divided by the distance of the bearing point from the axis of the mirror he calls the *grossissement;* so that had he used a fixed star in place of his scale, the *grossissement* would have been virtually infinite. From the given length of the lever it would appear that a movement of $0^{\mu}.03$ in the point would turn the mirror 4″. The aperture of the mirror is not stated, but it cannot be supposed that the error of observation would be less than this. It does not seem to me that the use of this mode of measurement, which magnifies the motion but little more than my method, is conducive to accuracy, especially in investigating the difference between statical and dynamical flexure. A certain finite force presses together the point and the lever. Dividing this force by the minute area of pressure, we find the pressure upon the metal is very great, approaching the crushing pressure. Now, the behavior of metals under great pressure is greatly influenced by the time. But my objection is not merely theoretical; I have myself made experiments upon this method, and, making them as skillfully as I could, I still found great uncertainty in the results.

The following table exhibits M. Plantamour's results:

M. Plantamour's flexure experiments.

	Flexure under swinging pendulum.	Flexure when weight is raised and lowered.	Statical flexure.
	μ	μ	μ
Support on floor, comparator removed	3.26 ± .05	3.17 ± .09	3.27 ± .04
On Geneva pier, comparator removed	3.17 ± .03	3.29 ± .08	3.48 ± .04
On Geneva pier, comparator in place	2.41 ± .06	2.50 ± .05	2.76 ± .04
On Berlin pier, comparator in place	2.51 ± .05	2.90 ± .04	3.24 ± .03
On wooden table, comparator in place	3.19 ± .03	3.26 ± .04	3.67 ± .02
On wooden table, comparator removed	4.42 ± .13	4.53 ± .04	4.98 ± .05
Excess:			
Geneva pier over floor	− .09 ± .06	+ .12 ± .12	+ .21 ± .06
Berlin over Geneva pier	+ .10 ± .08	+ .40 ± .06	+ .48 ± .05
Table over Geneva pier, comparator in place	+ .78 ± .07	+ .76 ± .06	± .91 ± .04
Table over Geneva pier, comparator removed	+1.16 ± .14	+1.36 ± .10	+1.71 ± .06
Effect of comparator:			
Geneva pier	− .76 ± .07	− .79 ± .09	− .72 ± .06
Table	−1.23 ± .14	−1.27 ± .06	−1.31 ± .05
Excess table over pier	− .47 ± .16	− .48 ± .11	− .59 ± .08

The table used is the same one shown in Fig. 26 of the Coast Survey Report for 1877. The numbers in the last line above should show the effect of the weight of 3 kilogrammes in diminishing the flexure of this table under a horizontal force of 100 grammes. The weights used in obtaining the first two numbers were about 100 grammes; but the last column is one-tenth the deflection produced by 1,000 grammes. It seems quite incredible that 3 kilogrammes, laid on the table, should really have an effect of this magnitude, so closely proportionate, too, to the deflecting force. It is highly desirable that this result should be confirmed by purely optical experiments; and until this is done, we must suspect that these large numbers indicate some error to which the method of observation is liable. It is certain that the comparator did not act as a brace to stiffen the instrument, and equally so that its weight is not sufficient to alter the modules of elasticity of the brass of the support. It would seem, however, that the effect might be due to a film of some semi-elastic substance under the feet of the tripod. When the tripod is on the floor, no such effect is observed; when it rests on the Geneva pier the dynamical flexure is the same as when it is on the floor, but the statical flexure is much larger. On the Berlin pier the excess of the statical flexure over that on the Geneva pier is five times the dynamical excess. On the other hand, the excess of the dynamical flexure on the table over that at Berlin is half as great again as the statical excess.

MEASURES OF FLEXURE.

My own measures form two series, those made previous to, and those made subsequent to the publication of M. Plantamour's memoir.

In the first series, I was simply occupied in measuring the flexure of the Repsold tripod, as well when properly put up as when the nuts of the bolts were not tightened, of the Geneva support as mounted at Hoboken, and of my "stiffest" support. All the precise measures are statical, and, being made with a filar micrometer, are superior in accuracy to the subsequent ones.

In the second series, the flexures are always measured dynamically as well as statically, and the statical flexure is always found to be the greatest. On the excessively flexible Repsold tripod the difference is sufficient to affect the length of the second's pendulum by 10^{μ}. Nevertheless, as the axis of motion is different for the two kinds of flexure, there are points at which the motion is *less* for dynamical than for statical flexure. And in point of fact, when the Geneva support rests on the Geneva tripod, the dynamical flexure of the center of the knife-edge is *greater* than the statical flexure.

Experiments were also made upon the effect of leaving the nuts of the Repsold tripod entirely loose, of tightening them as much as possible by the hand, and of tightening them by a wrench. It is found that there is little difference between leaving them loose and tightening them by hand, but the effect of the wrench is to produce a stiffening equivalent to a shortening of the pendulum by 20 microns.

Experiments were also made upon the effect of placing a weight of 6 pounds, and afterwards of 25 pounds, upon the head of the Repsold support. The first weight produced absolutely no effect; the second moved the axis of motion a little, and thus caused a slight difference of flexure at some points.

Experiments were also made upon the effect of resting the Repsold support upon blotting-paper, upon blocks of oak, and upon blocks of india-rubber. In every case the difference between the statical and dynamical flexure was much increased.

The pendulum has also been *swung* on all these different supports and the period of oscillation determined with a view of ascertaining whether the statical or dynamical flexure should be used in calculating the corrections to the periods. The result, as might have been predicted from the mathematical theory, shows that a value intermediate between the two is to be taken. But the best way is to make the support so solid that the difference of the two kinds of flexure must be inconsiderable.

EXPERIMENTS TO DETERMINE THE FLEXURE-CORRECTION.

A.—*Flexure of the Repsold stand.*

To determine the flexure, a known force was applied statically to the stand, and the resulting deflection was measured. The principal experiments were made in the cellar of the Stevens Institute at Hoboken. The floor of the cellar is of brick laid down in cement directly on the solid ledge. The floor having been carefully cleaned, the three brass pieces which support the screw-feet of the Repsold tripod were laid down upon it, and the tripod itself was set up. The binding-screws of the feet were screwed up very tight. The pendulum, comparator, and meter were not placed on the tripod, but a mass of iron about equal to them in weight was placed on blocks on the lower part of the tripod in order to ballast it. To apply the force, a silken cord was wound round the tongue upon which the pendulum usually rests, just in the slot over which is the middle of the knife-edge, in such a manner that the cord when stretched horizontally was exactly at the level of the knife-edge. The cord passed horizontally and perpendicular to the knife-edge to a pulley-wheel over which it passed, and from which it hung down vertically; and to its extremity was attached a kilogramme. The pulley-wheel was one which belonged to an Atwood's machine; it turned with very little friction and its rim was accurately planė and perpendicular to the axis. This wheel rested on a stout wooden tripod; its axis was carefully adjusted to be parallel to the knife-edge and the upper part of the rim was brought to the level of the knife-edge. The usual position of the knife-edge is here referred to; but the pendulum was not actually in position. In the measurements of flexure, one person gently raised and lowered this weight alternately. The measurement of the deflection was made by another person, as follows: A micrometer scale on glass was fixed, either to the tongue or to an arm solidly fixed to the tongue, in such a way that the direction of measurement was parallel to the force applied to the tripod. This micrometer scale was observed by a microscope magnifying about fifty diameters and provided with a filar micrometer. This microscope was mounted on a separate, very stiff, iron stand resting on the floor, and carrying at its head a brass apparatus for holding the microscope. The optical axis of the microscope was made exactly parallel to the knife-edge and the filar micrometer screw was made parallel to the force applied to the stand, and the microscope was focused on the micrometer scale. Each division of the scale usually employed was about 12^{μ}. The filar micrometer wire (which was vertical) was made to bisect one division of the scale and the micrometer was read; it was then made to bisect another division, by turning the screw through about one revolution, and the micrometer was read again. Thus, the value of the revolution was obtained. The weight was then put on, and pointings were made upon the same two divisions. Then, the whole process was repeated until the weight had been put on five times. This made one set of experiments.

The following experiments were made to determine the position of the axis of rotation of the knife-edge support during flexure.

HOBOKEN, *March* 10, 1877. Ther. 13° C.—The micrometer scale, attached to an arm, was placed on the line of the knife-edge 53^{mm} in front of the anterior extremity of the tongue. The following were the readings of the filar micrometer on one of the lines of the scale with the weight alternately on and off (ρ throughout signifies a revolution of the micrometer screw):

	Weight off.	Weight on.
	ρ	ρ
	10.955	11.324
	.968	.320
	.978	.324
Means....	10.967	11.323

Difference, $+0^{\rho}.356$.

The arm was now lengthened so that the scale was 318^{mm} in front of the end of the tongue. The following readings of the filar micrometer were now made:

	Weight off.	Weight on.
	ρ	ρ
	10.344	10.762
	.350	.776
	.341	.793
	.335	.778
	.330	.772
Means....	10.340	10.776

Difference, $+0^{\rho}.436$.

The micrometer-scale was next carried over to the other side of the instrument so as to be 496^{mm} behind the front end of the tongue. The following readings were now made:

	Weight off.	Weight on.
	ρ	ρ
	10.106	10.324
	.120	.334
	.141	.334
	.124	.346
	.136	.340
Means....	10.125	10.336

Difference, $+0^{\rho}.211$.

It will be understood that in all these experiments the arm to which the scale was fixed was attached to the tongue on which the pendulum rests, and that this arm was subjected to no force.

The above results are satisfied by supposing that the axis of rotation cuts the level of the knife-edge $1^{m}.258$ behind the end of the tongue. The following table shows the agreement of the observations with this supposition.

Distance forward of end of tongue.	Flexure.	
	Obs.	Calc.
m	ρ	ρ
+0.318	0.436	0.433
+0.053	0.356	0.361
−0.496	0.211	0.212

The scale was next (March 12, 1877, observer, Edwin Smith) fixed at 395^{mm} vertically below the end of the tongue. The following measures were then made:

	Weight off.	Weight on.
	ρ	μ
	13.739	13.260
	.700	.247
	.710	.261
	.700	.260
	.702	.243
	.710	......
Means....	13.710	13.254

Flexure, $+0^{\rho}.446$.

The filar micrometer was here in the reverse position from its usual one, and hence the reading with weight off is greater than with weight on.

The scale was next placed 44^{cm} above the point of support and the following measures were made:

Weight off.	Weight on.
ρ	ρ
10.523	10.737
.453	.645
.400	.578
10.459	10.653

Deflection, $-0^{\rho}.196$.

The filar micrometer was so shaky in this position that accurate measures could not be obtained, but the above answers the purpose.

The scale was next fixed on the end of the tongue and the three measures given below (series 18, 19, 20) were made. The mean of these gives a flexure of $0^{\rho}.340$. These measures show that the axis of rotation cuts a vertical from the end of the tongue at a height of 1.07 meters above the level of the knife-edge. Thus we have on this hypothesis:

Distance below knife-edge.	Flexure.	
	Obs.	Calc.
m	ρ	ρ
−0.44	0.196	0.196
0.00	0.340	0.332
+0.395	0.446	0.452

A large series of experiments were made at Hoboken to determine the amount of flexure. Of these, the following are chiefly relied upon:

HOBOKEN, *March* 7, 1877. Ther., 59°.15 F. 3h 12m P. M.

Möller's glass scale of hundredths of millimeters was fixed 3 millimeters above the end of the tongue. The filar micrometer wires remained fixed, and readings of the micrometer scale were made on the two wires, alternately with weight off and on.

FIRST SERIES.

	Weight off.		Weight on.	
	μ	μ	μ	μ
	844	893	878	931
	843	894	879	930
	844	894	879	931
	845	895	879	930
	844	896	879	931
Means,	844.0	894.2	878.8	930.4
Distance of wires,	50.2 μ		51.6 μ	
Flexure,		34.4 μ	36.2 μ	
Mean,		35.3 μ		

The following readings were then taken with the filar micrometer (temperature 59°.24 F.). The wire was set between lines 80 and 81, and between lines 90 and 91 of the scale.

SECOND SERIES.

	Weight off.		Weight on.	
	90–91	80–81	90–91	80–81
	ρ	ρ	ρ	ρ
	9.347	10.312	9.694	10.655
	344	.309	.693	.660
	348	.315	.692	.664
	348	.322	.699	.663
	352	.336	.713	.684
Means,	9.344	10.319	9.698	10.665
$\frac{1}{10}$ millimeter,	0.975 ρ		0.967 ρ	
Flexure,		0.354 ρ	0.346 ρ	
Mean,		0.350 ρ = 36.1 μ		

This last set was considered of inferior accuracy.

HOBOKEN, *March* 10, 1877. 0h 15m P. M. Temp., 11°.9 C.

A scale on glass by Rogers was observed in the same position as above. Each division is $\frac{1}{2000}$ of an inch (=127μ). The micrometer wire was placed between the first and second and between the tenth and eleventh lines. The observations were made alternately with the weight off and on.

THIRD SERIES.

	Weight off.		Weight on.	
	1–2	10–11	1–2	10–11
	$9.^{p}715$	$10.^{p}849$	$10.^{p}060$	$11.^{p}193$
	.720	.855	.058	.185
	.723	.845	.049	.186
	.715	.846	.055	.185
	.714	.841	.050	.176
Means,	9.719	10.847	10.054	11.185

$\frac{9}{2000}$ inch, $1.^{p}128$ $1.^{p}131$ $\therefore \frac{1}{10}$ mm. $=0.^{p}988$

Flexure, $0.^{p}335$ $0.^{p}338$

Mean, $0.^{p}336=34.^{\mu}1$

This series occupied seven minutes. The whole apparatus was readjusted and a new set was made, as follows:

FOURTH SERIES.

	Weight off.		Weight on.	
	1–2	10–11	1–2	10–11
	$9.^{p}117$	$10.^{p}239$	$9.^{p}459$	$10.^{p}580$
	.122	.236	.449	.586
	.125	.241	.464	.581
	.120	.244	.456	.584
	.128	.234	.456	.579
Means,	9.122	10.239	9.457	10.582

$\frac{9}{2000}$ inch, $1.^{p}117$ $1.^{p}125$ $\therefore \frac{1}{10}$ mm. $=0.^{p}982$

Flexure, $0.^{p}335$ $0.^{p}343$

Mean, $0.^{p}339=34.^{\mu}5$

At 2^{h} 55^{m} P. M. another set of experiments were made, giving the following results (temperature, 12°.2 C.):

FIFTH SERIES.

	Weight off.		Weight on.	
	1–2	10–11	1–2	10–11
	$9.^{p}641$	$10.^{p}745$	$9.^{p}980$	$11.^{p}082$
	.619	.748	.968	.084
	.612	.745	.962	.075
	.616	.735	.963	.089
	.626	.754	.976	.104
Means,	9.623	10.745	9.970	11.087

$\frac{9}{2000}$ inch, $1.^{p}122$ $1.^{p}117$ $\therefore \frac{1}{10}$ mm. $=0.^{p}980$

Flexure, $0.^{p}347$ $0.^{p}342$

Mean, $0.^{p}344=35.^{\mu}1$

After this set the focus was readjusted and two more sets were taken, as follows (temperature, 12°.2):

SIXTH SERIES.

	Weight off.		Weight on.		
	1–2	10–11	1–2	10–11	
	$\overset{\rho}{9.600}$	$\overset{\rho}{10.730}$	$\overset{\rho}{9.953}$	$\overset{\rho}{11.083}$	
	.602	.742	.956	.080	
	.605	.736	.945	.075	
	.594	.740	.953	.076	
	.602	.734	.951	.071	
Means,	9.601	10.736	9.952	11.077	
$\frac{9}{2000}$-inch,	$\overset{\rho}{1.135}$		$\overset{\rho}{1.125}$		∴ $\frac{1}{10}$ mm.=$\overset{\rho}{0.989}$
Flexure,		$\overset{\rho}{0.351}$	$\overset{\rho}{0.341}$		
Mean,		$\overset{\rho}{0.346}=\overset{\mu}{35.0}$			

SEVENTH SERIES.

Temp., 13° C.

	Weight off.		Weight on.		
	1–2.	10–11.	1–2.	10–11.	
	$\overset{\rho}{9.582}$	$\overset{\rho}{10.711}$	$\overset{\rho}{9.929}$	$\overset{\rho}{11.046}$	
	.575	.706	.921	.040	
	.570	.703	.922	.042	
	.575	.700	.918	.038	
	.561	.697	.912	.033	
Means,	9.573	10.703	9.920	11.040	
$\frac{9}{2000}$ inch,	$\overset{\rho}{1.130}$		$\overset{\rho}{1.120}$		∴ $\frac{1}{10}$ mm.=$\overset{\rho}{0.984}$
Flexure,		$\overset{\rho}{0.347}$	$\overset{\rho}{0.337}$		
Mean,		$\overset{\rho}{0.342}=\overset{\mu}{34.8}$			

Three sets were then taken, placing the micrometer wire between the second and third lines, instead of the first and second. The light had now become fainter. Temp., 13°.1 C.

EIGHTH SERIES.

	Weight off.		Weight on.		
	2–3.	10–11.	2–3.	10–11.	
	$\overset{\rho}{9.710}$	$\overset{\rho}{10.724}$	$\overset{\rho}{10.061}$	$\overset{\rho}{11.061}$	
	.712	.721	.057	.059	
	.719	.711	.052	.053	
	.713	.721	.052	.058	
	.725	.727	.060	.055	
Means,	9.716	10.721	10.056	11.057	
$\frac{8}{2000}$ inch,	$\overset{\rho}{1.005}$		$\overset{\rho}{1.001}$		∴ $\frac{1}{10}$ mm.=$\overset{\rho}{0.987}$
Flexure,		$\overset{\rho}{0.340}$	$\overset{\rho}{0.336}$		
Mean,		$\overset{\rho}{0.338}=\overset{\mu}{34.2}$			

After this set the focus was changed. Thermometer still 13°.1 C.

NINTH SERIES.

	Weight off.		Weight on.		
	2–3.	10–11.	2–3.	10–11.	
	ρ	ρ	ρ	ρ	
	9.669	10.677	10.027	11.022	
	.696	.696	.030	.035	
	.696	.693	.039	.045	
	.694	.695	.032	.038	
	.685	.693	.032	.035	
Means,	9.688	10.691	10.032	11.035	
$\frac{5}{2000}$ inch,	1.003 ρ		1.003 ρ		∴ $\frac{1}{10}$ mm.=0.987 ρ
Flexure,		0.344 ρ	0.344 ρ		
Mean,		0.344 ρ = 34.9 μ			

It was noted that this set ought to have double weight. The following set was then taken temperature, 13°.1 C.:

TENTH SERIES.

	Weight off.		Weight on.		
	2–3.	10–11.	2–3.	10–11.	
	ρ	ρ	ρ	ρ	
	9.688	10.689	10.019	11.029	
	.675	.674	.025	.031	
	.674	.676	.014	.020	
	.670	.675	.014	.020	
	.662	.660	.016	.016	
Means,	9.674	10.675	10.018	11.023	
$\frac{5}{2000}$ inch,	1.001 ρ		1.005 ρ		∴ $\frac{1}{10}$ mm.=0.987 ρ
Flexure,		0.344	0.348 ρ		
Mean,		0.346 ρ = 35.1 μ			

This set was also assigned double weight at the time.

Collecting the foregoing results, we have for the deflection of the end of the tongue under one kilogramme's weight—

		Diff. from mean.
	μ	μ
1st set, March 7, 1877	35.3	+0.4
2nd set, March 7, 1877	36.1	+1.2
3d set, March 10, 1877	34.1	−0.8
4th set, March 10, 1877	34.5	−0.4
5th set, March 10, 1877	35.1	+0.2
6th set, March 10, 1877	35.0	+0.1
7th set, March 10, 1877	34.8	−0.1
8th set, March 10, 1877	34.2	−0.7
9th set, March 10, 1877	34.9	+0.1
10th set, March 10, 1877	35.1	+0.2
Mean	34.9	±0.1

The middle of the knife-edge being 30^{mm} behind the end of the tongue, which is 1^{m}.258 forward of the point where the axis of rotation crosses the knife-edge produced, it follows that $\frac{30}{1258}$ of the flexure observed at the end of the tongue, or 0$^{\mu}$.8, has to be subtracted from that quantity to get the flexure of the middle of the edge. The latter is, therefore, 34$^{\mu}$.1.

Measures of the flexure were also made on the 8th and 12th of March, by Sub-assistant Edwin Smith. The following are his results:

ELEVENTH SERIES.

$1^h\ 15^m$ p. m. Temp., 60°.41 F.

	Weight off.		Weight on.	
	2–3.	7–9.	2–3.	7–9.
	ρ	ρ	ρ	ρ
	6.970	7.599	7.305	7.940
	.956	.571	.298	.940
	.963	.581	.295	.930
	.950	.573	.281	.915
Means,	6.960	7.581	7.295	7.931

$\frac{5}{2000}$ inch, $0^{\rho}.621$ $0^{\rho}.636$

Flexure, $0^{\rho}.335$ $0^{\rho}.350$

Mean, $0^{\rho}.342 = 34^{\mu}.5$

TWELFTH SERIES.

$1^h\ 45^m$ p. m. Temp., 60°.37 F.

	Weight off.		Weight on.	
	2–3.	10–11.	2–3.	10–11.
	ρ	ρ	ρ	ρ
	6.921	7.930	7.245	8.256
	.911	.915	.248	.249
	.915	.917	.248	.253
	.906	.913	.248	.248
	.902	.907	.247	.240
Means,	6.911	7.916	7.247	8.249

$\frac{8}{2000}$ inch, $1^{\rho}.005$ $1^{\rho}.002$

Flexure, $0^{\rho}.336$ $0^{\rho}.333$

Mean, $0^{\rho}.334 = 33^{\mu}.9$

THIRTEENTH SERIES.

$2^h\ 05^m$ p. m. Temp., 60°.52 F.

	Weight off.		Weight on.	
	2–3.	10–11.	2–3.	10–11.
	ρ	ρ	ρ	ρ
	6.943	7.949	7.271	8.282
	.945	.946	.279	.278
	.941	.941	.271	.275
	.932	.934	.270	.273
	.938	.939	.271	.273
Means,	6.940	7,942	7.272	8.276

$\frac{8}{2000}$ inch, $1^{\rho}.002$ $1^{\rho}.004$

Flexure, $0^{\rho}.332$ $0^{\rho}.334$

Mean, $0^{\rho}.333 = 33^{\mu}.7$

FOURTEENTH SERIES.

$2^h\ 25^m$ p. m. Temp., 60°.27 F.

	Weight off.		Weight on.	
	2–3.	10–11.	2–3.	10–11.
	ρ	ρ	ρ	ρ
	6.942	7.943	7.280	8.288
	.939	.945	.277	.287
	.942	.949	.281	.281
	.944	.950	.279	.280
	.940	.941	.279	.283
Means,	6.941	7.946	7.279	8.284

$\frac{8}{2000}$ inch, $1^{\rho}.005$ $1^{\rho}.005$

Flexure, $0^{\rho}.338$ $0^{\rho}.338$

Mean, $0^{\rho}.338 = 34^{\mu}.2$

FIFTEENTH SERIES.

$2^h\ 40^m$ p. m. Temp., 60°.23 F.

	Weight off.		Weight on.	
	1–2.	9–10.	1–2.	9–10.
	ρ	ρ	ρ	ρ
	6.825	7.825	7.159	8.159
	.823	.826	.159	.161
	.822	.829	.158	.158
	.819	.820	.157	.160
	.821	.823	.157	.160
Means,	6.822	7.825	7.158	8.160

$\frac{8}{2000}$ inch, $1^{\rho}.003$ $1^{\rho}.002$

Flexure, $0^{\rho}.336$ $0^{\rho}.335$

Mean, $0^{\rho}.336 = 34^{\mu}.0$

SIXTEENTH SERIES.

$2^h\ 55^m$ p. m. Temp., 60°.18.

	Weight off.		Weight on.	
	1–2.	9–10.	1–2.	9–10.
	ρ	ρ	ρ	ρ
	6.822	7.822	7.153	8.160
	.824	.826	.158	.156
	.826	.827	.157	.158
	.821	.823	.160	.161
	.820	.823	.161	.161
Means,	6.823	7.824	7.158	8.159

$\frac{8}{2000}$ inch, $1^{\rho}.001$ $1^{\rho}.001$

Flexure, $0^{\rho}.335$ $0^{\rho}.335$

Mean, $0^{\rho}.335 = 34^{\mu}.0$

SEVENTEENTH SERIES.

3^h 10^m p. m. Temp., 60°.04 F.

	Weight off.		Weight on.	
	1–2	9–10	1–2	9–10
	ρ	ρ	ρ	ρ
	6. 850	7. 847	7. 172	8. 175
	. 849	. 848	. 179	. 183
	. 849	. 852	. 186	. 183
	. 849	. 851	. 189	. 190
	. 850	. 850	. 180	. 181
Means,	6. 849	7. 850	7. 181	8. 182

$\frac{8}{2000}$ inch, 1. 001 ρ   1. 001 ρ

Flexure, 0. 332 ρ   0. 332 ρ

Mean, 0. 332 ρ = 33. 7 μ

1877, MARCH 12.

EIGHTEENTH SERIES.

Ther., 14°.1 C.

	Weight off.		Weight on.	
	12–13	20–21	12–13	20–21
	ρ	ρ	ρ	ρ
	8. 432	9. 452	8. 807	9. 805
	. 453	. 467	. 807	. 805
	. 448	. 457	. 797	. 801
	. 445	. 462	. 792	. 800
	. 440	. 450	. 780	. 779
Means,	8. 444	9. 458	8. 797	9. 798

$\frac{8}{2000}$ inch, 1. 014 ρ   1. 001 ρ

Flexure, 0. 353 ρ   0. 340 ρ

Mean, 0. 346 ρ = 34. 9 μ

After this the apparatus was readjusted.

NINETEENTH SERIES.

Ther., 14°.2 C.

	Weight off.		Weight on.	
	12–13	20–21	12–13	20–21
	ρ	ρ	ρ	ρ
	9. 421	10. 431	9. 755	10. 758
	. 419	. 435	. 753	. 750
	. 421	. 428	. 763	. 770
	. 428	. 428	. 757	. 770
	. 419	. 423	. 763	. 763
Means,	9. 422	10. 429	9. 758	10. 762

$\frac{8}{2000}$ inch, 1. 007 ρ   1. 004 ρ

Flexure, 0. 336 ρ   0. 333 ρ

Mean 0. 334 ρ = 34. 0 μ

TWENTIETH SERIES.

Ther., 14°.2 C.

	Weight off.		Weight on.	
	12–13	20–21	12–13	20–21
	ρ	ρ	ρ	ρ
	9.383	10.390	9.726	10.744
	.400	.402	.730	.738
	.395	.399	.728	.725
	.396	.405	.735	.743
	.398	.405	.735	.730
Means,	9.394	10.400	9.731	10.736
$\frac{8}{2000}$ inch,	1.006 ρ		1.005 ρ	
Flexure,		0.337 ρ	0.336 ρ	
Mean,		0.337 ρ = 34.1 μ		

During the last two sets the illumination was very poor.

Mr. Smith's results, being collected, are as follows:

	Flexure.	Difference from the mean.
	μ	μ
11th set, 1877, March 8	34.5	+0.4
12th set, 1877, March 8	33.9	−0.2
13th set, 1877, March 8	33.7	−0.4
14th set, 1877, March 8	34.2	+0.1
15th set, 1877, March 8	34.0	−0.1
16th set, 1877, March 8	34.0	−0.1
17th set, 1877, March 8	33.7	−0.4
18th set, 1877, March 12	34.9	+0.8
19th set, 1877, March 12	34.0	−0.1
20th set, 1877, March 12	34.1	±0.0
Mean	34.1	±0.1

It will be seen that there is a rather large difference between the results of the two observers. It will, of course, be understood that the discordances of single readings are due mainly to relative movements of the micrometer and the pendulum-support. As most of the sources of constant error tend to make the observed values too small, the larger result has been preferred. When the binding-screws of the feet were not perfectly tight the flexure was still greater, as is shown by the following means of sets of observations made under those circumstances:

	Flexure.	Diffence from the mean.
	μ	μ
21st set, 1877, February 17	36.5	−0.4
22d set, 1877, February 17	37.9	+1.0
23d set, 1877, February 17	36.1	−0.8
24th set, 1877, February 17	37.8	+0.9
25th set, 1877, February 17	36.9	0.0
26th set, 1877, February 19	35.3	−1.6
27th set, 1877, February 19	37.6	+0.7
Mean	36.9	±0.3

The flexure of the Repsold stand was also measured in Geneva, Paris, and Berlin. In Berlin the microscope was mounted on a wooden stand, which rested on the same pier as the tripod. This was to avoid including the flexure of the pier, which is best measured separately. The micrometer scale was simply fixed to a piece of wood, which was laid on the brass pieces at the ends of the tongue. This wooden piece projected 35^{mm} beyond the tongue, and consequently 1 .7 has to be subtracted from the observed results to get the flexure at the middle of the knife-edge. The following are the means of sets of ten measures:

	Flexure.	Difference from mean.
	μ	μ
1876, May 24, a. m	35.8	+0.1
1876, May 24, p. m	35.7	0.0
1876, May 24, p. m	35.8	+0.1
1876, May 24, p. m	35.9	+0.2
1876, May 25, a. m	35.5	−0.2
1876, May 25, a. m	35.4	−0.3
Mean	35.7	±0.1

This result agrees well with that obtained at Hoboken. Thus:

	Flexure of middle of knife-edge under 1 kilogramme	f_0.
	μ	μ
Hoboken (C. S. P., observer)	34.1	215.2
Berlin	34.0	214.5

The same value was found in a rough measure made at Geneva, September 13, 1875.

Much larger values were obtained in Paris, which agree with those found at Hoboken when the binding-screws were not tight. Thus we have

	Flexure with binding-screws loose.	
	μ	
Hoboken	36.1	227.8
Paris, 1876, January 18	36.3	229.1
Paris, 1876, March 7	37.1	234.1

B.—*Flexure of the stiffest support.*

This support was constructed in order to test the calculated effect of the flexure of the stand. The following table shows the results of measures of its flexure:

Distance of scale forward of center of knife.	Distance of scale below level of knife.	Deflection + in the direction of the force, − in the opposite direction.
mm.	*mm.*	μ
+ 30	− 33	+ 5.2
+ 30	− 33	+ 5.2
+ 30	+1003	−42.5
+335	+1003	−36.7

It follows from this that the axis of rotation cuts the line of the knife-edge 166^{mm} behind the center of the edge, and cuts the vertical from that center 68^{mm} below the edge. Also, that the deflection of the middle of the edge under a force of 1 kilogramme's weight is 3$^{\mu}$.1. This includes the flexure of the pier.

C.—*Flexure of the Geneva support and pier.*

In measuring this flexure, instead of a low-power microscope and filar micrometer a high-power microscope and eye-piece micrometer were used. A stage micrometer was always observed, and the value of the divisions of the eye-piece micrometer determined by it. In the following experiments the scale was 254^{mm} above the level of the knife-edge and 25^{mm} forward of the middle.

1878, OCTOBER 1.

FIRST SET.

	Weight off.	Weight on.
	10.4	5.8
	10.2	5.6
	10.3	5.4
	10.2	5.5
	10.0	5.4
Means,	10.2	5.5
Flexure,	4.7	

It was observed that 18.5 of eye-piece micrometer equals 9 of stage micrometer. 1 division of latter = $7^{\mu}.34$. ∴ observed flexure = $16^{\mu}.8$.

SECOND SET.

(Higher power.)

	Weight off.		Weight on.	
	12.4	39.2	18.7	45.8
	12.3	39.2	19.0	45.9
	12.3	39.3	18.8	46.0
	12.4	39.3	18.8	46.1
	12.2	39.4	18.7	46.2
Means,	12.3	39.3	18.8	46.0
	$66^{\mu}.1$ = 27.0		27.2	∴ 1 div. = $2^{\mu}.44$
Flexure,		6.5	6.7	
Mean,		6.6 = $16^{\mu}.1$		

THIRD SET.

	Weight off.	Weight on.
	39.8	46.4
	39.8	46.5
	39.9	46.5
	40.0	46.6
	40.1	46.7
Means,	39.9	46.5
Flexure,	6.6	

Nine spaces of stage micrometer were equal to 27.1 of eye-piece micrometer. Hence, observed flexure = $16^{\mu}.1$.

FOURTH SET.

	Weight off.	Weight on.
	39.9	46.4
	40.0	46.6
	40.1	46.6
	40.0	46.8
	40.2	46.8
Means,	40.0	46.6
Flexure,	6.6 = $16^{\mu}.1$	

1878, OCTOBER 19.

The scale was fixed 111cm below the knife-edge, and three sets of 10 gave for the deflection

	μ.
	−48. 8
	−47. 5
	−48. 1
Mean,	−48. 1

1878, OCTOBER 21.

The scale was fixed 244 millimeters above the knife-edge, and 356 millimeters forward of the middle. Two sets gave as the deflection

μ
+13. 5
+12. 2
+12. 8

From these measures we find the flexure at the middle of the knife-edge to be 4μ.05.

EXPERIMENTS AT PENNSYLVANIA GRAVITATION STATIONS.

ALLEGHENY.

Statical flexure of Geneva support on iron bars. Weight and pulley employed. Weight=2k; f_o denotes flexure produced by a horizontal force equal to the weight of pendulum (6k.308).

1879, FEBRUARY 18.

Scale ¾-inch (=2cm) above, and 12.5 inches (=32cm) forward of middle of knife-edge; 22.4 div. of scale=100μ. C. S. P., observer.

Scale readings.

	Wt. off.	Wt. on.	Diff.	f_o
	....	....	1. 2	μ
	....	....	1. 2	15. 1
	....	....	1. 1	
			div. 1. 17	

The following measures were made under a higher power of microscope; 58 div. of scale=100μ.

	Wt. off.	Wt. on.	Diff.	f_o
	37. 0	34. 0	2. 85	16
	36. 7			
	35. 7	33. 1	2. 6	15
	36. 0	33. 4		
	37. 7	35. 0	3. 0	16
	37. 9	35. 0		
	38. 0	35. 2		
	38. 6			
Mean,				16

In the following measures, 37 div. of scale=100μ; otherwise same as preceding.

24. 1	22. 2	1. 9	16μ 4

1879, February 20.

Scale $\frac{1}{2}$ inch ($=1^{cm}$) above, and $12\frac{1}{2}$ inches ($=32^{cm}$) behind middle of knife-edge; 23.5 div. ot scale$=100^{\mu}$.

Scale readings.

Wt. off.	Wt. on.	Diff.	f_0.
6.3	11.4	div.	μ
6.5	11.2	4.7	63
6.6	11.2		
7.0			

In the following, 28.3 div. of scale$=100^{\mu}$; otherwise the same.

	Wt. off.	Wt. on.	Diff.	f_0.
	35.4	40.5		
	35.6	40.4	4.8	54
	35.7	40.4		
	35.7			
	63.4	68.4		
	63.6	68.9	5.0	56
	64.0	68.9		
	64.0			
Mean,				55

In the following, 41.1 div. of scale$=100^{\mu}$; otherwise same.

Wt. off.	Wt. on.	Diff.	f_0.
20.8	25.0	4.4 *rej.*	
30.5	35.0		
71.5	78.0		
71.6	78.1	6.5	64
71.6	78.0		
71.4	78.8		

1879, March 4.

Scale 1 inch ($=2^{cm}.5$) above, and $13\frac{1}{2}$ inches ($34^{cm}.5$) behind middle of knife-edge; 26.7 div. of scale$=100^{\mu}$. H. Farquhar, observer.

....	....	5.4	64

Scale next put 2^{cm} above and 14 inches ($=35^{cm}$) forward of middle of knife-edge; 26.7 div. of scale$=100^{\mu}$.

....	....	1.0	12

1879, March 6.

Scale on level of knife-edge and 15 inches ($=38^{cm}$) forward; 38.5 div. of scale$=100^{\mu}$.

....	....	1.9	16

Scale next put 55 inches ($=140^{cm}$) below middle of knife-edge; 33.3 div. of scale$=200^{\mu}$.

....	....	1.6	302

The following is a summary of the above. F_0 here and elsewhere denotes the flexure at the middle of the knife-edge under a horizontal force equal to the weight of the pendulum. A=distance from middle point of knife-edge to its intersection with axis; B=distance from middle point of knife-edge to the intersection of axis with vertical line.

C. S. P.'s observations.

$\frac{3}{4}$ inch above, 32^{cm} forward of knife-edge, $f_0=16.1$
$\frac{1}{2}$ inch above, 32^{cm} behind, $f_0=55^{\mu}.7$
∴ Flexure at 2^{cm} above$=35^{\mu}.9$
$A=58^{cm}$.

H. F.'s observations.

$2^{cm}.0$ above, 35^{cm} forward of knife-edge, $f_0=12^{\mu}$
$2^{cm}.5$ above, $34^{cm}.5$ behind, $f_0=64^{\mu}$
∴ Flexure at 2^{cm} above$=38^{\mu}.18$
$A=51^{cm}$.
140^{cm} directly below middle of knife-edge, $f_0=156$.
∴ $F_0=40^{\mu}.66$; A (mean)$=54^{cm}$,
$B=29^{cm}$.

EBENSBURG.

At this station the Repsold tripod stood on a hard floor of clay. Statical flexure measured by means of weight of 1.0818^{k}. C. S. P., observer.

1879, SEPTEMBER 26.

The two screw-taps binding the front legs of the tripod to top of the stand were first tightened *by hand*. Scale on the level of the point of suspension, 50^{cm} to the right, and $18^{cm}.4$ forward of middle of knife-edge. 21.6 div. of scale=.001 inch.

Scale readings.

Wt. off.	Wt. on.	Mean diff.	f_0.
8.4	61.3		μ
9.4	61.3	52.3	358
9.6	61.6		

P. M.—Taps wrench-tightened; scale $18^{cm}.4$ *directly* forward of knife-edge; 20.7 div. of scale=.001 inch.

35.0	82.3		
35.0	81.8	47.2	337
34.5	82.0		
34.6	81.6		

Taps next loosened.

25.5	82.0		
28.0	82.3	54.7	391
28.3	82.1		
28.3	82.5		

Taps next hand-tightened.

16.0	67.6		
16.0	67.5		
16.2	67.6	51.5	368
16.0	67.7		
16.7			

1879, SEPTEMBER 27.

Scale $18^{cm}.4$ forward of middle of knife-edge, as before; 21.8 div. of scale=.001 inch. Taps tightened by Mr. F. about as tight as during last four days of pendulum swinging. (*Note.*—It had been discovered that during these days the taps had only been tightened by hand.)

Scale readings.

Wt. off.	Wt. on.	Mean diff.	*fo.*
			μ
19.0	47.8 rej.		
18.4	69.0	50.5	343
18.2	69.0		

In the following measures, 21.6 div. of scale=.001 inch. Microscope refocussed.

Wt. off.	Wt. on.	Mean diff.	*fo.*	Position of lines of stage micrometer, read on eye-piece micrometer.	
21.0	71.5				
20.9	71.5	50.4	346		
21.0	71.0				
20.5	71.0				
21.8	72.8			0.1	0.5
22.0	72.9	50.9	349	21.8	21.9
22.0	73.0			43.3	43.4
22.0				64.7	64.9
				86.5	86.8
Mean interval				21.55	21.55

Screws now somewhat loosened by Mr. F.

Wt. off.	Wt. on.	Mean diff.	*fo.*		
2.5	55.5			13.2	12.5
2.5	55.5	53.0	363	34.7	34.5
2.3	55.4			77.7	77.4
				99.5	99.1
Mean interval				21.55	21.58

Screw-taps now tightened as tight as possible with fingers; 21.6 div. of scale=.001 inch.

Wt. off.	Wt. on.	Mean diff.	*fo.*		
18.5	69.6			7.0	4.6
18.6	69.6	50.8	348	28.5	26.3
18.6	69.0			49.8	47.6
				71.5	69.5
				93.2	91.0
Mean interval				21.53	21.6

Screw-taps now tightened with wrench by Mr. P.; 21.2 div. of scale=.001 inch.

Wt. off.	Wt. on.	Mean diff.	*fo.*		
20.0	67.4			16.5	20.0
20.0	67.3	47.4	331	37.7	40.9
19.6	67.2			59.0	62.2
				81.6	83.5
Mean interval				21.25	21.2

New set. Screws entirely loose; 21.6 div. of scale=.001 inch.

Wt. off.	Wt. on.	Mean diff.	*fo.*			
21.5	75.8			16.6	11.2	10.8
22.0	76.0	54.0	370	38.5	33.2	32.7
21.8	75.7			59.7	54.8	54.6
21.7					76.6	76.1
				81.7	97.9	97.4
Mean interval				21.58	21.68	21.67

Screws now tightened by hand of Mr. F. "about right"; 21.6 div. of scale=.001 inch.

Scale readings.

Wt. on.	Wt. off.	Mean diff.	fo.	Position of lines of stage micrometer, read on eye-piece micrometer.	
17.0	67.6		μ	3.4	1.8
17.1	67.3	50.2	344	24.9	24.0
17.0	67.0			46.4	45.3
17.2				68.1	67.2
				89.5	88.6
Mean interval				21.55	21.66

Screws again hand tightened "about right"; scale as above.

20.4	70.0			8.5	5.0
20.4	70.0	49.8	341	30.6	26.8
20.0	69.9			57.8	48.5
20.1				73.3	69.9
				94.9	91.4
Mean interval				21.52	21.58

Screws again tightened by hand "about right"; 21.5 div. of scale=.001 inch.

11.5	61.5			0.7
11.4	61.2	50.0	344	22.9
11.2				43.9
				65.7
				87.0
Mean interval				21.52

The last set of measures were not regarded as being so satisfactory as the preceding.

Head of stand taken off, put on again, and tightened with wrench. Scale $18^{cm}.6$ forward of middle of knife-edge; 21.4 div. of scale=.001 inch.

36.4	83.4			2.6	7.9
36.4	83.1	46.8	324	13.4	18.7
36.4	83.1			23.0	29.2
36.4				34.5	40.0
				45.5	50.8
				56.1	61.6
				66.9	72.1
				77.4	82.6
				87.8	
Mean interval				21.47	21.41

Another set; 21.4 div. of scale=.001 inch. *N. B.*—In this and following sets the positions of several, generally three, lines of the stage micrometer are read off on the eye-piece micrometer, between all the changes of the weight. This explains the separation of the numbers in the first two columns into groups.

37.4	84.2			4.0	5.3
32.3	79.4			13.5	10.7
26.6	74.1	47.1	327	24.3	16.0
				35.0	21.3
36.9	84.2			46.2	26.4
31.8	79.3			56.6	31.9
26.4	74.1	47.5		67.7	36.9
				78.0	
				88.8	
Mean interval				21.43	21.07

Another set; 21.5 div. of scale=.001 inch.

Wt. on.	Wt. off.	Mean diff.	f_0.	Position of lines of stage micrometer read on eye-piece micrometer.	
28.3	74.5			32.1	31.1
33.2	80.0	46.6		37.9	37.1
38.4	85.3			43.3	42.3
				47.9	47.0
27.6	74.5			54.0	53.0
33.0	80.0	47.0		59.2	58.6
38.2	85.3			64.8	63.9
				70.0	68.9
27.6	74.6			75.1	74.5
33.0	80.0			80.8	79.8
38.2	84.9	46.9	μ 323	85.8	85.0
28.0	74.6	Mean interval.		21.51	22.55
32.9	80.0				
38.0	84.9	46.9			
28.0	75.1				
32.9	79.9				
38.0	85.1	47.1			
27.8	74.1				
33.0	79.9				
38.0	85.1	46.8			

Stand reversed. Scale 42cm.5 behind middle of knife-edge; 21.6 div. of scale=.001 inch.

Scale readings.

Wt. off.	Wt. on.	Mean diff.	f_0.	Lines of scale.		
3.0	28.8			5.0	18.4	6.6
24.4	50.6	26.0		26.8	39.5	27.5
				37.2	61.5	48.8
3.7	29.1		μ 177	47.9	83.0	70.6
25.0	50.9	25.6				92.6
46.5	72.0					
		Mean interval.		21.6	21.6	21.5

Microscope refocussed. Scale 21.6 div.=.001 inch.

5.6	31.0	25.7		13.0
26.7	52.7			34.0
				56.0
7.4	32.4		176	77.2
28.4	54.0	25.7		
48.8	75.4	Mean interval.		21.55

Again, 21.4 div. of scale=.001 inch.

12.5	38.6			17.0	18.6
33.4	60.2			38.6	40.2
54.0	81.4			60.0	61.6
				81.5	82.9
14.0	39.5	26.8	181 rej.		
35.0	61.2	Mean interval.		21.47	21.4
56.0	82.5	26.1			
14.4	39.5				
35.6	61.2				
56.5	82.5	25.6			

Screws next entirely loose; 21.5 div. of scale=.001 inch

Scale readings.

Wt. on.	Wt. off.	Mean diff.	f_0.	Lines of scale.
52.0	68.6			5.0
62.7	79.6		μ	16.1
73.2	90.4	16.9	115	28.8
				37.5
52.5	68.4			48.0
62.9	79.3			59.1
73.2	90.0	16.4		69.6
				80.6
				91.1
				21.5

Screws now hand-tightened "about right," by H. F.; 21.6 div. of scale=.001 inch.

1.5	21.3			6.0
1.3	21.4	20.0	137	92.2
1.2				
				21.55

Again, 21.6 div. of scale=.001 inch.

8.7	29.0			18.6
8.5	28.9	20.2	138	94.2
9.0	28.8			
				21.6

Three bricks were next put on the bottom of the stand; weight, 4 pounds $5\frac{1}{2}$ ounces, 4 pounds $6\frac{3}{4}$ ounces, 4 pounds $11\frac{1}{2}$ ounces, respectively. The following measures were taken at $41^{cm}.4$ behind the middle of knife-edge, and $0^{cm}.7$ above level of support. Screws hand-tightened, as in last observations; 21.4 div. of scale=.001 inch.

16.5	34.4	17.9	124	2.2
16.5				87.8
				21.4

Refocussed. Scale as above.

20.2	38.5	18.4	127	9.5
20.2	38.6			73.7
				21.4

Screws next tightened with wrench; 21.7 div. of scale=.001 inch. Measures taken $1^{cm}.4$ above level of support.

67.3	93.0			2.4	5.6
67.4	92.4			88.8	92.7
67.3	92.6	25.2	172		
67.8	92.7			21.6	21.7

Measure taken $4^{cm}.1$ below level of support; 21.6 div. of scale=.001 inch. Stage micrometer fixed to the top of the tripod, but not to the tongue on which the pendulum rests.

10.8	41.0			18.6
12.2	40.6	29.3	201	83.5
13.6	42.8			
				21.6

Again, 21.7 div. of scale=.001 inch.

16.0	45.0			11.8
16.9	46.0	29.0	197	98.5
18.0	46.8			
				21.7

Again, 21.6 div. of scale=.001 inch.

Scale readings.

Wt. off.	Wt. on.	Mean diff.	fo.	Lines of scale.
			μ	
20. 7	50. 0	29. 2	201	7. 0
21. 0	50. 2			93. 4
				21. 6

Again, scale 21.5 div.=.001 inch.

Wt. off.	Wt. on.	Mean diff.	fo.	Lines of scale.
23. 0	51. 7	29. 1	200	22. 2
	52. 5			86. 8
				21. 5

The bricks were now removed from base of support, and pendulum suspended heavy end down; 21.4 div. of scale=.001 inch. Screws wrench-tightened.

Wt. off.	Wt. on.	Mean diff.	fo.	Lines of scale.
8. 0	37. 4	29. 2	202	16. 0
8. 5	37. 5			80. 3
				21. 4

Again, 21.6 div. of scale=.001 inch. "Good."

Wt. off.	Wt. on.	Mean diff.	fo.	Lines of scale.
7. 5	37. 5			7. 9
7. 5	36. 8	29. 9	206	94. 3
	38. 0			
				21. 6

The following is a summary of the observations with weight and pulley at Ebensburg, on Repsold stand. F_0=flexure at middle point of knife edge, under a horizontal force equal to the weight of the pendulum; A=distance from middle point of knife-edge to intersection of axis with knife-edge.

Arrangement.	f_0. 18cm.4 forward, 0cm.7 above.	f_0. 42cm.5 back, 0cm.7 above.	f_0. 10cm from axis of rotation.	A.	F_0.
	μ	μ	μ	Cm.	μ
Front taps wrenched up	337	177			
	331	176			
	324	....			
	327	....			
	323	....			
	328	176. 5	24. 9	113. 5	283
Front taps hand-tightened	343	137			
	349	138			
	344	....			
	341	....			
	344	....			
	342	137. 5	33. 6	83. 4	281
Taps somewhat loose	363	....	....	....	....
Front taps loose	391	115	45. 3	67. 9	309
		41cm.4 back.	Calculated from above without bricks.		
		μ	μ		
Tripod loaded with bricks; taps hand-tightened	....	124			
		127			
		125. 5	141		
Taps wrenched	....	172	179		
		With bricks.	*W'thout bricks.*		
Flexure of tripod without that of tongue; taps wrenched.	....	201	202		
		197	206		
		201	....		
		200	....		
		200	204		

It will be seen that the effect of loosening the front taps is to increase the angular flexure about the instantaneous axis. But this axis is at the same time brought forward, and the consequence is that the flexure at the middle point of the knife-edge is not much changed. That the flexure of the tripod alone, without that of the tongue supporting the pendulum, appears, when measured, 40^{cm} behind the middle point, to be greater than the combined flexure of the two, is no doubt due to the axis of flexure of the tongue cutting the level of the knife-edge only a short distance behind the middle point. The effect of loading the base of the tripod with bricks was to make it slightly stiffer when wrench-tightened, and considerably stiffer when hand-tightened. All of these measures of flexure seem, however, to be in error, and it seems likely that the position of the scale, when in front of the stand, was not really 184^{mm} as recorded, but perhaps 584^{mm}. With that change, these measures would agree with others, which they do not now do.

The following are dynamical measurements. The pendulum swung heavy end down; 21.4 div. of scale=.001 inch. Arc expressed in ten-thousandths of the radius. Screws wrench-tightened; scale $41^{cm}.4$ behind, $0^{cm}.7$ above knife-edge.

1879, September 27.

	Arc.	Scale readings.		Diff.	f_0.
					μ
	292	7.6	13.7	6.1	178
	291	9.0	14.8	5.8	169
	290	6.7	12.8	6.1	179
Another swing.					
	381	6.6	14.7	8.1	179
	379	7.8	16.0	8.2	183
	377	6.6	13.8	7.2	162
	373	6.9	14.4	7.5	171
	370	0.0	7.8	7.8	179
		Mean			175.0

1879, September 28.

Scale 1.0 to 87.6=.004 inch. Other conditions same as before.

	Arc.	Scale readings.		Diff.	f_0.
					μ
	520	5.0	15.9	10.9	175
	512	4.0	14.8	10.8	178
	506	0.0	10.7	10.7	178
	503	0.5	11.3	10.8	181
	496	7.7	17.8	10.1	170
	492	3.8	14.3	10.5	177
	487	2.3	12.0	9.7	166
	479	2.6	12.8	10.2	177
	465	8.7	17.8	9.1	163
	459	5.1	14.8	9.7	178
	455	2.2	11.4	9.2	168
	290	3.9	10.0	6.1	175
		Mean			173.8
Stopped, and started again.					
	392	3.6	11.7	8.1	173
	383	0.3	8.2	7.9	173
		Mean			173.0

The following are statical measures of the flexure produced by drawing the pendulum to one side over a meaured arc; steel tongue used instead of wooden strip before employed. Scale $44^{cm}.4$ behind middle of knife-edge, and $2^{cm}.5$ below its level; 1.0 to 87.6 div. of scale=.004 inch.

	Scale readings; pend. vertical.		Scale readings; pend. inclined.		Arc.	Mean diff.	f_0.
							μ
	9.7	74.5	16.6	81.1	474	6.75	233
	10.4	74.8	16.0	80.9	465	5.85	207
Again.							
	10.7	75.3	16.9	81.6	474	6.25	217
	11.3	76.3	16.1	80.7	475	4.60	159
	11.6	76.5	15.8	80.4	438	4.05	151
	10.8	75.6	16.1	80.8	486	5.25	177
	11.1	76.0	16.2	80.7	489	4.90	164
	10.8	75.8	16.2	80.1	429	4.85	186
	9.6	74.3	15.9	80.8	450	6.40	233
	7.3	71.9	12.9	77.7	501	5.70	187
	9.5	74.5	15.0	79.7	500	5.35	175
	10.1	74.8	16.0	80.6	515	5.85	187
	9.6	74.2	15.0	79.7	490	5.45	182
							189

Dynamical measurements; 21.6 div. of scale=.001 inch.

Arc.	Scale readings.		Diff.	f_0.
				μ
496	8.8	19.7	10.9	185
493	1.6	11.3	9.7	166
490	12.3	22.6	10.3	177
488	13.6	23.9	10.3	178
485	8.7	19.4	10.7	186
482	0.9	11.0	10.1	177
480	12.6	22.7	10.1	177
477	13.7	23.8	10.1	179
474	9.0	19.3	10.3	182
472	0.9	10.8	9.9	177
469	12.5	22.7	10.2	183
466	13.8	23.7	9.9	179
463	3.8	13.7	9.9	180
460	0.7	10.6	9.9	181
458	2.4	11.9	9.5	174
455	3.5	13.3	9.8	181
				178.8

Tongue readjusted. Scale $44^{cm}.6$ behind middle of knife-edge, and same height as before. Screws hand-tightened by H. F.; 3.3 to 89.2 div. of scale=.004 inch.

				μ
466	10.6	18.9	8.3	150
463	11.6	19.9	8.3	150
461	3.6	12.2	8.6	157
460	5.4	13.7	8.3	151
457	0.4	8.8	8.4	155
450	1.7	9.9	8.2	154
447	4.9	13.3	8.4	158
445	5.7	13.9	8.2	155
				153.8

Statical flexure with same arrangement.

Scale readings; pend. vertical.		Scale readings; pend. inclined.		Arc.	Mean diff.	f_0.
						μ
2.3	89.6	8.2	94.3	464	4.80	174
3.6	89.8	7.8	93.9	471	4.15	149
2.3	88.6	7.9	94.1	482	5.55	195
2.5	88.2	5.9	93.2	475	4.70	167
2.3	88.4	6.9	93.0	488	4.60	159
0.8	87.8	6.1	91.9	494	4.20	144
1.4	87.6	6.2	92.0	488	4.60	159
						158.7

Screws now loosened, and again tightened by hand.

						μ
4.6	78.9	8.8	83.4	482	4.35	151
4.6	79.0	9.4	83.8	507	4.80	160
4.6	79.1	8.9	83.5	473	4.35	155
4.7	79.2	8.9	83.8	482	4.45	155
						155.3

Dynamical measures with last arrangement; 21.3 div. of scale=.001 inch.

Arc.	Scale readings.		Diff.	f_0.
				μ
512	0.3	9.4	9.1	151
508	1.3	10.5	9.2	154
506	1.6	10.8	9.2	155
504	2.7	11.7	9.0	152
503	3.7	12.4	8.7	147
500	4.8	13.6	8.8	150
498	0.2	8.9	8.7	149
496	1.1	9.8	8.8	150
495	1.8	10.7	8.9	153
				151.2

Screws retightened by H. F.

				μ
520	5.6	15.3	9.7	159
516	1.7	16.3	9.6	158
513	8.7	13.1	9.4	156
512	0.4	9.7	9.3	155
509	6.0	15.7	9.7	162
507	7.3	16.6	9.3	156
504	9.0	18.4	9.4	159
501	0.7	9.9	9.2	156
				157.6

Statical flexure. Same arrangement; 21.6 div. of scale=.001 inch.

Pend. vertical.		Pend. inclined.		Arc.	Mean diff.	f_0.
						μ
18.6	83.3	23.1	87.7	464	4.45	162
19.6	83.5	23.4	88.1	514	4.70	153
18.6	83.5	23.3	87.8	488	4.50	155
18.4	83.4	23.3	87.9	467	4.70	170
18.6	83.5	23.6	88.3	513	4.90	162
						160.4

Focussed. Same arrangement.

Pend. vertical.		Pend. inclined.		Arc.	Mean diff.	f_0.
						μ
20.0	84.7	24.4	89.3	483	4.50	157
20.1	84.8	24.9	89.3	504	4.65	155
20.6	85.1	25.1	89.2	506	4.30	143
21.1	85.6	25.1	88.9	509	....	...
20.6	85.4	25.5	89.9	478	4.70	165
						155.

The stand turned around; tongue now projects in front of middle of knife-edge $33^{cm}.9$; height approximately as before; nuts wrenched up; dynamical.

Arc.	Scale readings.		Diff.	f_0.
				μ
472	7.3	23.4	16.1	287
470	5.6	21.3	15.7	281
467	4.0	20.3	16.3	293
465	7.6	22.9	15.3	277
462	5.7	20.9	15.2	277
460	7.7	22.9	15.2	278
457	5.8	21.0	15.2	280
				281.9

Statical flexure. Same arrangement; 21.4 div. of scale=.001 inch.

Scale readings; pend. vertical.		Scale readings; pend. inclined.		Arc.	Mean diff.	f_0.
						μ
5.7	80.2	13.0	87.1	398	7.20	307
15.4	79.6	23.8	88.1	460	8.45	312
15.4	79.7	23.7	87.9	476	8.25	294
15.4	79.6	23.7	87.7	485	8.20	287
14.8	79.3	23.6	87.9	496	8.70	297
14.9	79.3	23.7	88.0	480	8.75	309
14.6	79.1	23.4	87.6	488	8.65	300
14.7	79.2	23.5	87.5	491	8.55	353
14.7	78.9	23.5	87.6	500	8.75	297
14.6	78.7	23.5	87.7	504	8.95	302
						300.5

Dynamical flexure; 21.4 div. of scale=.001 inch; nuts tightened by hand of H. F.

Arc.	Scale readings.		Diff.	f_0.
				μ
426	6.1	22.6	16.5	328
422	2.2	18.6	16.4	330
420	6.3	22.6	16.3	329
417	4.7	21.2	16.5	336
414	5.5	21.7	16.2	332
412	3.8	19.8	16.0	329
410	6.3	22.2	15.9	329
				330.4

Statical flexure; last arrangement.

Scale readings; pend. vertical.		Scale readings; pend. inclined.		Arc.	Mean diff.	f_0. μ
13.9	78.3	23.9	87.8	490	9.75	340
14.0	78.1	24.1	88.3	502	10.15	345
14.1	78.3	24.3	88.4	511	10.15	338
14.5	78.5	24.4	88.4	503	9.90	334
14.5	78.4	24.0	87.9	480	9.50	336
14.5	78.6	24.8	88.7	521	10.20	333
14.2	78.2	23.5	87.7	476	10.40	370
14.0	78.4	23.6	87.7	485	9.45	331
14.0	77.9	23.9	87.9	504	9.45	319
14.5	78.6	23.9	88.0	490	9.40	326
						333.6

Nuts readjusted by hand. Same arrangement as before.

24.1	88.2	32.9	97.0	437	8.80	341
24.2	88.3	34.3	98.5	504	10.15	341
23.5	88.3	33.8	97.8	475	10.40	372
23.8	87.7	33.1	97.4	466	9.50	346
23.5	87.5	32.5	96.7	455	9.10	340
23.5	87.6	33.4	97.7	496	10.00	343
23.6	87.6	34.4	98.6	542	10.90	341
23.4	87.5	33.2	97.1	475	9.70	346
23.4	87.6	32.3	96.5	433	8.90	350
23.5	87.6	32.9	97.0	474	9.40	336
						342.7

Dynamical flexure. Same arrangement.

Arc.	Scale readings.		Diff.	f_0. μ
384	0.7	15.7	15.0	331
382	6.6	21.7	15.1	334
380	8.3	23.0	14.7	329
379	10.3	24.7	14.4	322
484	8.5	27.7	19.2	336
479	6.1	24.7	18.6	329
474	6.6	24.8	18.2	326
471	8.2	26.6	18.4	331
				329.8

Screws again tightened by hand of H. F.

412	3.0	19.4	16.4	337
410	4.1	20.6	16.5	341
408	2.6	19.3	16.7	347
407	4.2	20.4	16.2	337
405	3.2	19.3	16.1	337
404	4.4	20.2	15.8	332
523	0.5	21.5	21.0	340
520	0.6	21.6	21.0	342
518	5.5	25.9	20.4	334
514	3.8	24.3	20.5	338
511	1.0	21.3	20.3	336
506	5.5	25.7	20.2	338
504	5.4	24.8	19.4	328
				337.4

Statical flexure; same arrangement; 21.5 div. of scale=.001 inch.

Scale readings; pend. vertical.		Scale readings; pend. inclined.		Arc.	Mean diff.	f_0.
10. 6	75. 4	20. 9	85. 1	492	10. 00	346
10. 9	75. 1	20. 1	84. 4	457	9. 25	344
10. 6	74. 9	20. 3	84. 7	485	9. 75	341
10. 4	74. 9	20. 8	82. 0	509	10. 25	342
10. 5	74. 9	21. 2	85. 9	505	....	...
10. 6	75. 3	21. 0	84. 2	494	....	...
10. 6	75. 3	21. 0	84. 8	501	9. 95	337
10. 4	75. 0	21. 3	85. 7	517	10. 80	355
10. 4	74. 6	20. 4	84. 9	496	10. 15	347
10. 3	74. 7	20. 5	84. 9	500	10. 20	346
					$10^{div}.03$	344. 7

Summary of observations with pendulum, dynamical and statical, made at Ebensburg on Repsold stand.

Nuts wrenched.

	Dynam.	Stat.
$44^{cm}.4$ behind knife-edge,	$f_0=178^{\mu}.8$	$189^{\mu}.0$
$33^{cm}.9$ forward,	$f_0=281^{\mu}.9$	$300^{\mu}.9$
	$\therefore F_0=237^{\mu}.2$	$252^{\mu}.7$
	$A=182^{cm}.9$	$176^{cm}.5$

Nuts hand-tightened.

$44^{cm}.6$ behind knife-edge,	$f_0=154^{\mu}.2$	$158^{\mu}.6$
$33^{cm}.9$ forward,	$f_0=332^{\mu}.4$	$339^{\mu}.4$
	$\therefore F_0=255^{\mu}.4$	$261^{\mu}.3$
	$A=112^{cm}.5$	$113^{cm}.4$

YORK.

H. F., observer. All observations at this station, made in two positions, carefully brought to the level of the knife-edge plane. Ebensburg weight=$1^{k}.0818$.

1879, NOVEMBER 8.

Scale 47^{cm} in front of middle agate; 76.2 div. of scale=.003 inch; Geneva support.

Scale readings.

Wt. on.	Wt. off.	Diff.	f_0.
28. 5	37. 0		
28. 0	40. 0		
28. 0	38. 0	10. 3	$58^{\mu}.7$
28. 0	38. 5		
28. 5	39. 0		
29. 0	39. 0		
23. 5	34. 0		
24. 0	35. 0		
24. 0	35. 0	10. 6	60. 4
24. 0	34. 0		

22.5	33.5	6.0	18.0		
23.9	33.0	6.0	16.5		
23.0		6.5	17.0	10.6	60.4
		7.0	18.0		
		7.0	17.5		
		7.5	18.0		
		7.5	18.0		
		8.0			

Scale put on 46^{cm} *behind* middle of agate; 85.5 div. of scale=.003 inch. Measures not very good, on account of jarring of machinery.

Scale readings.

Wt. on.	Wt. off.	Diff.	f_0.
31.0	49.5		
31.0	49.5		
30.5	49.0		μ
30.0	48.0	18.4	93.5
29.5	48.0		
29.5	47.0		
29.0	47.0		
28.5			
13.5	32.5		
12.0	31.5		
13.0	31.5		
14.0	32.0	18.7	95.0
14.0	32.5		
14.0	33.0		
14.0	32.0		
12.5			

1879, NOVEMBER 9.

Sunday. Shops all still. Scale 46^{cm}.6 *behind* the middle of the knife-edge; 91.87 div. of scale=.003 inch.

Scale readings.

0.003 inch.		Wt. on.	Wt. off.	Diff.	f_0.
91.8	91.8	29.5	11.2		
92.0	91.7	29.7	11.1		
92.1	92.0	29.9	12.1		
92.1	91.9	31.3	13.0		
92.1	91.8	30.9	13.0		μ
	92.0	31.7	14.0	18.1	85.4
		32.7	14.2		
		33.3	14.9		
		32.9	15.1		
		33.8	16.1		

Again. 93.47 div. of scale=.003 inch.

.003 inch.	Scale readings. Wt. on.	Wt. off.	Diff.	f_0.
93.9	41.7	22.8		
93.5	41.9	22.7		μ
93.3	41.7	22.9	18.85	87.4
93.4	41.2	22.5		
93.3	41.5	22.9		
93.4	41.8			

Again. 93.51 div. of scale=.003 inch.

93.4	37.3	18.7		
93.3	37.2	18.1		
93.8	36.9	18.0		
93.7	36.6	18.1	18.8	87.2
93.4	36.6	17.7		
93.3	36.7	18.0		
93.7	37.5	18.3		
	36.9	18.1		
	36.9			

Scale put on $46^{cm}.6$ *in front* of middle of knife-edge; 70.63 div. of scale=.002 inch.

.002 inch.	Scale readings. Wt. on.	Wt. off.	Diff.	f_0.
70.8	27.0	37.0		
70.3	26.6	37.9		μ
70.9	26.8	37.0	10.4	42.6
70.6	26.4	37.0		
70.9	26.7	37.0		
70.3	——	——		
	27.8	38.2		
	27.7	38.0		
	27.9			

Again. 70.57 div. of scale=.002 inch.

70.8	22.7	33.1		
70.2	24.7	35.4		
70.7	25.4	35.8		
70.7	25.9	36.3		
70.4	25.3	35.9		
70.6	25.5	36.1	10.24	42.0
	26.0	36.2		
	26.4	36.1		
	26.2	36.5		
	26.0	35.9		
	25.8	36.8		
	27.0	37.0		
	27.5			

Again. Draw-tube shortened; 97.25 div. of scale=.004 inch.

.004 inch.		Wt. on.	Wt. off.	Diff.	f_0.
97.1	96.8	24.1	32.1		
97.3	96.9	25.0	31.9		
96.8	97.2	24.1	31.3		
97.5	97.3	24.0	31.3	7.49	44.6
97.4	97.7	23.8	31.5		
97.6	97.7	23.9	31.8		
97.2	97.3	24.6	31.9		
97.0		24.2	31.7		
		23.9			

1879, NOVEMBER 13.

5.30 p. m., and machinery stopped. Scale put on 46^{cm}.6 *in front* of agate; 98.38 div. of scale= .004 inch.

Scale readings.

.004 inch.	Wt. on.	Wt. off.	Diff.	f_0.
98.3	2.6	10.0		
98.3	3.4	9.2		
98.5	2.8	9.0		
98.2	2.2	8.8		
99.0	3.1			
98.2	——	——		μ
98.2	2.8	8.9	6.43	37.8
98.3	2.1	8.9		
	2.0	8.6		
	2.9	8.3		
	2.8	9.0		
	2.0	10.0		
	3.1	9.1		
	3.0			

Again. Draw-tube lengthened; 97.84 div. of scale=.003 inch.

.003 inch.	Wt. on.	Wt. off.	Diff.	f_0.
97.8	26.8	36.7		
97.6	28.2	38.1		
98.1	27.8	37.2		
98.0	28.0	36.8		
97.9	27.9	37.6	9.01	40.0
97.8	29.2	37.2		
97.6	28.0	37.4		
97.9	29.7	37.2		
	——	——		
	29.8	39.1		
	28.3	37.7		
		36.8		

Again.

Wt. on.	Wt. off.	Diff.	f_0.
17.4	26.2		
18.1	27.7		
19.0	27.2		
18.4	27.0	8.65	38.4
18.1	26.9		
17.8	26.6		
17.8	25.9		
17.9			

1879, NOVEMBER 16.

Morning. Draw-tube=1.35; 82.28 div. of scale=.003 inch.

Scale readings.

.003 inch.	Wt. on.	Wt. off.	Diff.	f_0.
82.0	4.0	10.0		
82.8	4.3	10.5		
82.2	4.5	10.2		
82.1	4.8	9.9		
82.0	4.1	9.8	5.93	31.3

Scale readings.

.003 inch.	Wt. on.	Wt. off.	Diff.	f_0.
82.0	4.6	10.4		
82.5	4.1	10.2		
82.5	4.3	10.9		
82.4	4.1	10.9		
	5.2			

Draw-tube lengthened to 5.5; 76.14 div. of scale=.002 inch.

.002 inch.	Wt. on.	Wt. off.	Diff.	f_0.
75.8	10.9	19.8		
76.5	11.0	20.2		
76.3	11.1	20.3		
76.1	11.3	20.3		
76.2	12.2	21.9	8.94	μ 34.0
76.0	13.8	24.0		
76.1	13.2	22.0		
	13.2	20.8		
	13.0			

Again. Draw-tube 4.0; 67.39 div. of scale=.002 inch.

.002 inch.		Wt. on.	Wt. off.	Diff.	f_0.
67.2	67.8	27.2	34.8		
67.0	67.7	27.3	35.0		
67.4	66.9	26.9	34.2	7.70	33.1
67.3	67.2	27.2	35.1		
67.5	67.9	27.7	35.1		
		27.2	35.9		
		28.0			

Again.

Wt. on.	Wt. off.	Diff.	f_0.
28.9	37.0		
28.5			
28.0	36.0		
28.1	36.0		
28.2	37.0		
28.3	37.8		
29.9	38.0		
30.0	38.1	8.23	35.4
30.0	38.1		
30.1	38.0		
30.5	38.0		
29.1	37.9		
30.0	40.0		
32.3	41.0		
33.0	41.1		
33.2			

Scale put on 46^{cm}.6 behind middle of knife-edge. Draw tube=0; 98.02 div. of scale=.004 inch.

.004 inch.	Wt. on.	Wt. off.	Diff.	f_0.
97.9	38.2	22.4		
97.7	37.8	22.2		
97.9	38.2	22.2		
98.3	38.4	22.7	15.46	μ 91.3
98.4	38.2	23.0		
98.2	38.1	23.1		
97.9	38.1	23.2		
97.9				

Again, tube=5.4; 37.94 div. of scale=.001 inch.

.002 inch.		Wt. on.	Wt. off.	Diff.	f_0.
76.0	75.9	33.1	10.1		
76.2	75.5	33.2	9.0		
75.9	76.1	33.0	9.7	23.20	μ 88.5
76.2	76.2	33.1	9.1		
	76.6	33.1	10.8		
		33.7	10.7		
		33.9			

Again.

	Wt. on.	Wt. off.	Diff.	f_0.
75.1	41.6	18.3		
75.3	41.5	18.1		
75.8	41.6	18.0	23.40	89.2
76.0	42.0			
76.1	42.2	19.0		

1879, NOVEMBER 19.

Scale 46^{cm}.6 *behind* agate. Tube 0; 96.96 div. of scale=.004 inch.

Scale readings.

.004 inch.	Wt. on.	Wt. off.	Diff.	f_0.
97.0	28.9	14.4		
96.9	29.3	14.7		
97.1	29.1	15.0		
96.8	29.8	15.8	14.44	μ 86.3
97.0	30.1	15.7		
	30.3	16.0		
	30.4	15.7		
	30.2	15.8		
	30.0			

Again, tube=5.4; 74.69 div. of scale=.002 inch.

.002 inch.	Wt. on.	Wt. off.	Diff.	f_0.
74.0	22.7	0.2		
74.7	22.9	0.0		
74.8	22.8	0.9		
74.9	23.1	1.2	22.36	86.6
74.9	23.3	0.3		
74.7	23.2	1.0		
	22.3	0.5		
	23.2	0.8		
	23.2			

Again, tube 3.6; 98.7 div. of scale=.003 inch.

.003 inch.	Wt. on.	Wt. off.	Diff.	f_0.
98.7	36.9	17.1		
98.7	36.8	16.8		
98.7	36.3	16.8		
	35.9	16.3	19.80	87.2
	36.0	16.2		
	36.1	15.9		
	35.0	15.3		

Again, tube 1.8; 85.78 div. of scale=.003 inch.

.003 inch.	Wt. on.	Wt. off.	Diff.	f_0.
85.7	36.1	19.1		
86.0	36.0	19.0		
85.8	36.8	19.9	17.01	86.1^{μ}
85.5	36.1	19.0		
85.9	36.1	19.0		
	36.0			

NOTE.—When a wagon passes by on the street (ground somewhat frozen), the agitation of the apparatus is so violent that the lines wholly disappear. Tremor estimated at 5^{div} (sometimes even 12^{div}, when a wagon is moving very rapidly and is exactly opposite); $1^{\text{div}}=0^{\mu}.889$.

Scale $46^{\text{cm}}.6$ in front. Tube 1.8; 85.08 div.=.003 inch.

85.0	30.0	37.0		
85.3	29.1	37.3		
85.1	29.0	36.7		
85.0	29.1	36.8		
85.0	28.8	35.9	7.81	39.9
	28.0	36.7		
	29.0	36.1		
	28.8	36.7		
	28.8	37.3		
	29.2			

Again. Tube 4.8; 70.94 div. of scale=.002 inch.

.002 inch.	Wt. on.	Wt. off.	Diff.	f_0.
70.9	7.3	16.9		
71.0	7.0	16.2		
70.7	6.2	15.0		
71.1	5.9	14.3		
71.0	5.9			
	6.0	15.9	9.38	38.3
	5.8	14.4		
	4.3	14.2		
	5.3	14.3		
	5.2	15.2		
	4.3	14.3		
	5.6	14.2		
	5.0			

1879, NOVEMBER 23.

Flexure apparatus readjusted. Pieces of hoop iron substituted for heavier strips. So much agitation that experiments were postponed.

1879, NOVEMBER 26.

Scale $43^{\text{cm}}.5$ *forward* of middle of knife-edge, and 111^{cm} *below*. Tube 5, with ⅔ objective; 98.9 div. of scale=.009 inch.

.009 inch.	Scale readings. Wt. on.	Wt. off.	Diff.	f_0.
98.8	11.1	23.0		
98.4	10.9	23.7		
98.9	9.7	22.2		
99.0	9.2	20.7	12.73	167.6^{μ}
99.0	7.9	20.0		
99.1	6.5	19.4		
99.0	6.2	18.9		
99.0	5.7			

Summary of statical observations with weight and pulley made at York upon Geneva support. F_0=flexure at middle point of knife-edge; A=distance of middle point to intersection of axis with knife-edge; B=distance in a vertical line from middle point to axis.

$46^{cm}.6$ forward of knife-edge, f_0= $38^{\mu}.6$; Nov. 9, $43^{\mu}.1$; Nov. 13, $38^{\mu}.7$; Nov. 16, $33^{\mu}.5$; Nov. 19, $39^{\mu}.1$.
$46^{cm}.6$ back of knife-edge, f_0= 87.7; Nov. 9, 86.7; Nov. 16, 89.7; Nov. 19, 86.6.
$43^{cm}.5$ forward, 111^{cm} below, f_0=167.6.

$\therefore F_0 = 63.1.$

$F_0 : A = 10^{-4}\, 0.527 \quad A = 119.8.$

$F_0 : B = 10^{-4}\, 1.148 \quad B = 54.9.$

1879, DECEMBER 7.

Dynamical flexure. Scale $52^{cm}.5$ *behind* middle of knife-edge. In these and the following experiments the silver arc is always carefully placed with its zero exactly under the pendulum point at rest; 90.39 div. of scale=.003 inch.

.003 inch.	Arc.	Scale readings.		Diff.	f_0.
					μ
90.0	.0350	9.2	13.7	4.5	77.6
90.7	346	10.1	14.6	4.5	78.3
90.5	341	10.3	13.9	3.6	63.2
90.0	330	13.0	16.8	3.8	69.2
90.4	328	12.4	16.5	4.1	75.2
90.9	325	12.4	15.8	3.4	62.6
90.2	319	12.3	16.1	3.8	71.6
	296	16.2	20.0	3.8	77.1
	294	14.8	18.2	3.4	69.2
	292	15.1	18.8	3.7	76.5
	290 rej.	13.8	18.2		....
	285	16.3	19.1	2.8	....
	282	15.4	18.4	3.0	63.8
	280	17.0	20.2	3.2	68.6
	278	17.0	20.2	3.2	71.6
	269	17.0	20.4	3.4	75.9
					71.5

1879, DECEMBER 14.

Scale $52^{cm}.5$ *behind*; 1 div. of scale=$0^{\mu}.843$; dynamical.

Arc.	Scale readings.			Diff.	f_0.
					μ
.0374	29.5	24.7		4.8	77.1
369	32.3	27.8		4.5	73.4
365	6.8	2.7		4.1	67.4
361	8.9	4.2		4.7	78.9
352	11.9	8.2		3.7	63.2
348	13.1	8.8		4.3	74.0
344	14.7	10.8		3.9	68.0
341	16.2	12.3		3.9	68.6
321	11.2	7.2		4.0	74.6
319	11.6	7.8		3.8	71.6
317	11.3	7.5		3.8	72.2
314	11.3	7.3		4.0	76.5
310	11.0	7.1		3.9	75.9
309	12.0	7.9		4.1	80.1
308	10.8	7.0		3.8	74.0
306	10.8	7.2	good	3.6	70.2
305	10.8	7.1	good	3.7	72.8
					72.9

Again, evening; 81 div. of scale=.003 inch.

.003 inch.	Arc.	Scale readings.		Diff.	f_0.
					μ
80.9	.0353	4.2	0.3	3.9	73.9
81.0	351	3.9	0.2	3.7	70.6
81.0	350	4.9	1.1	3.8	73.2
	348	5.0	1.2	3.8	73.2
	345	5.0	1.4	3.6	69.9
	342	13.3	9.7	3.6	70.6
	339	13.1	9.8	3.3	65.2
	335	13.4	9.8	3.6	71.9
	333	13.6	10.0	3.6	72.6
	330	14.4	10.7	3.7	75.3
	328	14.0	11.0	3.0	61.2
	325	14.8	11.2	3.6	74.6
					71.2

Statical flexure. Same position as before; 35 div. of scale=.001 inch. Readings taken in two positions of pendulum, zero and .0370 out.

.002 inch.	Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
70.0	.0370	0.7	3.2		
70.0		0.2	2.9		
		0.2	2.3		
		0.0	3.0	div.	μ
		0.2	3.0	2.50	70.0
		0.1	2.4		
		0.3	2.9		
		0.2	2.3		
		−0.1	2.0		
		−0.1	1.9		
		−0.9	1.7		
		−0.8	1.6		
		−0.9			

Scale 46^{cm}.0 in front of knife-edge; 69.74 div.=.002 inch.

.002 inch.	Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
69.8	.0370	27.0	25.1		
70.4		26.3	24.9		
69.1		25.8	23.9		
69.7		25.3	23.8		
69.6		25.3	23.2	div.	μ
69.2		23.9	22.2	1.65	46.6
70.2		23.2	22.2		
69.9		23.1	21.6		
69.8		23.0	21.0		
		23.0	21.5		
		22.6	20.8		
		22.4	20.9		

Dynamical flexure. Scale 46^{cm}.0 in front of middle of knife-edge; 34.72 div. of scale=.001 inch.

.002 inch.	Arc.	Scale readings.		Mean diff.	f_0.
					μ
69. 6	.0363	25. 3	22. 3	3. 0	43. 2
69. 3	359	24. 8	21. 8	3. 0	43. 7
69. 4	356	24. 3	21. 4	2. 9	42. 6
	350	22. 6	20. 0	2. 6	39. 0
	343	21. 9	19. 7	2. 2	33. 5
	334	20. 1	17. 9	2. 2	34. 5
	329	19. 5	17. 0	2. 5	39. 7
	323	18. 8	16. 3	2. 5	40. 5
	319	18. 5	15. 7	2. 8	45. 9
	309	25. 0	21. 8	3. 2	....
	305	22. 0	19. 3	2. 7	46. 3
	293	22. 9	20. 3	2. 6	46. 4
	279	22. 0	20. 2	1. 8	33. 7
	276	21. 1	19. 5	1. 6	....
	274	21. 7	19. 4	2. 3	43. 9
	270	23. 0	21. 2	1. 8	34. 9
	261	23. 7	21. 6	2. 1	42. 0
					41. 0

1879, DECEMBER 15.

Statical flexure. Scale 46^{cm}.0 in front of middle of knife-edge; 34.57 div. of scale=.001 inch.

.002 inch.	Arc.	*Scale readings.* Out.	Zero.	Mean diff.	f_0.
69. 1	.0370	16. 0	17. 3		
69. 1		13. 5	14. 7		
69. 2		12. 7	13. 4		
		11. 5	12. 8		
		11. 0	12. 1		
		10. 8	11. 9	div.	μ
		9. 2	10. 5	1. 125	32. 0
		9. 8	11. 0		
		9. 1	10. 0		
		8. 2	9. 1		
		7. 6	8. 7		
		22. 9	24. 1		
		20. 9	22. 2		
		20. 1	21. 2		
		18. 9	20. 0		
		17. 8	19. 0		

Dynamical flexure; 46^{cm}.0 in *front;* 34.57 div. of scale=.001 inch.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0356	18. 0	20. 7	2. 7	39. 8
352	16. 3	18. 8	2. 5	37. 3
348	14. 0	16. 8	2. 8	42. 3
341	13. 8	16. 3	2. 5	38. 5
338	14. 9	17. 1	2. 2	34. 2
334	12. 8	15. 2	2. 4	37. 7
323	13. 5	16. 0	2. 5	40. 6

Arc.	Scale readings.		Diff.	f_0.
				μ
321	12.5	14.8	2.3	39.5
319	12.0	14.4	2.4	43.6
313	11.2	13.8	2.6	39.1
309	11.6	13.9	2.3	39.6
305	10.9	13.2	2.3	36.9
299	9.9	12.0	2.1	
				39.1

Dynamical flexure; $52^{cm}.5$ *behind* middle of knife-edge; 35.35 div. of scale=.001 inch.

.002 inch.	Arc.	Scale readings.		Diff.	f_0.
					μ
70.7	.0355	11.9	16.7	4.8	69.3
70.7	351	10.4	14.8	4.4	64.1
	347	10.1	14.4	4.3	63.6
	345	9.9	14.2	4.3	64.1
	341	9.2	13.7	4.5	67.7
	338	9.6	13.9	4.3	65.2
	334	8.8	13.3	4.5	69.3
	331	8.8	13.2	4.4	68.2
	316	8.3	12.7	4.4	71.3
	312	8.7	12.9	4.2	68.7
	310	8.8	12.8	4.0	66.2
	308	8.0	12.2	4.2	69.8
	305	8.0	12.1	4.1	69.3
	302	8.5	12.4	3.9	66.2
					67.4

Statical flexure. Same conditions as above.

Arc.	*Scale readings.* Out.	Zero.	Mean diff.	f_0.
.0370	9.6	7.4		
	9.3	7.2		
	9.2	6.9		
	9.0	6.8		
	8.4	6.4	div.	μ
	8.4	6.2	2.21	61.3
	9.3	7.1		
	9.8	7.2		
	7.0	4.8		
	6.9	4.7		
	6.9	4.8		
	6.7	4.5		

Statical flexure. Scale $52^{cm}.5$ behind middle of knife-edge; 34.3 div. of scale=.001 inch. *Binding-screws all loosened.*

.002 inch.	Arc.	*Scale readings.* Out.	Zero.	Mean diff.	f_0.
68.6	.0370	27.4	25.1		
68.6		27.7	25.2	div.	μ
		27.1	24.6	2.48	70.9
		26.8	24.3		
		26.8	24.4		
		26.4	23.7		

Dynamical flexure. Same condition as above.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0335	21.9	26.4	4.5	71.0
330	21.7	26.3	4.6	73.7
327	21.7	26.1	4.4	71.2
321	21.2	25.5	4.3	70.9
318	21.3	25.2	3.9	64.9
312	21.0	24.8	3.8	64.4
				69.4

Scale 46cm.0 in front of middle of knife-edge; 33.83 div. of scale=.001 inch. *Binding-screws loose;* dynamical flexure.

.002 inch.	Arc.	Scale readings.		Diff.	f_0.
					μ
67.6	.0362	15.0	17.7	2.7	40.0
67.7	358	12.7	15.8	3.1	46.4
	335	6.7	8.8	2.1	33.6
	334	4.7	7.0	2.3	36.9
	329	2.1	4.7	2.6	42.4
	325	3.0	5.7	2.7	44.5
					40.6

Statical flexure under same conditions.

Arc.	*Scale readings.* Out.	Zero.	Mean diff.	f_0.
.0370	29.0	29.9		
	23.9	25.0		
	19.7	20.6	div.	μ
	19.7	19.2	1.10	31.9
	17.1	18.1		
	15.6	16.7		
	14.4	15.6		

Summary of observations, dynamical and statical, made at York, with pendulum on Geneva support.

Screws wrenched.

		Dynam.	Stat.
46cm.0 forward of knife-edge,	$f_0=$	40 μ.0	39 μ.3
52cm.5 back of knife-edge,	$f_0=$	70 μ.7	65 μ.7
	∴ $F_0=$	56 μ.4	53 μ.4
	A=	181cm	199cm
	F_0 : A	0.312	0.268

Screws loose.

		Dynam.	Stat.
46cm.0 forward of knife-edge,	$f_0=$	40 μ.6	31 μ.9
52cm.5 back of knife-edge,	$f_0=$	69 μ.4	70 μ.9
	∴ $F_0=$	59 μ.9	52 μ.7
	A=	192cm	133cm
	F_0 : A	0.292	0.396

The statical measures are evidently unreliable. The dynamical measures show that the binding screws have no effect.

1879, DECEMBER 21.

Repsold stand; the three legs hand-tightened both above and below. Statical observations with weight and pulley; weight used=1ᵏ; 27.78 div. of scale=.001 inch. Scale 56ᶜᵐ.7 in front of knife-edge.

Scale readings.

.003 inch.		Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
82.8	83.2	16.6	80.7	16.8	div.	μ
83.8	83.3	10.1	73.9	9.9	63.95	368.8
84.1	83.2	4.0	67.5	2.8		
83.0		4.2	67.9	3.8		
		13.2	76.7	12.4		

Now wrench-tightened below and hand-tightened above; 27.78 div. of scale=.001 inch.

Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
16.7	76.9	15.3	div.	μ
12.2	73.9	12.8	61.16	352.7
8.1	69.2	7.9		

Microscope refocused; 27.47 div. of scale=.001 inch.

.003 inch.			Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
82.2	82.6	82.6	20.3	80.4	19.1	div.	μ
81.9	82.4	82.7	19.8	80.0	20.2	60.2	351.2
			20.7	80.2	19.9		

Nuts again wrench-tightened both above and below; 27.77 div. of scale=.001 inch.

.003 inch.	Wt. on	Wt. off.	Wt. on.	Mean diff.	f_0.
83.3	27.2	88.8	27.4		
83.3	27.2	88.4	26.8	div.	μ
83.3	23.4	84.2	23.4	61.22	353.3
	20.6	81.8	20.0		
	19.1	79.8	18.7		

Scale 50ᶜᵐ.1 behind middle of knife-edge. Nuts still wrench-tightened; 27.74 div. of scale=.001 inch.

.003 inch.	Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
83.2	14.3	43.5	14.9		
83.2	15.1	44.0	15.9	div.	μ
83.3	16.3	45.4	16.5	28.82	166.5
83.2	17.5	46.3	17.5		
	19.0	47.9	19.0		

Wrench-tightened below, hand-tightened above.

Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
26.7	58.3	27.3		
3.7	36.2	3.7	div.	μ
2.3	34.8	2.5	32.05	185.2
1.8	34.0	1.6		
1.4	33.2	1.2		
1.1	33.0	1.1		

Evening. Nuts hand-tightened above and below; 27.72 div.=.001 inch.

.003 inch.	Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
83.1	25.5	57.2	25.1		
83.2	23.8	56.0	23.8	div.	μ
83.2	23.3	55.2	22.9	32.08	185.4
	22.0	54.2	21.8		
	21.5	53.3	21.3		

Nuts wrench-tightened below, also on hind leg above, but the two front legs hand-tightene
Feet tightened very slightly. 27.72 div. of scale=.001 inch.

.003 inch.	Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_o
	11.5	36.7	11.5		
	10.4	35.2	10.0	div.	μ
	10.1	34.9	9.9	25.08	145.0
	9.0	34.2	9.2		
	8.7	33.9	8.7		

Again. Wrench-tightened above and below; 27.58 div. of scale=.001 inch.

82.6	10.1	41.4	9.7		
82.9	9.8	41.1	9.8	div.	μ
82.7	9.0	40.3	9.2	31.20	181.2
	9.2	40.2	9.2		
	9.0	40.1	9.2		

Again. Binding-screws of the three feet quite loose. 27.58 div. of scale=.001 inch.

	6.3	39.2	7.1		
	6.3	38.2	6.3	div.	μ
	5.0	36.9	4.4	32.24	187.3
	4.0	36.1	4.0		
	3.7	36.0	3.3		

Binding-screws tight as possible.

	29.9	59.8	29.7		
	29.1	59.0	28.5	div.	μ
	27.6	57.4	27.8	29.94	173.9
	27.3	57.1	26.9		
	26.5	56.3	26.5		

Weight of 2.7 kilos put on stand above.

	12.4	42.1	11.5	div.	μ
	9.2	38.9	9.0	29.84	173.4
	8.0	37.5	7.2		
	6.0	35.3	5.2		
	3.8	33.4	3.4		

Scale 56^{cm}.7 in front of knife-edge. Nuts tightened. Same weight on top of stand; 27.83

83.6	20.9	81.0	19.7		
83.3	13.8	74.5	13.6	div.	μ
83.6	11.0	71.7	10.6	60.66	349.2
	6.4	66.4	5.2		
	1.3	61.4	0.9		

Weight removed from top of stand; otherwise same as above; 27.83 div.=.001 inch.

	32.0	92.4	31.4		
	26.8	87.4	25.8	div.	μ
	23.1	83.7	22.3	60.90	350.6
	13.3	73.8	12.7		
	10.8	71.5	10.4		

Binding-screws loosened; 28.09 div. of scale=.001 inch.

84.6	30.0	95.1	28.2		
84.0	27.2	92.5	26.2		
84.3	23.6	88.7	22.4	div.	μ
84.2	19.8	84.9	18.6	65.66	374.6
	17.1	82.3	16.5		
	15.1	80.4	14.5		
	13.6	78.8	13.4		

Binding-screws *moderately tightened* (about as in earlier experiments).

Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
32.5	93.2	31.9		
31.4	92.0	30.8		
30.4	91.0	30.2	div.	μ
28.1	88.9	27.7	61.03	348.1
25.8	86.7	...		
27.0	88.4	26.8		
26.7	87.7	26.5		

Nuts at top of two front legs hand-tightened; 28.11 div. of scale=.001 inch.

.003 inch.	Wt. on.	Wt. off.	Wt. on.	Mean diff.	f_0.
84.0	15.4	88.6	15.2		
84.7	13.4	87.2	13.6	div.	μ
84.3	16.5	90.2	16.1	73.76	420.5
84.3	15.7	88.7	14.7		
	13.6	87.8	13.2		

NOTE.—The weight put on stand was a very heavy paper weight. Observer troubled all day by tremor; every passer by on the street, every one entering the building, or even the adjoining building, agitates the apparatus so as to make it impossible to read the scale.

Summary of observations with weight and pulley, on Repsold stand, at York.

	Nuts hand-tightened above and below.	Nuts hand above, wrench below.	Nuts wrenched above and below.	Binding-screws extra tight.	Binding-screws loose.	Weight on.
$56^{cm}.4$ forward,	$368^{\mu}.8$	$351^{\mu}.9$	$350^{\mu}.7$	$350^{\mu}.6$	$374^{\mu}.6$	$349^{\mu}.2$
$50^{cm}.1$ behind middle point,	$185^{\mu}.4$	$185^{\mu}.2$	$173^{\mu}.9$	$173^{\mu}.9$	$187^{\mu}.3$	$173^{\mu}.4$
$50^{cm}.1$ behind middle point,	$271^{\mu}.4$	$263^{\mu}.6$	$256^{\mu}.9$	$256^{\mu}.8$	$275^{\mu}.2$	$255^{\mu}.9$
A,	158^{cm}	168^{cm}	156^{cm}	156^{cm}	147^{cm}	156^{cm}
F_0 : A,	1.72	1.57	1.65	1.65	1.75	1.65

1879, DECEMBER 23.

Evening. Still very tremulous; especially when vehicles pass, as they frequently do. Scale $56^{cm}.6$ in front. All tightly wrenched up. No weight on top of stand. Measures all very uncertain. 27.89 div. of scale=.001 inch. Dynamical flexure.

.003 inch.	Arc.	Scale readings.		Diff.	f_0. μ
83.5	.0461	4.2	28.8	24.6	348
83.8	450	5.7	29.2	23.5	340
83.7	433	5.2	27.8	22.6	340
	427	5.3	27.7	22.4	342
	420	6.0	27.5	21.5	333
	393	6.5	27.0	20.5	340
	377	6.2	26.1	19.9	344
	368	5.3	24.7	19.4	343
	353	7.3	25.8	18.5	341
	347	7.3	25.6	18.3	342
	323	7.6	24.4	16.8	339
					342.4

Statical flexure, with same arrangement.

Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
.0500	3.2	16.6		
	3.7	17.0		
	2.9	16.2		
	27.2	39.9		μ
	25.9	38.7	13.08	340.4
	26.3	39.0		
	24.9	38.4		
	24.8	38.0		
	26.4	39.2		

1879, DECEMBER 25.

Morning. Arrangement same as in last observation. Scale 56^{cm}.6 in front of middle of knife-edge; 25.67 div. of scale=.001 inch. Statical flexure.

.003 inch.	Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
77.2	.0500	22.3	23.8		
77.0		21.3	32.2		
76.9		22.2	33.8		
76.9		20.8	32.1	div.	μ
		21.4	32.9	11.53	326.4
		20.6	32.5		
		20.7	32.5		
		27.4	39.0		
		20.5	32.2		
		20.8	32.3		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0485	17.7	39.4	21.7	316
478	17.1	38.8	21.7	321
440	17.3	38.4	21.1	339
431	19.2	39.0	19.8	325
325	19.7	33.3	13.6	296
320	19.8	34.8	15.0	332
315	20.8	35.1	14.3	321
269	5.4	17.4	12.0	315
506	20.8	44.8	24.0	335
502	22.3	45.8	23.5	331
500	22.6	45.1	22.5	318
497	20.7	43.9	23.2	330
494	21.9	44.4	22.5	322
472	22.2	43.4	21.2	317
469	22.0	43.9	21.9	330
465	21.7	43.2	21.5	327
462	22.8	43.6	20.8	318
458	23.0	43.6	20.6	318
				323.5

1880, JANUARY 4.

Same arrangement and position as in last observed; 25.72 div. of scale=.001 inch; statical flexure.

Scale readings.

.300 inch.	Arc.	Zero.	Out.	Mean diff.	f_0.
77.3	.0500	21.1	9.8	div.	μ
77.1		21.2	10.0	11.43	323.9
77.1		20.1	8.4		
		15.0	3.5		

Statical flexure again. Scale 50^{cm}.5 behind middle of knife-edge.

	Arc.	Zero.	Out.	Mean diff.	f_0.
	.0500	16.5	22.3		
		16.3	22.3		
		16.4	22.5	div.	μ
		19.0	25.2	6.09	172.4
		18.8	24.8		
		18.8	24.9		
		19.1	25.4		
		18.9	25.1		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0427	22.6	13.9	8.7	145
419	99.4	90.3	9.1	154
386	23.8	15.7	8.1	149
360	23.8	16.0	7.8	154
357	23.7	16.0	7.7	153
351	74.5	66.8	7.7	155
348	74.3	66.8	7.5	153
480	75.3	64.9	10.4	154
474	75.1	64.7	10.4	155
463	74.8	65.0	9.8	151
455	100.0	90.3	9.7	151
448	100.2	90.8	9.4	149
441	74.2	65.1	9.1	146
438	74.0	64.9	9.1	148
433	73.6	64.6	9.0	148
				150.6

Two front nuts at top hand-tightened; 25.64 div. of scale=.001 inch. Statical flexure.

Scale readings.

.003 inch.	Arc.	Zero.	Out.	Mean diff.	f_0.
77.0	.0500	18.0	22.3		
76.9		18.0	22.2	div.	μ
76.9		19.1	23.5	4.30	121.7
		71.2	75.4		
		19.0	23.4		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0509	22.8	15.6	7.2	100
500	23.0	15.5	7.5	106
489	22.3	15.3	7.0	102
484	22.4	15.3	7.1	104
479	48.4	41.3	7.1	105
467	48.2	41.3	6.9	105
460	48.1	41.2	6.9	106
				104.1

Stand wrench-tightened above, two front feet loosened below; 25.54 div. of scale=.001 inch. Statical flexure.

.003 inch.	Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
76.6	.0500	21.1	27.7		
76.6		21.0	27.6		
76.7		21.0	27.8	div.	μ
		72.9	80.0	6.81	193.0
		73.0	79.8		
		21.2	28.1		
		21.0	27.9		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0496	48.2	35.8	12.4	177
469	24.3	12.4	11.9	180
466	24.7	13.0	11.7	178
457	24.7	13.0	11.7	181
450	25.0	13.8	11.2	176
442	25.1	14.2	10.9	175
437	25.2	14.4	10.8	175
427	25.4	14.8	10.6	176
				177.1

All wrench-tightened. Weight of 2,700^g put on top of stand; otherwise same as preceding. Statical flexure.

Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
.0500	3.8	10.0		
	30.3	36.5		
	31.2	36.8		
	6.2	12.3	div.	μ
	6.9	12.6	5.82	164.9
	7.5	13.0		
	33.6	39.2		
	8.4	14.2		
	8.6	14.3		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0500	14.8	3.8	11.0	156
497	40.3	29.8	10.5	149
491	40.5	30.2	10.3	149
484	40.2	30.3	9.9	145
465	14.7	5.0	9.7	148
460	40.2	30.6	9.6	148
451	14.4	4.9	9.5	149
448	14.4	5.1	9.3	147
444	39.8	30.9	8.9	142
				148.1

Evening. Scale $56^{cm}.8$ in front of middle of knife-edge. Statical flexure.

Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
.0500	15.2	3.6		
	15.3	3.6		
	16.0	4.0	div.	μ
	15.8	4.0	11.75	332.7
	15.8	3.9		
	67.3	55.9		
	67.6	55.8		
	67.7	55.9		

Dynamical flexure; with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0501	29.2	5.4	23.8	336
496	28.6	5.7	22.9	327
486	28.1	5.4	22.7	331
475	27.8	5.8	22.0	328
471	27.8	6.0	21.8	328
469	27.7	5.9	21.8	329
462	27.6	6.1	21.5	32)
454	26.7	5.8	20.9	326
423	26.6	6.9	19.7	330
416	26.3	7.0	19.3	329
411	26.0	6.9	19.1	329
				329.1

Weight taken off from top of stand. Two front feet loosened; 25.63 div. of scale=.001 inch. Statical.

.003 inch.	Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
76.5	.0500	23.6	11.3		
77.0		23.5	11.2	div.	μ
76.9		23.5	11.5	12.17	344.6
		23.4	11.3		
		23.2	10.9		
		22.7	10.7		

Dynamical flexure; with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0500	34.0	9.8	24.2	343
496	34.7	10.5	24.2	346
487	34.8	11.2	23.6	343
480	34.7	11.4	23.3	343
475	33.8	10.9	22.9	341
469	33.8	11.2	22.6	341
456	33.7	11.3	22.4	348
452	33.6	11.3	22.3	349
				344.3

Feet of stand tightened, and two front legs hand-tightened above. Statical flexure; 25.54 div. of scale=.001 inch.

.300 inch.	Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
76.6	.0500	29.8	15.3		
76.7		32.1	17.7	div.	μ
76.6		32.3	18.0	14.34	406.0
		31.8	17.4		
		32.7	18.6		

Dynamical flexure; with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0512	47.0	18.2	28.8	398
506	47.7	19.0	28.7	401
500	47.7	19.7	28.0	396
488	47.5	20.1	27.4	397
475	46.6	20.3	26.3	392
468	47.0	20.7	26.3	398
463	46.7	20.8	25.9	396
451	46.7	21.4	25.3	397
443	47.7	22.7	25.0	399
				397.4

Three thicknesses of blotting-paper put under each foot of pendulum stands. All nuts tight Otherwise same as preceding. Statical flexure.

Arc.	Scale readings. Zero.	Out.	Mean diff.	f_0.
.0500	21.8	8.7		
	22.0	8.5		
	22.1	8.6		
	19.3	6.6	div.	μ
	21.5	8.1	13.36	379.6
	21.7	7.9		
	21.1	7.4		
	22.3	8.8		
	21.3	8.3		

Dynamical flexure; with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
0.509	27.7	1.3	26.4	367
494	53.9	28.7	25.2	361
482	54.0	29.2	24.8	365
477	53.7	29.4	24.3	360
461	53.3	29.7	23.6	362
457	53.7	30.0	23.7	367
446	52.9	30.2	22.7	360
435	53.2	30.7	22.5	366
				365.0

Scale 50cm.5 behind knife-edge, with blotting-paper arrangement, etc., as above. Statical flexure; 25.62 div.=.001 inch.

Arc.	*Scale readings.* Zero.	Out.	Mean diff.	f_0.
.0500	17.3	24.8		
	16.7	24.9		
	16.9	25.3		
	16.6	24.7		
	16.4	24.6	div.	μ
	15.8	24.3	8.28	234.4
	15.4	24.1		
	15.3	23.9		
	15.1	23.4		
	14.7	23.0		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0478	19.2	5.0	14.2	210
469	18.8	5.2	13.6	205
460	18.8	5.4	13.4	206
447	18.7	6.0	12.7	201
439	18.6	5.8	12.8	207
434	18.4	5.7	12.7	207
428	17.8	5.3	12.5	207
418	18.5	5.2	12.3	208
409	16.7	5.0	11.7	202
400	16.3	4.8	11.5	204
394	16.2	4.7	11.5	207
				205.9

1880, January 11.

Flexure in third position, about 52cm.7 in front of knife-edge and 118cm.5 below. All clamped. No weight on stand. Statical flexure. 84.4 div.=.01 inch.

Arc.	*Scale readings.* Zero.	Out.	Mean diff.	f_0.
.0500	72.3	66.6		
	72.8	66.3		
	71.4	66.2	div.	μ
	71.5	65.6	5.87	506
	71.2	65.1		
	71.0	65.1		
	70.8	65.0		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0484	62.8	73.3	10.5	467
476	63.9	74.9	11.0	498
470	64.1	74.7	10.6	487
466	63.3	73.7	10.4	481
461	63.2	73.7	10.5	491
454	63.8	74.0	10.2	485
				484

All tight. Weight of 2,700g on top of stand; otherwise the same. 84.4 div. of scale=.01 inch Statical.

.001 inch.	Arc.	*Scale readings.* Zero.	Out.	Mean diff.	f_0.
84.7	.0500	93.6	87.3		
84.3		93.0	86.6	div.	μ
84.2		92.5	86.4	6.26	540
84.4		92.5	86.3		
		92.4	86.1		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0498	85.2	98.8	13.6	588
474	86.3	98.0	11.7	532
469	86.0	98.0	12.0	552
462	86.0	98.0	12.0	560
442	83.0	94.7	11.7	571
419	86.3	96.8	10.5	541
409	85.3	95.8	10.5	554
401	85.8	96.0	10.2	547
				556

Hand-tightened above; otherwise the same. Statical.

Arc.	*Scale readings.* Zero.	Out.	Mean dift.	f_0.
.0500	75.2	68.7		
	75.0	68.1		
	75.1	68.0	div.	μ
	80.1	73.7	6.79	585
	84.1	77.1		
	84.3	77.4		
	84.4	77.7		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
.0424	25.8	38.7	12.9	655
417	25.4	38.8	13.4	692
413	25.3	38.6	13.3	699
408	25.3	38.1	12.8	677
400	25.7	37.0	11.3	609
388	26.3	37.2	10.9	606
379	26.4	37.9	11.5	653
				656

Statical flexure. Feet unclamped; otherwise same.

Arc.	Zero.	Out.	Mean diff.	f_0.
.0500	48.1	41.7		
	47.4	41.7	div.	μ
	47.4	41.5	6.02	519
	47.9	41.8		
	47.7	41.7		

Scale readings. (Zero, Out)

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0414	38.2	49.6	11.4	593
405	38.2	48.9	10.7	569
398	37.7	48.6	10.9	590
389	38.6	48.7	10.1	560
377	38.6	48.9	10.3	588
				580

Statical flexure, with blotting-paper under feet; otherwise same; 84.16 div.=.01 inch.

.01 inch.	Arc.	Zero.	Out.	Mean diff.	f_0.
84.4	.0500	73.2	65.8		
84.1		72.9	66.2	div.	μ
84.2		73.1	66.0	7.00	603
84.1		73.3	66.1		
84.0		72.7	66.1		

Scale readings. (Zero, Out)

Dynamical flexure; with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
.0369	67.8	78.1	10.3	601
361	67.7	77.9	10.2	610
360	68.0	77.2	9.2	552
340	66.8	75.8	9.0	571
338	67.0	76.3	9.3	582
333	67.0	76.0	9.0	552
328	67.8	76.2	8.4	
				583

1880, JANUARY 21.

Third position; about 53^{cm} in front of middle of knife-edge, and 120^{cm} below; 84.72 div. of scale=.01 inch. Statical flexure.

.01 inch.	Arc.	Scale readings.		Diff.	f_0.
84.7	.0500	81.9	88.7		
84.3		82.3	88.6	div.	μ
84.6		82.0	88.4	6.38	546.4
85.0		82.9	88.9		
85.0		82.4	88.8		

Dynamical flexure; with same arrangement.

				μ
.0507	82.2	94.9	12.7	537
500	82.2	96.0	13.8	591
496	82.3	95.2	12.9	556
488	82.8	94.8	12.0	526
478	82.8	94.8	12.0	537
464	82.7	94.7	12.0	554
				550.8

Same as above, except that a weight of 25 pounds is put on top of stand. Statical flexure.

Arc.	Scale readings.		Diff.	f_0.
.0500	35.5	41.7		
	35.9	41.8		
	35.5	41.6	div.	μ
	34.8	41.3	6.15	526.9
	35.0	41.1		
	34.9	41.0		

Dynamical flexure; with same arrangement.

.0488	34.8	47.1	12.3	539
481	35.2	46.8	11.6	516
476	35.0	46.8	11.8	531
468	35.0	46.9	11.9	544
459	34.3	45.8	11.5	537
439	35.0	45.4	10.4	507
				529.5

1880, JANUARY 23.

Second position; 50cm.8 behind knife-edge. No weight on; 26.27 div.=.001 inch. Statical flexure.

.003 inch.	Arc.	Scale readings.		Diff.	f_0.
78.9	.0500	4.7	10.6		
79.0		3.0	7.2		
78.7		9.6	16.0		
78.7		24.7	32.0		
		25.1	31.8		
		26.5	33.7	div.	μ
		6.0	12.4	6.23	172.1
		11.0	17.9		
		13.2	19.0		
		1.3	6.8		
		21.3	27.9		
		22.4	28.3		
		28.3	34.3		
		25.1	30.9		
		23.2	30.0		

Dynamical flexure; with same arrangement.

.0329	12.7	19.1	6.4	134
309	12.4	19.4	7.0	121
304	18.4	24.7	6.3	143
298	17.1	24.3	7.2	166
259	23.6	29.8	6.2	165
389	23.8	33.7	9.9	175
338	23.4	32.3	8.9	181
319	27.3	35.4	8.1	175
294	2.5	9.4	6.9	162
289	3.2	9.2	6.0	143
278	4.2	10.4	6.2	154
276	3.5	10.2	6.7	168
265	3.9	10.0	6.1	159
259	5.2	11.3	6.1	162
				157.6

The mean of the last swinging is 164μ.3.

1880, JANUARY 25.

Second position; 25 pounds weight on stand. Tube lengthened; 27.44 div. of scale=.001 inch Statical flexure.

Arc.	Scale readings.		Diff.	f_0.
.0500	8.4	2.3		
	7.4	1.4		
	8.3	1.7		
	8.3	1.8	div.	μ
	7.2	0.1	6.57	173.8
	9.9	3.2		
	9.6	3.2		
	10.0	3.3		
	11.3	4.5		
	11.5	4.7		

Dynamical flexure; with same arrangement.

				μ
.0478	7.7	20.6	12.9	178
467	8.8	20.8	12.0	170
455	9.4	20.9	11.5	167
439	10.4	21.2	10.8	163
432	10.2	21.2	11.0	169
422	10.2	20.7	10.5	165
416	10.3	20.6	10.3	164
392	11.8	21.5	9.7	163
384	12.0	22.2	10.2	176
				168.3

Weight taken off; otherwise same. Statical flexure.

.003 inch.	Arc.	Scale readings.		Diff.	f_7.
82.4	.0500	14.1	7.3		
82.8		14.5	8.3		
82.5		15.2	8.7		
82.3		16.7	9.6	div.	μ
82.1		17.0	10.4	6.59	174.3
82.2		16.9	10.4		
82.2		23.4	16.7		
82.1		23.1	16.8		

Dynamical flexure, with same arrangement.

				μ
.0482	18.3	30.9	12.6	173
474	19.2	31.6	12.4	173
461	18.3	30.6	12.3	176
457	18.3	29.8	11.5	167
450	20.0	31.7	11.7	172
444	19.8	31.1	11.3	168
437	20.2	31.2	11.0	167
424	21.0	31.6	10.6	165
				170.3

First position: about $57^{cm}.0$ in front of knife-edge; 26.85 div. of scale=.001 inch. Statical. No weight on.

0.500	15.5	3.4		
	15.8	4.0	div.	μ
	16.5	4.5	12.07	326.3
	17.1	4.9		
	17.8	5.5		
	17.6	5.6		

Dynamical flexure, with same arrangement.

.003 inch.		Arc.	Scale readings.		Diff.	f_0.
80.7	80.3	.0481	11.3	34.2	22.9	322$^\mu$
80.5	80.6	473	11.3	33.8	22.5	322
80.8	80.9	467	12.0	34.0	22.0	318
80.1	80.6	459	11.8	34.0	22.2	327
		448	12.2	33.9	21.7	327
		441	13.3	34.4	21.1	324
		437	13.4	34.2	20.8	322
						322.9

Weight on stand. Statical flexure. Otherwise same.

.0500	20.0	32.3		
	21.8	34.1	div.	μ
	22.5	34.6	12.17	329.0
	22.7	35.1		
	23.4	35.4		
	23.7	35.6		

Dynamical flexure, with same arrangement.

.0483	10.8	34.1	23.3	326$^\mu$
467	11.3	33.8	22.5	326
458	10.9	32.4	21.5	317
446	11.7	33.3	21.6	327
438	11.7	32.7	21.0	324
429	12.7	33.6	20.9	329
419	12.6	32.7	20.1	324
406	13.0	32.4	19.4	323
				324.6

Summary of dynamical and statical observations, with pendulum on Repsold stand, made at York.

All tight; no load.

		Statical.	Dynamical.
56^{cm}.6 forward,	f_0	326$^\mu$.4	323$^\mu$.5
50^{cm}.5 back,	f_0	172$^\mu$.4	150$^\mu$.6
52^{cm}.7 forward, 118^{cm}.5 below,	f_0	504$^\mu$.4	482$^\mu$.9
Inclination of axis to knife-edge,		42°.9	49°.0
Distance of axis from middle of knife-edge,		1^m.159	1^m.086
f_0 at 1 cm. from axis,		2$^\mu$.114	2$^\mu$.137
	F_0	245$^\mu$.0	232$^\mu$.2

The same arrangement.

57^{cm}.0 forward,	f_0	326$^\mu$.3	322$^\mu$.9
50^{cm}.8 back,	f_0	173$^\mu$.2	167$^\mu$.3
53^{cm}.0 forward, 120^{cm} below,	f_0	546$^\mu$.4	550$^\mu$.8
Inclination of axis to knife-edge,		37°.0	36°.5
Distance of axis from middle of knife-edge,		1^m.040	0^m.993
f_0 at 1 cm. from axis,		2$^\mu$.359	2$^\mu$.424
	F_0	245$^\mu$.3	240$^\mu$.6

All tight; load of $2^{k}.7$.

		Statical.	Dynamical.
$56^{cm}.6$ forward,	f_0	$332^{\mu}.7$	$329^{\mu}.1$
$50^{cm}.5$ back,	f_0	$164^{\mu}.9$	$148^{\mu}.1$
$52^{cm}.7$ forward, $118^{cm}.5$ below,	f_0	$539^{\mu}.6$	$556^{\mu}.0$
Inclination of axis to knife-edge,		$41^{\circ}.1$	$40^{\circ}.6$
Distance of axis from middle of knife-edge,		$1^{m}.023$	$0^{m}.899$
f_0 at 1 cm. from axis,		$2^{\mu}.385$	$2^{\mu}.597$
	F_0	$244^{\mu}.0$	$233^{\mu}.4$

All tight; load of $11^{k}.3$.

$57^{cm}.0$ forward,	f_0	$329^{\mu}.0$	$324^{\mu}.6$
$50^{cm}.8$ back,	f_0	$173^{\mu}.8$	$168^{\mu}.3$
$53^{cm}.0$ forward, 120^{cm} below,	f_0	$526^{\mu}.9$	$529^{\mu}.4$
Inclination of axis to knife-edge,		$40^{\circ}.3$	$39^{\circ}.5$
Distance of axis from middle of knife-edge,		$1^{m}.009$	$1^{m}.063$
f_0 at 1 cm. from axis,		$2^{\mu}.226$	$2^{\mu}.277$
	F_0	$246^{\mu}.9$	$242^{\mu}.0$

Front taps above hand-tightened.

$56^{cm}.6$ forward,	f_0	$406^{\mu}.0$	$397^{\mu}.4$
$50^{cm}.5$ back,	f_0	$121^{\mu}.7$	$^{\mu}.1$
$52^{cm}.7$ forward, $118^{cm}.5$ below,	f_0	$584^{\mu}.9$	$656^{\mu}.0$
Inclination of axis to knife-edge,		$58^{\circ}.9$	$50^{\circ}.3$
Distance of axis from middle of knife-edge,		$0^{m}.850$	$0^{m}.681$
f_0 at 1 cm. from axis,		$3^{\mu}.099$	$3^{\mu}.559$
	F_0	$255^{\mu}.7$	$242^{\mu}.4$

Tight above; binding-screws of front feet loose.

$56^{cm}.6$ forward,	f_0	$344^{\mu}.6$	$344^{\mu}.3$
$50^{cm}.5$ back,	f_0	$193^{\mu}.0$	$177^{\mu}.1$
$52^{cm}.7$ forward, $118^{cm}.5$ below,	f_0	$518^{\mu}.9$	$580^{\mu}.3$
Inclination of axis to knife-edge,		$43^{\circ}.0$	$37^{\circ}.4$
Distance of axis from middle of knife-edge,		$1^{m}.273$	$0^{m}.995$
f_0 at 1 cm. from axis,		$2^{\mu}.077$	$2^{\mu}.572$
	F_0	$264^{\mu}.5$	$255^{\mu}.9$

All tight; blotting-paper under the feet.

$56^{cm}.6$ forward,	f_0	$379^{\mu}.6$	$365^{\mu}.0$
$50^{cm}.5$ back,	f_0	$234^{\mu}.4$	$205^{\mu}.9$
$52^{cm}.7$ forward, $118^{cm}.5$ below,	f_0	$603^{\mu}.4$	$580^{\mu}.1$
Inclination of axis to knife-edge,		35 .0	$38^{\circ}.7$
Distance of axis from middle of knife edge,		$1^{m}.282$	$1^{m}.179$
f_0 at 1 cm. from axis,		$2^{\mu}.362$	$2^{\mu}.384$
	F_0	$302^{\mu}.9$	$281^{\mu}.0$

These measures are interesting as showing that while the dynamical flexure at the point of application of the force is constantly less than the statical, yet the angular flexure is less in the statical experiments than in the dynamical ones. This fact probably indicates the cause of the difference between the two kinds of flexure, namely, that in the dynamical experiments the flexure-wave has not had time to fully reach the distant parts of the apparatus.

1880, MARCH 17.

Top of Repsold stand fastened to an oak plank, which is bolted to top of Geneva tripod. Statical observations with weight (1[k]) and pulley; 76.3 div. of scale=.003 inch. Scale 56[cm].8 in front of middle of knife-edge.

Scale readings. Wt. on.	Wt. off.	Mean diff.	f_{μ}
4.0	29.0		
2.3	26.7		
3.9	28.3	div. 24.62	μ 154.7
1.6	26.2		
3.4	28.1		

Scale 50[cm].5 behind knife-edge; 76.7 div.=.003 inch.

6.1	0.3		
1.9	6.7		
21.3	25.3		
25.7	29.8	div. 4.42	μ 28.9
23.0	27.6		
21.2	25.8		

1880, APRIL 5.

Experiments on oak plank support as last used. (It has been removed and put on again since last experiment.) Statical flexure; with weight and pulley. Scale about 50[cm].0 behind knife-edge. Filar micrometer, 1 revolution=100[μ].

Micrometer readings. Wt. on.	Wt. off.	Diff.	f_{μ}
4.186	4.150	ρ 0.036	
.218	.180	.038	
.242	.216	.026	
.291	.256	.035	
4.425	4.390	.035	
.451	.415	.036	μ 21.9
.484	.452	.032	
.523	.492	.031	
4.566	4.528	.038	
.595	.562	.033	
.625	.591	.034	
.650	.611	.039	
		.035	

Scale about 56[cm].7 in front of knife-edge; otherwise the same.

3.200	3.022	0.178	
4.427	4.247	.180	
.405	.216	.189	
.385	.215	.170	
.368	.205	.163	
.345	.175	.170	μ 109.1
.269	.092	.177	
.231	.064	.167	
.236	.178	.178	
.236	.165	.165	
.221	.178	.178	
.210	.177	.177	
.203	.157	.157	
		0.173	

Scale about 44cm in front, and about 120cm below the middle point of knife-edge; otherwise same as before. The flexure is now in the reverse direction from the pull.

Micrometer readings.

Wt. on.	Wt. off.	Diff.	f_0.
3.735	3.308	0.427	
.760	.597	.163	
.622	.424	.198	
.808	.343	.465	
4.121	.624	.497	
3.740	.270	.470	
.448	.160	.288	
.926	.277	.649	
.676	.189	.487	
.759	.386	.373	
.660	.480	.180	μ
.585	.378	.207	229.6
4.785	4.550	.235	
5.913	5.668	.245	
.873	.384	.489	
.639	.545	.094	
.438	.186	.252	
.306	4.567	.739	
.073	.910	.163	
.037	.831	.206	
.066	.661	.405	
.112	.640	.472	
.113	.652	.461	
4.943	.714	.229	
5.158	.607	.551	
.103	.756	.347	
.187	.646	.541	
		0.364	

1880, April 17.

Repsold stand supported on oak blocks, six inches high, and braced together. Microscope as in former experiments. Scale 56cm.7 in front of middle of knife-edge; 21.93 div. of scale=.001 inch. Statical flexure with pendulum.

.004 inch.		Arc.	Scale readings.		Diff.	f_0.
87.7	87.5	.0500	5.7	24.0		
87.1	87.9		22.6	40.9		
86.8	88.3		18.9	37.9		
87.6	87.7		10.9	30.9		
87.9	88.0		3.5	21.7	div.	μ
87.6			2.2	20.3	18.72	619.6
88.0			21.8	41.2		
87.9			17.3	36.1		
			15.0	32.6		
			6.5	25.2		
			4.9	23.7		
			17.8	37.2		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0494	42.7	9.6	33.1	554
490	41.8	7.5	34.3	579
484	40.7	7.0	33.7	576
454	32.3	2.8	29.5	538
448	32.7	3.6	29.1	538
438	32.4	3.3	29.1	549
433	30.1	1.2	28.9	552
				556

Scale 50cm.4 behind knife-edge. Otherwise same. Statical.

.0500	22.3	31.7		
	3.8	13.4		
	5.3	14.6		
	5.4	14.0		
	10.0	18.5	div.	μ
	11.0	20.3	9.11	301.5
	18.3	26.0		
	20.0	28.9		
	22.5	31.4		
	3.4	12.2		
	4.7	14.4		
	8.1	17.8		
	10.0	19.5		
	11.0	20.6		

Dynamical flexure, with same arrangement.

.0489	28.4	9.6	18.8	318
484	27.7	8.9	18.8	321
477	26.2	7.8	18.4	319
470	24.8	6.9	17.9	296
465	23.7	6.7	17.0	303
				315

1880, APRIL 18.

Repsold stand raised up on pieces of rubber 2¼ inches thick. Scale 50cm.3 behind knife-edge; 21div.94 of scale=.001 inch. Statical flexure.

Arc.	Scale readings.		Diff.	f_0.
.0500	13.8	42.1		
	16.4	39.3 rej.		
	19.8	48.7		
	20.9	48.6	div.	μ
	23.7	51.8	28.36	939.2
	20.9	49.7		
	23.1	51.6		
	26.0	54.2		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0. μ
.0460	*49.6	2.4	47.2	849
452	49.6	2.8	46.8	856
447	49.3	3.9	45.4	840
438	48.7	4.6	44.1	833
426	49.2	5.3	43.9	853
				846

57cm.0 in front of knife-edge; 21.9 div. of scale=.001 inch. Statical flexure.

.0500	36.8	2.8		
	52.0	17.5		
	48.6	15.7		
	45.6	11.9		
	43.5	12.3	div.	
	41.8	8.3	33.2	1099
	40.8	6.7		
	38.6	6.5		
	37.2	5.1		
	36.3	2.5		
	35.4	1.9		

Dynamical flexure, with same arrangement.

.004 inch.		Arc.	Scale readings.		Diff.	f_0. μ
88.0	87.7	.0509	75.8	14.0	61.8	1008
88.4	87.6	498	71.8	12.7	59.1	985
87.9	87.3	487	70.3	12.4	57.9	987
88.3	87.2	482	68.4	11.3	57.1	984
88.1	87.3	473	66.8	10.0	56.8	997
		467	66.8	11.0	55.8	992
		458	64.4	11.2	53.2	964
						989

Third position; 81.1div. of scale=0.1 inch. Statical flexure.

.0500	19.0	26.3		
	18.2	26.3	div.	μ
	18.7	26.0	7.9	709.8
	18.3	27.1		
	18.3	26.5		

Dynamical flexure, with same arrangement.

.01 inch.	Arc.	Scale readings.		Diff.	f_0. μ
81.2	.0320	15.8	23.6	7.8	545
80.9	304	15.8	22.8	7.0	514
81.2	288	15.3	22.3	7.0	543
	278	15.1	22.2	7.1	570
	270	15.2	21.8	6.6	545
	262	15.2	21.6	6.4	545
					543

* The first figure in this column is everywhere recorded as 5; the observer, however, notes that it should be 4.

1880, APRIL 21.

Flexure observed again on rubber feet. Scale $50^{cm}.2$ behind knife-edge; 22.17 div. of scale = .001 inch. Statical flexure.

.004 inch.	Arc.	Scale readings.		Diff.	f_m
88.6	.0500	15.6	38.9		
88.8		13.2	37.8		
88.8		13.8	38.8		
88.6		18.8	43.5		μ
		21.0	44.5	24.5	803.8
		19.7	43.7		
		19.3	44.9		
		18.9	43.9		
		19.8	43.8		
		19.2	44.1		

Dynamical flexure, with same arrangement.

				μ
.0472	45.8	5.2	40.6	707
463	48.7	8.4	40.3	715
456	48.7	9.5	39.2	707
450	48.7	9.4	39.3	718
443	48.0	9.4	38.6	716
				713

$57^{cm}.1$ in front of knife-edge; 21.99 div. of scale = .001 inch. Statical flexure.

.004 inch.	Arc.	Scale readings.		Diff.	f_m
88.7	.0500	47.9	18.7		
88.6		50.0	20.0		
87.6		48.8	18.2		
88.3		48.0	18.3		
87.5		46.3	16.6		
87.8		44.0	12.8	div.	μ
87.9		43.0	11.1	30.1	989.5
88.0		41.2	12.4		
88.0		38.7	7.9		
88.0		35.5	6.2		
		31.2	2.8		
		33.7	2.4		
		31.1	0.6		

Dynamical flexure, with same arrangement.

				μ
.0495	61.7	6.3	55.4	921
488	61.2	6.8	54.4	918
480	58.7	5.3	53.4	915
476	59.4	6.2	53.2	920
467	56.7	4.0	52.7	928
459	55.8	4.0	51.8	929
				922

Block feet again. Scale 56^{cm}.9 forward of middle of knife-edge; 22.19 div. of scale=.001 inch Statical flexure first.

Arc.	Scale readings.		Diff.	f_0.
.0500	22.5	11.3		
	28.5	17.3		
	31.2	20.2	div.	μ
	12.3	1.1	11.14	365.2
	14.9	3.6		
	14.7	3.9		
	13.8	2.5		
	13.6	2.5		

Dynamical flexure, with same.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0459	30.5	10.2	20.3	362
450	36.8	17.3	19.5	355
445	37.2	17.7	19.5	359
439	35.8	16.4	19.4	362
430	36.0	16.7	19.3	368
420	33.7	15.4	18.3	358
414	34.0	16.2	17.8	353
				360

Scale 50^{cm}.1 behind middle point of knife-edge; 22.08 div. of scale=.001 inch. Statical.

0.004 inch.		Arc.	Scale readings.		Diff.	f_0.
88.7	88.7	.0500	21.1	31.0		
88.2	88.8		18.6	28.8		
88.7	88.8		17.7	27.2	div.	μ
87.9			16.5	26.4	10.10	331.2
88.2			13.3	23.9		
88.2			12.4	22.5		
			11.3	21.8		

Another set.

Arc.	Scale readings.		Diff.	f_0.
.0500	12.4	23.6		
	11.4	22.4	div.	μ
	10.6	21.3	10.8	354.1
	11.5	21.8		
	11.5	22.3		

Dynamical flexure, with same arrangement.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0495	31.3	8.0	8.3	386
489	31.0	7.4	23.6	396
483	29.7	7.3	23.4	380
478	29.6	6.8	22.8	391
469	28.8	6.4	22.4	392
467	28.7	6.8	21.9	385
				388

Scale 56^{cm}.9 again in front of knife-edge; 22 div. of scale=.001 inch. Statical flexure.

Arc.	Scale readings.		Diff.	f_0.
.0500	35.9	23.9		
	18.7	8.3	div.	μ
	27.7	18.3	10.8	354.1
	31.0	19.8		
	31.7	20.7		

Dynamical flexure, with same.

Arc.	Scale readings.		Diff.	f_0.
				μ
.0484	31.9	11.7	20.2	342
476	33.6	13.8	19.8	341
469	34.3	14.8	19.5	341
460	36.0	16.8	19.2	342
456	36.8	17.6	19.2	345
				342

1880, APRIL 25.

Observations on *wooden stand.* Much troubled by tremor, probably due in great measure to the irregular heating of the wooden blocks by the illuminating flame. Third position=55cm in front of knife-edge and 118cm below; 71.72 div. of scale=.01 inch. Statical flexure.

Arc.	Scale readings.		Diff.	f_0.
.0500	32.8	36.5 rej.		
	25.0	30.3		
	24.8	30.2		
	32.3	37.4		
	32.4	37.2		
	32.6	38.0		
	32.3	37.8	div.	μ
	35.2	41.1	5.52	558.6
	35.0	41.0		
	36.4	41.6		
	34.4	42.1 rej.		
	36.6	42.3		
	36.2	42.4		
	37.0	42.5		
	36.7	42.3		
	36.8	42.5		
	37.0	42.8		
	36.6	41.4		
	36.9	42.2		
	36.9	43.0		

Dynamical flexure, with same arrangement.

				μ
.0478	32.3	42.2	9.9	524
469	32.0	41.8	9.8	529
453	30.3	40.1	9.8	546
442	30.1	40.1	10.0	572
438	31.2	42.1	10.9	630
437	31.9	41.9	10.0	579
433	31.1	41.0	9.9	579
426	31.8	42.0	10.2	605
419	31.8	41.2	9.4	567
416	32.2	41.2	9.0	546
413	32.3	41.4	9.1	557
412	32.2	41.3	9.1	559
409	32.7	42.1	9.4	582
403	33.3	42.2	8.9	559
400	33.3	42.3	9.0	569
				567

Scale in front of knife-edge about 56cm.8 (not measured). Statical flexure; 21.67 div. of scale=.001 inch.

Arc.	Scale readings.		Diff.	f_0.
.0500	30.0	16.8		
	25.7	12.6		
	25.4	12.4		
	27.2	13.7		
	27.7	14.1		
	30.0	16.6	div.	μ
	32.1	18.4	13.4	448.7
	33.3	20.1		
	27.4	13.4		
	31.4	18.8		
	32.4	19.7		
	56.4	43.2		
	56.7	41.9		

Dynamical flexure, with same arrangement.

				μ
.0480	55.4	32.3	23.1	403
459	57.7	35.8	21.9	399
451	58.7	38.8	19.9	369
443	59.3	38.4	20.9	395
425	59.3	39.6	19.7	388
416	60.7	41.2	19.5	393
399	63.7	45.8	17.9	376
				389

Scale 50cm.4 behind knife-edge. Statical flexure.

.0500	22.8	15.8		
	24.7	18.2		
	36.7	29.8		
	52.8	29.2		
	16.3	47.4		
	22.7	10.8	div.	μ
	23.6	17.2	6.49	217.0
	27.6	17.0		
	27.4	21.0		
	31.4	25.3		
	36.0	29.3		
	41.0	33.3		
	43.4	36.8		

Dynamical flexure, with same arrangement.

				μ
.0495	46.3	30.6	15.7	265
488	47.7	31.4	16.3	280
483	49.3	33.7	15.6	270
478	50.3	34.4	15.9	279
469	51.3	36.3	15.0	268
463	52.8	37.3	15.5	280
426	52.8	38.5	14.3	281
414	53.4	41.3	12.1	244
410	54.9	41.0	13.9	284
407	56.0	43.0	13.0	267
				272

The above experiments of April, 1889, were made with a view of testing experimentally the question of whether the statical or dynamical flexures should be used in reducing the periods of oscillation, or whether some intermediate value would be preferable. The pendulum was actually swung upon all these supports. Unfortunately, the measures of flexure of the excessively flexible supports are extremely discordant.

Repsold support on rubber blocks.

	f_0 forward. Statical.	f_0 forward. Dynamical.	f_0 behind. Statical.	f_0 behind. Dynamical.	f_0 below. Statical.	f_0 below. Dynamical.	F_0 Statical.	F_0 Dynamical.
	μ	μ	μ	μ	μ	μ	μ	μ
April 18,	1099	989	939	846	710	543	1014	913
April 21,	989	922	804	713			891	811

From the circumstances of the experiment, the results of the second day are to be absolutely preferred. The pendulum was swung on this support on April 18 and 20. The measurements of the flexure of the Repsold tripod on an oaken support are still more utterly discordant. The second experiment of April 21 seems to be the best, and this gives $F_0=354^\mu$ for the statical flexure, and the dynamical flexure appears to be very little less. For the stiffest support we have $F_0=62^\mu.9$. statical; dynamical not measured.

The details of the experiments to determine the periods of oscillation on these supports will be given in another report. The following are the results:

Stiffest support.

(Method of transits.)

	Heavy end down.	Heavy end up.
	s.	s.
March 31,	1.006435	1.006473
April 2,	439	478
April 4,	434	483
April 4,	444	475
Mean,	1.006438	1.006477

(Method of eye and ear coincidences.)

	Heavy end down.	Heavy end up.
March 26,	1.006443	1.006468
March 27,	447	475
March 28,	437,	467
March 29,	435	460
Mean,	1.006440	1.006470

Repsold support.

(Method of transits.)

	Heavy end down.	Heavy end up.
	s.	s.
April 7,	1.006499	1.006506
April 30,	491	485
May 2,	500	528
May 3,	500	516
Mean,	1.006498	1.006509

Repsold support.

(Method of eye and ear coincidences.)

	Heavy end down.	Heavy end up.
	s.	s.
March 19,	1.006523	1.006523
March 21,	490	473
March 22,	508	527
March 23,	492	533
June 4,	495	505
June 5,	489	483
June 6,	507	511
June 6,	515	505
Mean,	1.006502	1.006508

Oaken support.

(Method of eye and ear coincidences.)

	Heavy end down.	Heavy end up.
	s.	s.
April 24,	1.006545	1.006530
April 25,	542	526
April 27,	536	521
April 28,	538	539
Mean,	1.006540	1.006529

On India-rubber blocks.

April 18,	1.006706	1.006612
April 20,	703	610
Mean,	1.006705	1.006611

Let us now try the dynamical correction of these periods. For the Repsold support the dynamical $F_0 = 237^\mu$; thence we deduce the corrections for the other supports, as follows:

	Heavy end down.	Heavy end up.
	s.	s.
Period, Repsold support,	1.006500	1.006509
Dynamic correction,	−84	−36
Corrected period,	1.006416	1.006473
Period, stiffest support,	1.006439	1.006475
Period, oaken support,	1.006540	1.006529
Period, on India rubber,	1.006705	1.006611
Apparent corrections:		
Stiffest support,	23	2
Oaken support,	124	56
India rubber,	289	138

The values of F_0, calculated from these corrections, are as follows:

	μ	μ
Stiffest support,	65	13
Oaken support,	352	371
India rubber,	821	915

Of course, extremely little weight is to be attached to the values calculated from "heavy end up." For the stiffest support and the oaken support the result is in very good accord with the statically observed flexure. For the India rubber support the dynamically observed flexure seems to be indicated, or rather something between this and the results of statical measures.

General conclusions.

1. The flexibility of almost any pendulum support has an important effect on the time of oscillation, and should be measured.

2. The flexure rotates the knife-edge about an axis, sometimes not over 60 cm. distant. It is, therefore, altogether erroneous to measure the flexure at any other point than the middle of the knife-edge, unless it be measured at a number of points and reduced to that point.

3. On a properly constructed support the difference between the statical and dynamical flexure should be immaterial. The dynamical flexure is less than the statical, owing to the time required for the transmission of the wave of strain to the more distant parts of the apparatus. The true correction seems to be intermediate between that calculated from the statical and the dynamical flexures, but pretty decidedly nearer to the latter.

4. A support like the Repsold tripod will grow more flexible with time, owing probably to the slight loosening of some parts.

5. Any dirt, cement, or other elastic film under the feet of such a tripod may greatly increase the flexure, as well as the difference between the two kinds.

6. If the flexure is considerable, it is likely to vary from day to day, or even during the course of an experiment.

7. The tightening of the parts may or may not greatly affect the flexure.

8. The loading of the support has no sensible effect.

9. Experiments made with weight and pulley give a larger value for the flexure than those made with the pendulum drawn to one side.

Note on Hardy's Noddy.

The theory of Hardy's noddy is very simple. When two pendulums oscillate on the same support in parallel planes, I have shown (Am. Jour. Sci., third series, xviii, 113) that one of the differential equations is

$$\lambda D_t^2\varphi + D_t^2 s = -\gamma\varphi,$$

where

t is the time;

φ, the instantaneous angle of inclination of one pendulum;

s, the instantaneous linear displacement of its knife-edge from the position of repose;

λ, the virtual length of the pendulum;

γ, the vertical acceleration of each particle, or the constant of force of restoration of the pendulum.

The Hardy's noddy is a pendulum placed on the support of another pendulum so as to oscillate in a parallel plane. Its natural period $\tau = ☉\sqrt{\frac{\lambda}{\gamma}}$ is as nearly as possible equal to T, the period of the main pendulum; but γ, instead of being gravity, is the excess of the force of a spring over gravity and is made to be as small as possible, λ being correspondingly small, so as to give τ the right value. The noddy being very light, the value and changes of s are determined entirely by the main pendulum. We may, therefore, write

$$s = S\cos\frac{t}{T}☉$$

Substituting this value in the differential equation, the solution of the latter is

$$\varphi = \Phi\cos\frac{t-t_0}{\tau}☉ - \frac{1}{\gamma}\frac{S☉^2}{\tau^2-T^2}\cos\frac{t}{T}☉.$$

But the noddy has no oscillation to begin with. This fact is represented by the equations

$$t_0 = 0 \qquad \Phi = \frac{1}{\gamma}\cdot\frac{S☉^2}{\tau^2-T^2}$$

We thus have

$$\varphi=\frac{1}{\gamma}\cdot\frac{S\odot^2}{\tau^2-T^2}\left(\cos\frac{t}{\tau}\odot-\cos\frac{t}{T}\odot\right)=\frac{2}{\gamma}\cdot\frac{S^2\odot^2}{\tau^2-T^2}\cdot\sin\frac{\tau-T}{2\tau T}t\odot\cdot\sin\frac{\tau+T}{2\tau T}t\odot.$$

This equation shows that the noddy oscillates with a period that is a sort of mean between its natural period and that of the large pendulum. The amplitude of oscillation increases from nothing at an initial rate equal to

$$\frac{S\odot^3}{\gamma(\tau+T)\tau T};$$

a rate not much affected by the value of $(\tau-T)$. But the amplitude increases more and more slowly, and reaches its maximum when

$$t=\frac{\tau T}{\tau-T},$$

after which it again diminishes and after the lapse of an equal time vanishes. At the beginning, the phase of motion of the support is $\frac{1}{2}\odot$, and that of the noddy is 0, so that the support is one quadrant ahead. At the time of the first maximum the phase of the support is

$$\left(\frac{1}{2}+\frac{\tau}{\tau-T}\right)\odot$$

and that of the noddy is

$$\frac{1}{2}\frac{\tau+T}{\tau-T}\odot.$$

Subtracting the second from the first we see that the two motions are in opposition. When the motion of the noddy vanishes its phase is a quadrant in advance of that of the support. The motion immediately recommences, but

$$\sin\frac{\tau-T}{2\tau T}t\odot$$

is now negative, and this shows that the difference of phase changes to the opposite quadrant, and that the two oscillations again proceed toward opposition.

We have thus far not taken account of the resistance to the motion of the noddy, although this must evidently be large. In consequence of it, the natural motion of the noddy would be of the form

$$\varphi=\Phi\circledcirc^{-\frac{t}{\theta}\odot}\cos\frac{t}{\tau}\odot.$$

From this we easily infer that the differential equation is

$$D_t^2\,\varphi+2\frac{\odot}{\theta}D_t\varphi+\frac{\odot^2}{\tau^2}\,\varphi=\frac{S}{\lambda}\frac{\odot^2}{T^2}\cos\frac{t}{T}\odot.$$

The solution of this is

$$\varphi=\frac{S}{\lambda}\,\frac{1}{\dfrac{4\tau^2T^2}{\theta^2(\tau^2-T^2)}+\dfrac{\tau^2-T^2}{\tau^2}}\left(\circledcirc^{-\frac{t}{\theta}\odot}\cos\frac{t}{\tau}\odot-\cos\frac{t}{T}\odot\right)+\frac{S}{\lambda}\,\frac{\dfrac{2\tau^2T}{\theta(\tau^2-T^2)}}{\dfrac{4\tau^2T^2}{\theta^2(\tau^2-T^2)}+\dfrac{\tau^2-T^2}{\tau^2}}\sin\frac{t}{T}\odot.$$

The signification of this is that the noddy approaches indefinitely toward settling down to an oscillation strictly synchronous with that of the support. Its ultimate amplitude is very little less than half what the maximum amplitude would be without resistance. But the phase may differ very much from that of the motion of the support. Namely, if the noddy is in precise adjustment to the period of the large pendulum, its phase will be one quadrant behind that of the support. If the noddy naturally oscillates slower than the large pendulum, its phase may be anywhere from one quadrant in arrear to opposition; if the noddy naturally oscillates faster than the pendulum, it may be anywhere from one quadrant behind, to coincidence.

If, then, γ is one tenth of gravity, τ^2-T^2 one thousandth of a second, and S one tenth of a micron, the amplitude of movement of the noddy will be one thousandth of the radius, a quantity easily measured with a microscope.

ON THE INFLUENCE OF THE FLEXIBILITY OF THE SUPPORT ON THE OSCILLATION OF A PENDULUM.

(Translated from French into English by the author.)

NEW YORK, *July* 13, 1877.

DEAR SIR: On taking charge of the Coast Survey researches upon gravity, I ordered of Messrs. Repsold a reversible pendulum, to be a copy of that of the Prussian Geodetical Institute. But the instrument makers were at that time so taken up with the construction of instruments for the Transit of Venus, that the pendulum was only ready in the spring of 1875. I then went to Hamburg to receive it; and from Hamburg I went on to Berlin, where I found General Baeyer rather dissatisfied with the results obtained with the Prussian instrument. He specially mentioned the flexibility of the tripod, a source of error which pendulum experimenters have surely never overlooked. The pendulum apparatus that I had carried with me from America having been ruined in transportation, I was under the necessity of employing the new instrument, and therefore undertook to measure and take account of the error in question.

A pendulum support might be rickety, so that the pendulum in its oscillations should throw the knife-edge plane from one position to another, without its undergoing any resistance to the motion other than inertia and friction, between two fixed points. This, however, does not happen in the case of any of the supports that I have examined; for, upon observing their behavior under a high-power microscope, I have always found that they spring back exactly to their original position after every flexure that I have applied to them. In short, the movement with which we have to do is the oscillatory flexure of an elastic body. The amplitude of the oscillation is, at most, about $\frac{1}{5000}$ of that of the lower knife-edge of the pendulum, so that its square may be neglected.

The plane of support of the knife is itself undoubtedly bent during the movement; but I neglect this and limit myself to the consideration of the movement of its middle point. When to this middle point is applied a horizontal force perpendicular to the knife edge, the latter describes a movement of revolution around an axis which, in the case of the Repsold apparatus, is situated behind and above the tripod at a distance of about a meter from the knife-edge. We can neglect the difference between this movement and a translation, until we come to measure its amount. There is also a minute variation in the vertical pressure of the pendulum on the support, but this is very far from producing any sensible effect on the period of oscillation.

Let us denote by

m the mass of a particle,

r its distance from the knife-edge,

ω the inclination, at rest, to the vertical of the perpendicular let fall from the particle on to the knife-edge,

M the mass of the pendulum,

l the length of the corresponding simple pendulum,

h the distance of the center of mass from the knife-edge,

T the period of the oscillation,

g the acceleration of gravity,

ε the elasticity of the support,

φ the instantaneous inclination of the pendulum to its position of rest,

s the instantaneous displacement of the middle point of the knife-edge from its position of rest,

t the time.

Then, the horizontal velocity of a particle will be

$$r \cos (\varphi + \omega) \mathrm{D}_t \varphi + \mathrm{D}_t s$$

and its vertical velocity will be

$$r \sin (\varphi + \omega) \mathrm{D}_t \varphi.$$

Its living force will, therefore, be

$$\frac{1}{2}mr^2(D_t\varphi)^2+mr\cos(\varphi+\omega)D_t\varphi.D_ts+\frac{1}{2}m(D_ts)^2,$$

and that of the pendulum will be

$$\frac{1}{2}Mlh(D_t\varphi)^2+Mh\cos\varphi.D_t\varphi.D_ts+\frac{1}{2}M(D_ts)^2.$$

The living force of the motion of the support itself may be left out of account since it involves the square of an excessively small velocity.*

The differential of the potential energy is

$$Mgh\sin\varphi.\,d\varphi+\varepsilon s.\,ds.$$

There is really a third term to be added to this expression dependent on the molecular friction of the matter of the support. But I think we may neglect this term; for its effect cannot be very great, and its coëfficient is, at any rate, unknown.†

From the expressions for the living force and potential we deduce the Lagrangian equations

$$lD_t^2\varphi+\cos\varphi.\,D_t^2s=-g\sin\varphi$$

$$-h\sin\varphi.\,(D_t\varphi)^2+h\cos\varphi.\,D_t^2\varphi+D_t^2s=-\frac{\varepsilon}{M}s,$$

or, neglecting terms of the second degree,

$$lD_t^2\varphi+D_t^2s=-g\varphi$$

$$hD_t^2\varphi+D_t^2s=-\frac{\varepsilon}{M}s.$$

[NOTE.—1882, July 24. I omit the solution of these equations as originally given, and substitute the following, which is perhaps less inelegant. Subtracting the second equation from the first, we get

$$(l-h)D_t^2\varphi+g\varphi=\frac{\varepsilon}{M}s$$

or

$$D_t^2s=\frac{M}{\varepsilon}(l-h)D_t^4\varphi+\frac{Mg}{\varepsilon}D_t^2\varphi$$

Substituting this value in the first differential equation, we have

$$\frac{M}{\varepsilon}(l-h)D_t^4\varphi+\left(l+\frac{Mg}{\varepsilon}\right)D_t^2\varphi+g\varphi=0.$$

Separating the operator into factors, and using the abbreviation

$$i=4\frac{Mg}{\varepsilon l}\cdot\frac{1-\frac{h}{l}}{\left(1+\frac{Mg}{\varepsilon l}\right)^2}$$

we get

$$\left[D_t^2+\frac{\varepsilon l+Mg}{2M(l-h)}(1+\sqrt{1-i})\right]\cdot\left[D_t^2+\frac{\varepsilon l+Mg}{2M(l-h)}(1-\sqrt{1-i})\right]\varphi=0.$$

* It is easy to see that the effect of this would be to increase the last term of the living force; this would affect the second of the differential equations just as if M had been multiplied and h divided by the same quantity. But this would not affect the final result. [1882.]

† This is the point to which the greatest objection to my work has been made. [1882.]

The solution of this is

$$\varphi = A_1 \cos\left(\sqrt{\frac{\varepsilon l + Mg}{2M(l-h)}(1-\sqrt{1-i})}\,.\,t+\eta_1\right) + A_2 \cos\left(\sqrt{\frac{\varepsilon l + Mg}{2M(l-h)}(1+\sqrt{1-i})}\,.\,t+\eta_2\right)$$

where A_1, A_2, η_1, η_2 are arbitrary constants. On neglecting the square of $\frac{Mg}{\varepsilon l}$, this reduces to

$$\varphi = A_1 \cos\left(\sqrt{\frac{g}{l}}\left(1-\frac{Mg}{\varepsilon l}\right).\,t+\eta_1\right) + A_2 \cos\left(\sqrt{\frac{\varepsilon}{M}\cdot\frac{1+\frac{Mgh}{\varepsilon l^2}}{1-\frac{h}{l}}}\,.\,t+\eta_2\right)$$

The second term represents a mere tremor, for its period is very short, owing to the large value of ε. The period of the first harmonic constituent is

$$T = \sqrt{\frac{l}{g}+\frac{M}{\varepsilon}}$$

From the value of φ and the first equation of this note, we deduce the following value of s:

$$s = \frac{Mg}{2\varepsilon}\left(-\frac{\varepsilon l}{Mg}(1-\sqrt{1-i})+1+\sqrt{1-i}\right)A_1 \cos\left(\sqrt{\frac{\varepsilon l+Mg}{2M(l-h)}(1-\sqrt{1-i})}\,.\,t+\eta_1\right)$$

$$+\frac{Mg}{2\varepsilon}\left(-\frac{\varepsilon l}{Mg}(1+\sqrt{1-i})+1-\sqrt{1-i}\right)A_2 \cos\left(\sqrt{\frac{\varepsilon l+Mg}{2M(l-h)}(1+\sqrt{1-i})}\,.\,t+\eta_2\right)$$

It thus appears that the amplitude of the principal constituent of s is nearly

$$h\frac{Mg}{\varepsilon l}A_1,$$

while that of the other constituent is nearly $-lA_2$.

To find the best way of starting the pendulum so as to make the ratio of A_2 to A_1 as small as possible, we must consider how to make the initial value of s as nearly as possible $h\frac{Mg}{\varepsilon l}$ times the initial value of φ. Now, it is easy to see that if the pendulum is supported at a point at a distance x from the knife-edge, any yielding of the support will diminish the value of φ as expressed by the equation

$$ds = -x \sec\varphi.\,d\varphi.$$

Substituting this in the expression for the differential of the potential energy, this last becomes

$$Mgh \sin\varphi.\,d\varphi - \varepsilon s\, x \sec\varphi.\,d\varphi.$$

Equating this to zero, we find

$$s = h\frac{Mg}{\varepsilon x}\sin\varphi.\cos\varphi.$$

In order that this should be equal to $h\frac{Mg}{\varepsilon l}\varphi$, it is only necessary to put $x=l$, so that in starting the pendulum the finger or trigger should be applied at the lower knife-edge or center of gyration.]

The elasticity, ε, may be measured by observing the deflection, S, of the support produced by a horizontal force equal to the unit of weight. For

$$\varepsilon = \frac{g}{S}$$

Substituting this value, we find

$$\varphi=\frac{A}{h}\cos\left(\sqrt{\frac{g}{\left(l+MS\frac{h}{l}\right)}}\,t\right)$$

Accordingly, the effect on the pendulum is to give it a virtual length greater than what it would have on a rigid support by $MS\frac{h}{l}$.

Let us denote the duration of an oscillation by T, and let Δ be used to indicate the effects of flexure. Then, since

$$T^2=\varpi^2\frac{l}{g}$$

we have

$$\Delta T^2=\frac{\varpi^2}{g}MS\frac{h}{l}.$$

If we distinguish by subjacent letters the two positions of a reversible pendulum, we have

$$\frac{\varpi^2 l}{g}=\frac{T_d^2h_d-T_u^2h_u}{h_d-h_u}$$

and

$$\Delta l=MS,$$

or putting λ for the length of the second's pendulum

$$\Delta\lambda=MS\frac{\lambda}{l}.$$

To determine the flexure, I fasten in the slot in the plane of suspension of the Repsold apparatus a fish-line passing horizontally in the direction of the pendulum's movement over an Atwood's machine pulley, and on the end of this cord I hang a kilogramme. [With a stronger support, the pendulum itself may conveniently replace the kilogramme.] On the extremity of the plane of suspension, or at the end of an arm attached thereto,* I stick a glass stage micrometer, turned so as to measure in a direction parallel to the impressed force. This scale is looked at by a microscope carrying a filar micrometer, and solidly mounted upon an independent support, the standard of which is a piece of gas pipe about 10 centimeters in diameter.

I now give a brief *résumé* of my results, beginning with the experiments to determine the position of the fixed axis about which the head of the Repsold support rotates during flexure.

A.—*Experiments made on a level with the suspension plane.*

HOBOKEN, *March* 10, 1877. Temperature 13° C.

+ = forward; − = back.

Distance of scale from end of plane.	Flexure in revolutions of the micrometer screw.	
	Observed.	Calculated.
m.		
−0.496	+0.211	+0.209
+0.053	+.0356	+0.358
+0.318	+0.436	+0.431

The calculated quantities suppose that the axis pierces the suspension plane at a distance of $1^m.355$ behind the forward end of the suspension plane.

* This arm is best made of brass tubing, which may be cut out to make it lighter. [1882.]

B.—*Experiments in the vertical of the forward end.*

HOBOKEN, *March* 12, 1877. Temperature 14° C. Observer, Sub-assistant SMITH.

+ = below; − = above.

Position of the scale relative to the suspension plane.	Flexure in revolutions of the micrometer screw.	
	Observed.	Calculated.
m.		
−0.44	+0.196	+0.196
0.000	+0.340	+0.332
+0.395	+0.446	+0.454

The calculated quantities suppose the axis to pierce the vertical of the forward end of the suspension plane $1^{m}.07$ above this plane. It is not at all surprising that the instantaneous axis is above the suspension plane. Let us suppose that the flexure existed exclusively in three feet of the support. In this case the movement of the upper end of each foot would be perpendicular to the general direction of the foot, and at the same time perpendicular to the radius of the circle of revolution, so that the foot would be directed directly towards the fixed axis. The axis is without doubt behind the support, on account of the flexure of the plane itself.

I made experiments at Geneva, Paris, Berlin, and New York, in order to determine S numerically. The experiment at Geneva, made the 13th of September, was only a trial. But I had a good pulley which I had borrowed from the workshop of the Geneva society for the construction of physical instruments, and I got as an approximate value—

$$S = 0^{mm}.034$$

The pulley that I used at Paris had considerable friction, to which can be attributed the fact that the numbers found differ sensibly from those obtained with the aid of better apparatus.

These are the figures—

January 18, 1876, at Messrs. Brünner, Temp. 1° C $S=0^{mm}.0363$
March 7, 1876, at the Paris Observatory, Temp. 9° C $S=0^{mm}.0371$

At Berlin I used a very delicate pulley which turned on friction-whels, in order to diminish the friction. It belonged to the Physical Cabinet of the Institute of Technology of Berlin, and was put at my disposition by the kindness of Professor Paalzow. The micrometric readings were made alternately with and without the weight, making but one reading each time, in order to avoid any error arising from the support of the micrometer, this being made of wood. In the readings made alternately with and without the weight, I ended with the arrangement with which I began (11 for one, and 10 for the other), in order that the mean instant of the observations should be the same for the two arrangements. The value of 1 division of the micrometer screw was measured separately.

Below are the results of the different series—

May 24, 1876, a. m.,	$S=0^{mm}.0340$
Temp. 13° C., p. m.,	$0^{mm}.0339$
	$0^{mm}.0340$
	$0^{mm}.0341$
May 25, 1876, Temp. 13°,	$0^{mm}.0337$
	$0^{mm}.0336$
Mean,	$S=0^{mm}.0339 \pm 0^{mm}.001$

At Hoboken (near New York) I obtained, through the kindness of Professor Morton, an excellent pulley, made in the workshop of the Stevens Institute of Technology. I always made a reading on each one of the lines of the scale before changing the disposition of the weight.

The results of the separate series are—

March 7, 1877, Temp. 15° C.,	$S=0^{mm}.0342$
March 10, 1877, Temp. 12°.	$0^{mm}.0332$
	$0^{mm}.0337$
	$0^{mm}.0343$
	$0^{mm}.0342$
	$0^{mm}.0339$
	$0^{mm}.0334$
These two series should have double weight in the reduction,	$0^{mm}.0342$
	$0^{mm}.0342$
Mean,	$S=0^{mm}.0340\pm 0^{mm}.0001$

In all the experiments made in the different positions of the scale the flexure obtained has been reduced to the center of the knife, and this last is what is called S.

It is to this last value that I give the preference.

It follows, from the experiments described on pages 430–431, made to determine the position of the axis of rotation, that the forward end of the suspension plane is distant from that axis by $\sqrt{1^m.355\times 1^m.07}=1^m.20$. And, since the movement of this end with the weight of a kilogramme is $S+0^{mm}.0008$, the correction $+0.0008$ arising from the reduction from the center of knife to the forward end, it follows that the torsion of the support by that force is $\frac{0^{mm}.0348}{1^m.20}=0.0000290=5''.8$. Although there is nothing to be suspected in this result, I wished to check it by a direct experiment. I attached a mirror at the extremity of the suspension plane, and, with the aid of a telescope, I measured the torsion by the reflection of a scale, and I found it 6″. This method, of course, is not as exact as the other.

In order to arrive at another confirmation of the theory, I made the following observations on the flexure produced by the oscillation of the pendulum itself in its two positions, using a tolerably high-power microscope (*i. e.*, magnifying 500 diameters). The scale used was made by Mr. Rodgers, of Harvard College Observatory. It is divided with extreme exactness, the interval between two lines being $\frac{1}{4000}$ of an English inch. It was fixed 70 millimeters before the center of the knife, which gives a correction to S of $+.0019$.

If Φ is the amplitude of oscillation on each side of the vertical, the double amplitude of the vibration of the scale should be

$$2\ M\ (S+0^{mm}.0019)\frac{h}{l}\Phi$$

in which $M=6.308$ and $\frac{h}{l}=\frac{17}{56}$ or $\frac{39}{56}$ according as the pendulum is suspended by the knife nearest or farthest from the center of gravity. I used this formula in calculating the quantities now given.

DYNAMICAL FLEXURE.

A.—Pendulum suspended by the knife farthest from the center of gravity.

HOBOKEN, *March* 20, 1877.

Φ	Amplitude of the movement of the scale. 1 div. = $\frac{1}{4000}$ inch. Observed.	Calculated.
° ′	*Divisions.*	*Divisions.*
2 32	2.2	2.2
2 30	2.1	2.1
2 24	2.0	2.1
2 22	1.9	2.0
2 20	1.9	2.0
2 19	1.95	2.0
1 43	1.5	1.5
0 47	0.8	0.7

B.—*Pendulum suspended by the knife nearest the center of gravity.*

Φ	Amplitude of the movement of the scale. 1 div. = $\frac{1}{4000}$ in. Observed.	Calculated.
° ′	*Divisions.*	*Divisions.*
2 39	1.0	1.0
2 34	0.9	1.0
2 29	0.9	0.9
2 25	0.9	0.9
2 22	0.8	0.9
2 14	0.8	0.8
2 12	0.8	0.8
2 06	0.7	0.8
2 04	0.75	0.8
1 57	0.75	0.7
1 51	0.75	0.7

In making these observations, I saw distinctly the little subsidiary vibration at the end of each oscillation arising from the second term of the formula.

Finally, I swung the pendulum on two supports of different flexibility—one was the metallic tripod, by Repsold, to which refer the flexure measurements given above; the other was made by fixing the upper part of the Repsold tripod to a thick wooden plank by means of bronze bolts passing through the three holes through which the feet pass. These holes are conical, and the bolts fit exactly. I put on each bolt between the head of the support and the plank a leaden washer, so that, in tightening the bolts and compressing the washers, great stability was obtained and at the same time a horizontal position. The plank (which was 5 centimeters thick) was cut in order to make a place for the pendulum, and it was placed by force between a stone wall and a brick pillar. A slit was then cut, in which a pulley of an Atwood machine was placed to measure the flexure.

Experiments on the flexure of this support.

HOBOKEN, *May* 21, 1877.

Distance of scale before +, behind − of the center of knife in English inches.	Distance of scale to suspension plane in English inches + above, − below.	Flexure in millimeters under a weight 1 kilogramme.	Temperature C.	Observer.
Inches.	*Inches.*	*mm.*	°	
+ 1.2	− 1.3	+0.0052	18.3	E S.
+ 1.2	− 1.3	+ .0052	18.9	E S.
+ 1.2	+39.5	− .0425	20.0	C. S. P.
+13.2	+39.5	− .0367		C. S. P.

It follows that for this apparatus $S^1 = 0^{mm}.0031$, and that the difference between the values of S for the two supports is $0^{mm}.0309$. Now I find $\frac{\pi^2 l}{g} = 1.0125$ sidereal seconds and $l = 1^m$. Hence, we conclude that the difference of $\frac{\pi^2 l}{g}$ according as the pendulum oscillates on one or the other supports must be equal to

$$\frac{\pi^2 l}{g} \frac{M(S-S^1)}{l} = \frac{81}{80} \times 6.308 \times 0.0309 = 0.000197$$

I swung the pendulum three times on the less solid support and once on the most solid to verify the theory. I observed 10 consecutive passages of the pendulum across the vertical at intervals of 5 minutes, using a relay that I invented for this purpose.

A.—*Oscillations on the Repsold metallic support.*

HOBOKEN, *April* 1, 1877.

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
300	301.9652	—0.0130	301.9522	1.006507
296	297.9408	—0.0084	297.9324	528
298	299.9533	—0.0060	299.9473	535
* Mean				1.0065238

PENDULUM SUSPENDED BY THE KNIFE FARTHEST FROM THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
296	297.9094	—0.0092	297.9002	1.006420
302	303.9374	—0.0081	303.9295	389
296	297.9060	—0.0066	297.8994	417
Mean				1.0064067

Hence, we have

$$T_1^2=1^s.0128544$$
$$T_2^2=1^s.0130902$$

And since $h_1 : h_2 = 101 : 44$ we have

$$\frac{\pi^2 l}{g}=\frac{T_1^2 h_1 - T_2^2 h_2}{h_1 - h_2}=1.013\left(1-\frac{101\times 0.0001456+44\times 0.0000902}{57}\right)=1.012672$$

This value is to be corrected for rate of chronometer and temperature. The chronometer lost $0^s.86$ per day, which gives a correction to T^2 of $+0^s.000020$. The temperature during the time the heaviest mass was above was 12°.7 in the mean, and 12°.9 when this mass was below. Hence, to reduce to 13° we must apply a correction of

$$\frac{0.1\times 101-0.3\times 44}{57}=0.0000186=-0.000001$$

Hence we conclude

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ \text{ C.}=1.012691$$

April 7, 1877.

PENDULUM SUSPENDED BY THE KNIFE FARTHEST FROM THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
290	291.8794	—0.0103	291.8691	1.006445
296	297.9131	—0.0086	297.9045	434
298	299.9241	—0.0073	299.9168	432
298	299.9241	—0.0060	299.9181	437
358	360.3090	—0.0058	360.3032	434
Mean				1.0064357

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected interval.	Period.
	s.	s.	s.	s.
288	289.9026	—0.0132	289.8894	1.006560
298	299.9648	—0.0092	299.9556	562
300	301.9760	—0.0067	301.9693	564
298	299.9564	—0.0051	299.9513	548
298	299.9591	—0.0037	299.9524	552
Mean				1.0065578

* When we have a series of equal consecutive intervals if n is the number of intervals and i is the position of one of them we should, in taking the mean, give to this interval the weight $-i\ (i-1)$.

Hence we have

$$T^2_1 = 1^s.0129128$$
$$T^2_2 = 1^s.0131586$$
$$\frac{\pi^2 l}{g} = 1^s.012723$$

Daily correction, $+0^s.44$ $+\ .000010$

Temp. $15^\circ.8$ (both positions) $-\ .000052$

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ \text{ C.} = 1.012681$$

April 8, 1877.

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Number of oscillations.	Interval by chronometer.	Reduction to infinitely small arc.	Corrected time.	Period.
	s.	s.	s.	s.
298	299.9647	−0.0175	299.9472	1.006534
298	299.9549	−0.0111	299.9438	523
298	299.9539	−0.0080	299.9459	530
298	299.9484	−0.0055	299.9429	520
298	299.9481	−0.0039	299.9442	526
Mean				1.0065261

PENDULUM SUSPENDED BY KNIFE FARTHEST FROM CENTER OF GRAVITY.

	s.	s.	s.	s.
298	299.9229	−0.0066	299.9163	1.006431
298	299.9213	−0.0058	299.9155	426
297	298.9125	−0.0049	298.9076	423
299	300.9236	−0.0042	300.9194	419
298	299.9174	−0.0035	299.9136	422
Mean				1.0064246

s.

$$T_1^2 = 1.0128905$$
$$T_2^2 = 1.0130948$$
$$\frac{\pi^2 l}{g} = 1.012733$$

Daily correction, $-0^s.41$ $-\ .000009$

Temp. heavy end up, $13^\circ.2$ }
Temp. heavy end down, $13^\circ.5$ } $-\ .000016$

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ = 1.012708$$

Hence the three experiments on the Repsold support give for the value of $\frac{\pi^2 l}{g}$ at 13° C.

s.

April 1, 1.012691
April 7, 1.012681
April 8, 1.012708

Mean, 1.012693

B.—*Experiments made on the stiffest support.*

HOBOKEN, *May* 14, 1877.

PENDULUM SUSPENDED BY THE KNIFE FARTHEST FROM THE CENTER OF GRAVITY.

Mean instant of 10 transits.			Interval of 298 oscillations.	Reduction to infinitely small arc.	Corrected interval.	Interval of 298 oscillations.	Reduction to infinitely small arc.	Corrected interval.
h.	*m.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*
14	6	22.4307	....	....	....	....	....	....
..	7	22.8245	....	....	....	....	....	....
..	11	22.3337	299.9030	—0.0132	299.8898	....	....	....
..	12	22.7213	....	....	....	299.8968	—0.0126	299.8842
..	16	22.2313	299.8976	—0.0110	299.8866	....	....	....
..	17	22.6209	....	....	....	299.8996	—0.0106	299.8890
..	22	22.5145	....	....	....	299.8936	—0.0087	299.8849
..	23	22.9017	....	....	....	....	....	....
..	27	22.4055	....	....	....	299.8910	—0.0074	299.8836
..	28	22.7949	299.8932	—0.0072	299.8860	....	....	....
..	33	22.6896	299.8947	—0.0060	299.8887	....	....	....

PENDULUM SUSPENDED BY THE KNIFE NEAREST THE CENTER OF GRAVITY.

Mean instant of 10 transits.			Intervals of 298 oscillations.	Reduction to infinitely small arc.	Corrected intervals.
h.	*m.*	*s.*	*s.*	*s.*	*s.*
15	53	22.4041	....	....	....
..	58	22.3579	299.9538	—0.0198	299.9340
..	3	22.3058	299.9479	—0.0131	299.9348
..	8	22.2531	299.9473	—0.0087	299.9386
..	13	22.2119	299.9588	—0.0062	299.9526
..	18	22.1554	299.9435	—0.0044	299.9391
..	23	22.1011	299.9457	—0.0031	299.9426

Mean $T_2 = 1^s.0065104$.

Hence we find—

$$T_1^2 = 1.0127144 \text{ s.}$$
$$T_2^2 = 1.0130632$$
$$\frac{\pi^2 l}{g} = 1.012445$$

Daily corr. to chron. + 2ˢ.59 — .000060

Temp. heavy end down, 14°.18 }
Temp. heavy end up, 15°.00 } — .000010

$$\frac{\pi^2 l}{g} \text{ at } 13^\circ = 1.012495$$

Comparing this value with the one obtained with the other support we find a difference of 0.000198. The difference, according to the computations of the experiments on flexure, ought to have been 0.000197,* which shows a sufficient agreement.

Yours, most faithfully,

[Signed] C. S. PEIRCE,
Assistant United States Coast Survey.

* In the original publication, owing to an erroneous value for the mass of the pendulum, this is erroneously calculated as 0.000191. The agreement of the experiments with theory is, therefore, much better than was supposed.

ON THE INFLUENCE OF INTERNAL FRICTION UPON THE CORRECTION OF THE LENGTH OF THE SECONDS PENDULUM FOR THE FLEXIBILITY OF THE SUPPORT.

It has been shown by Prof. A. M. Mayer that the only sensible resistance to the motion of a tuning-fork is proportional to the velocity. In the case of a slowly vibrating body the chief effect is probably due to that lagging of the strain after the stress, which Weber has called the elastic after-effect (*Nachwirkung*). The influence of the former mode of resistance upon the period of oscillation of a pendulum oscillating on an elastic tripod is easily calculated. The same thing cannot, in my opinion, be effected for the other kind of resistance, in the present state of our knowledge; nevertheless, the main characteristics of the motion may be made out. Put

t, for the time;

φ, for the instantaneous angle of deflection of the pendulum;

s, for the instantaneous horizontal displacement of the knife-edge from its position of equilibrium, in consequence of the flexure of the support;

l, for the length of the corresponding simple pendulum;

h, for the distance from the knife-edge to the center of mass of the pendulum;

g, for the acceleration of gravity;

γ, for the ratio of g to the statical displacement of the point of support, which would be produced by a horizontal force equal to the weight of the pendulum;

a, for the coefficient of internal friction supposed proportional to the velocity.

Then the differential equations are

$$lD^2{}_t\varphi+D^2{}_ts=-g\varphi$$
$$hD^2{}_t\varphi+D^2{}_ts=-\gamma s-aD_ts$$

The solution of these equations will be of the form (using $\circledcirc$ for the Neperian base and $\circledcirc'$ for the ratio of circumference to diameter):

$$\left.\begin{aligned}\varphi&=A_1\circledcirc^{z_1t}+A_2\circledcirc^{z_2t}+A_3\circledcirc^{z_3t}+A_4\circledcirc^{z_4t}\\ s&=B_1\circledcirc^{z_1t}+B_2\circledcirc^{z_2t}+B_3\circledcirc^{z_3t}+B_4\circledcirc^{z_4t}\end{aligned}\right\}\quad(1)$$

where z_1, z_2, z_3, z_4 are the roots of the equation

$$(l-h)z^4+alz^3+(\gamma l+g)z^2+agz+\gamma g=0,$$

where, for each subscript letter,

$$B=-(l+\frac{g}{z^2})\,A,$$

and where four arbitrary constants are determined by the initial conditions.

The roots of the biquadratic equation are all imaginary, and may be written

$$z_1=-\xi_1+\eta_1\sqrt{-1}\qquad z_3=-\xi_2+\eta_2\sqrt{-1}$$
$$z_2=-\xi_1-\eta_1\sqrt{-1}\qquad z_4=-\xi_2-\eta_2\sqrt{-1}$$

Expressing the coefficients in terms of the real and imaginary parts of the roots, the equation becomes

$$z^4+2(\xi_1+\xi_2)z^3+(4\xi_1\xi_2+\xi_1^2+\xi_2^2+\eta_1^2+\eta_2^2)z^2$$
$$+2[(\xi_1^2+\eta_1^2)\xi_2+(\xi_2^2+\eta_2^2)\xi_1]z+(\xi_1^2+\eta_1^2)(\xi_2^2+\eta_2^2)=0.$$

If the terms in z^3 and z were neglected, that is, if a were neglected, the solution of the false equation so obtained would be as follows (where observe the varying sign of η_1):

$$\text{False } z^2=-\tfrac{1}{2}(4\xi_1\xi_2+\xi_1^2+\xi_2^2+\eta_1^2+\eta_2^2)$$
$$\pm\tfrac{1}{2}(4\xi_1\xi_2+\xi_1^2+\xi_2^2-\eta_1^2+\eta_2^2)\sqrt{1+4\frac{4\xi_1\xi_2\eta_1^2-\xi_1^2(\eta_2^2-\eta_1^2)-\xi_1^2\xi_2^2}{(4\xi_1\xi_2+\xi_1^2+\xi_2^2-\eta_1^2+\eta_2^2)^2}}$$

Now, in the actual case, η_2 will be at least 100 times η_1, ξ_2 will be quite large, and ξ_1 very small. We may, therefore, neglect the square of the fraction under the radical; and we have very closely

$$\text{False } z_1^2 = \text{false } z_2^2 = -\eta_1^2 + \frac{4\xi_1\xi_2\eta_1^2 - \xi_1^2(\eta_2^2 - \eta_1^2) - \xi_1^2\xi_2^2}{4\xi_1\xi_2 + \xi_1^2 + \xi_2^2 - \eta_1^2 + \eta_2^2}$$

$$\text{False } z_3^2 = \text{false } z_4^2 = -\eta_2^2 - \xi_1^2 - \xi_2^2 - 4\xi_1\xi_2 \frac{4\xi_1\xi_2\eta_1^2 - \xi_1^2(\eta_2^2 - \eta_1^2) - \xi_1^2\xi_2^2}{4\xi_1\xi_2 + \xi_1^2 + \xi_2^2 - \eta_1^2 + \eta_2^2}$$

$$\text{False } z_1 = -\text{false } z^2 = \eta_1\left(1 - \frac{1}{2\eta_1^2}\,\frac{4\xi_1\xi_2\eta_1^2 - \xi_1^2(\eta_2^2 - \eta_1^2) - \xi_1^2\xi_2^2}{4\xi_1\xi_2 + \xi_1^2 + \xi_2^2 - \eta_1^2 + \eta_2^2}\right)\sqrt{-1}.$$

We thus see that, by neglecting the resistance, we get for the value of z_1 a quantity which requires only a minute correc ion in order to give the imaginary part of the true z_1. The same thing is not true for z_3 and z_4. Now, η_1 is $\odot$ divided by the principal period of oscillation of the pendulum upon the flexible stand. This is the quantity which we wish to determine; the others have only to be known approximately for the purpose of calculating the small correction to this. The logarithmic decrement of the amplitude of oscillation of the pendulum in the unit of time, so far as it is due to internal friction, is the quantity ξ_1. After these two quantities have been approximately ascertained, we may approximate to the quantity $(\xi_2^2+\eta_2^2)$ by means of the equation

$$(\xi_1^2+\eta_1^2)(\xi_2^2+\eta_2^2) = \frac{\gamma g}{l-h}.$$

Then, by eliminating a between the two equations

$$2(\xi_1+\xi_2) = \frac{al}{l-h},$$

$$2[(\xi_1^2+\eta_1^2)\xi_2 + (\xi_2^2+\eta_2^2)\xi_1] = \frac{ag}{l-h},$$

we obtain ξ_2, and consequently η_2. The values so obtained must satisfy the equation

$$4\xi_1\xi_2 + \xi_1^2 + \xi_2^2 + \eta_1^2 + \eta_2^2 = \frac{\gamma l + g}{l-h}.$$

Before proceeding to the consideration of the elastic after-effect, I propose to apply the equations thus obtained to the calculation of the correction of the seconds' pendulum for the flexure of the stand, supposing the internal friction to be proportional to the velocity.

For the pendulum used by me we have the approximate values—

$l=1.00$; h (heavy end up)$=0.30$; h (heavy end down)$=0.70$; g (New York)$=0.993\times\odot^2=9.89$; $\gamma=\frac{1}{0.0002125}=4706$; $\eta_1=1.00$.

The accompanying table shows that $\xi_1=0.000008$. From this we calculate that with heavy end up $\xi_2=0.08$, $\eta_2=257$; with heavy end down $\xi_2=0.17$, $\eta_2=392$. From this it appears that the correction of η_1 is absolutely insensible, or, in other words, the effect of resistance (supposed proportional to the velocity) vanishes. That this is nearly, in fact, the case for my instrument is shown by the circumstance that the times of oscillation upon stands of different rigidities agree with the values calculated in leaving the internal friction out of account.

U. S. Coast Survey. Pendulum. Decrement of Arc due to internal friction of brass of tripod. Pendulum was swung on brass tripod in Paris, Geneva, and Kew. On a stand ten times as stiff in Hoboken. The times of decrement given are the SUM *of the times with the heavy end up and heavy end down.*

Half amplitude.	Time decrement on— Flexible stand.	Stiff stand.	Time shortened by internal friction.	Ratio of shortening.	Decrement in one second.	Decrement due to internal friction in one second.	Mean arc.	Natural logarithmic decrement due to internal friction.
100′								
80	1073s	1095s	+22s	.022	0′.0186	.00023	90′	.0000025
70	706	762	+56	.080	.0142	.00114	75	.0000152
50	1927	1969	+42	.020	.0104	.00037	60	.0000062
40	1377	1254	Reject.				Mean	.000008

The last interval is probably affected by an error in the graduation of the scale used on one of the stands.

M. Plantamour proposes to determine the effect of the internal friction of the pendulum stand upon the correction for flexure, by means of the difference between the statical and dynamical flexure. He has made numerous observations, which, according to his own interpretation of them, would show that, if a pendulum be supported in a certain inclined position until the stand has had time to take its position of equilibrium under this force, and then be let go, the ratio of the amplitude of oscillation of the stand to that of the pendulum is not the initial one, but is very different from that. If this were the case, the motion of the stand and pendulum could not be represented, even approximately, in the form (1), for by those equations the logarithmic decrement of the oscillation of the stand is the same as that of the pendulum. It is true that the two parts of the oscillation (nearly in the natural periods of the pendulum and of the stand) have different logarithmic decrements; and, as the ratio of their amplitudes is not the same for the stand and for the pendulum, a certain change in the total relative amplitude might occur in this way, but only an excessively minute one, nothing like what M. Plantamour thinks he has observed. But it is so improbable that the motions of the stand and pendulum depart much from the forms (1) that it would be wrong to accept M. Plantamour's results until they are confirmed by a purely optical observation free from any possible influence from the machinery attached to the stand. Such an observation has been made by me, and, though I admit it was rather rough, it is entirely opposed to M. Plantamour's conclusions. Should the latter be confirmed, they would totally nullify the attempt to correct for the effect of flexure, as they would show the inapplicability of the analysis which has been proposed for the solution of that problem without affording us much hope of being able to replace it; and it would seem to be necessary in that case to reject all the work which has been done with the reversible pendulum.

If the pendulum were started in the manner proposed, and if for any cause the amplitudes of pendulum and stand were altered in different ratios, there would be a perpetual force at work tending to restore the old ratio, so long as the phases of the motion were the same in the pendulum and stand. But, if the phases differed, a part of this force would go to diminishing the amplitudes, and would act so strongly in this way that there would be a rapid decrement on account of this circumstance. Suppose, for instance, that in the differential equations we were to put instead of D_t^2s, $D_t^2s_1$, where s is the value of s at a time later than t by a constant. The result of this would be (neglecting terms involving a) that instead of the square of the exponent of the Naperian base being the sum of two negative quantities, one of them very small compared with the other, the smaller of these quantities would be multiplied by an imaginary root of unity. This would have but little effect on the imaginary part of the exponent of base, which determines the period; but it would add a considerable real part, which would represent a corresponding decrement of arc.

It seems difficult to conceive of a force which should greatly change the relative amplitudes of oscillation of the pendulum and stand, without at the same time producing an enormous decrement of the amplitude of oscillation, such as certainly does not exist. It is for those who believe that the existence of such a force has been experimentally proved to show how great an effect it would have upon the period of oscillation. M. Plantamour supposes that the formula given by me in my paper, "De l'influence de la flexibilité du trépied sur l'oscillation du pendule à reversion," would still apply to such a case; but I am unable to see upon what ground.

Meantime, in the present state of the question, it appears to me that we must appeal to direct experiment to determine the difference between the time of oscillation on a stiff and on a flexible stand. Such experiments were given by me in the paper above mentioned, and I have since greatly multiplied experiments on a stiff stand, with the general result there announced, namely, that the difference is slightly greater than my theory supposes (owing, perhaps, to neglecting the energy of movement of the support), and not smaller, as M. Plantamour's views would require.

ON THE EFFECT OF THE VERTICAL ELASTICITY OF A PENDULUM SUPPORT.

Let s = the amount of depression of the support below its mean position;
e = the force of restitution;
l = the length of the simple equivalent pendulum;
h = the distance of the point of support from the center of mass of the pendulum;
φ = the angle which the axis of the pendulum makes with the vertical;
r = the perpendicular distance of a given particle of the pendulum from the knife-edge;
ω = the angle which r makes with the plane through the knife-edge and the axis of the pendulum.

The horizontal velocity of a particle m is

$$r\cos(\varphi+\omega)\,\mathrm{D}_t\varphi.$$

The vertical velocity is

$$r\sin(\varphi+\omega)\,\mathrm{D}_t\varphi-\mathrm{D}_t s.$$

The vis viva of the particle is, then,

$$\frac{1}{2}mr^2(\mathrm{D}_t\varphi)^2-mr\sin(\varphi+\omega)\mathrm{D}_t\varphi\,.\,\mathrm{D}_t s+\frac{1}{2}m(\mathrm{D}_t s)^2.$$

The vis viva of the pendulum is

$$\frac{1}{2}\mathrm{M}lh(\mathrm{D}_t\varphi)^2-\mathrm{M}h\sin\varphi\mathrm{D}_t\varphi\,.\,\mathrm{D}_t s+\frac{1}{2}\mathrm{M}(\mathrm{D}_t s)^2.$$

The potential energy of the pendulum is

$$\mathrm{M}g(h-h\cos\varphi)+\frac{1}{2}es^2.$$

Lagrange's equations are consequently

$$\mathrm{M}lh\mathrm{D}_t^2\varphi-\mathrm{M}h\sin\varphi\mathrm{D}_t^2 s=-\mathrm{M}gh\sin\varphi,$$
$$-\mathrm{M}h\sin\varphi\mathrm{D}_t^2\varphi-\mathrm{M}h\cos\varphi(\mathrm{D}_t\varphi)^2+\mathrm{M}\mathrm{D}_t s=-es.$$

If S be the amount by which the stand is statically compressed by the weight of the pendulum, then $e=\mathrm{M}g:\mathrm{S}$, so that the equations become

$$l\mathrm{D}^2_t\varphi-\sin\varphi\mathrm{D}^2_t s=-g\sin\varphi, \qquad (1)$$
$$-h\sin\varphi\mathrm{D}_t\varphi-h\cos\varphi(\mathrm{D}_t\varphi)^2+\mathrm{D}^2_t s=-\frac{g}{\mathrm{S}}s. \qquad (2)$$

It is evident that small changes in φ will affect s insensibly, so that in determining s we may assume

$$\sin\varphi=\varphi,\qquad \cos\varphi=1,\qquad \varphi=\Phi\cos\sqrt{\frac{g}{l}}t.$$

The second equation then becomes

$$\mathrm{D}^2_t s+\frac{g}{\mathrm{S}}s=-\frac{gh\Phi^2}{l}\cos 2\sqrt{\frac{g}{l}}.t,$$

whence

$$s=\frac{h\mathrm{S}}{4\mathrm{S}-l}\Phi^2\cos 2\sqrt{\frac{g}{l}}t+\mathrm{C}\cos\sqrt{\frac{g}{\mathrm{S}}}(t-t_0).$$

The second term may obviously be neglected, and 4S may be neglected in comparison with l, so that

$$s=-\frac{h\mathrm{S}}{l}\Phi^2\cos 2\sqrt{\frac{g}{l}}t,$$

$$\mathrm{D}^2_t s=\frac{4gh\mathrm{S}}{l^2}\Phi^2\cos 2\sqrt{\frac{g}{l}}t=\frac{4gh\mathrm{S}}{l^2}(2\varphi^2-\Phi^2).$$

Then the first equation becomes

$$\mathrm{D}^2_t\varphi=-\frac{g}{l}\left(1+4\frac{\mathrm{S}h}{l^2}\Phi^2\right)\varphi+\frac{g}{6l}\left(1+48\frac{\mathrm{S}h}{l^2}\right)\varphi^3,$$

after substituting $\varphi-\frac{1}{6}\varphi^3$ for sin φ, &c. Or, more briefly,

$$D_t^2\varphi=-g'\varphi+\tfrac{1}{6}g''\varphi^3.$$

Putting $\varphi=\sqrt{\frac{g'}{g''}}\theta$, we get

$$D_t^2\theta=-g'\theta+\tfrac{1}{6}g'\theta^3,$$

whence

$$T=\odot\sqrt{\frac{l}{g'}}\left(1+\tfrac{1}{16}\Phi^2\right)$$

$$=\odot\sqrt{\frac{l}{g'}}\left(1+\tfrac{1}{16}\frac{g''}{g'}\Phi^2\right)$$

$$=\odot\sqrt{\frac{l}{g}}\left(1-\frac{2Sh}{l^2}\Phi^2\right)\left(1+\tfrac{1}{16}\Phi^2+\frac{3Sh}{l^2}\Phi^2\right)$$

$$=\odot\sqrt{\frac{l}{g}}\left(1+\tfrac{1}{16}\Phi^2+\frac{Sh}{l^2}\Phi^2\right).$$

If $\frac{S}{l}=.0001$, $\frac{h}{l}=.7$, and $\Phi=.05$, then $\frac{Sh}{l^2}\Phi^2=.000000175$.

so that the effect of the vertical elasticity of the support is insensible in ordinary cases.