UNITED STATES COAST SURVEY
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# APPARATUS FOR RECORDING

# A MEAN OF OBSERVED TIMES.

APPENDIX No. 15—REPORT OF 1875.

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[FROM UNITED STATES COAST SURVEY REPORT FOR 1875.]

# APPENDIX No. 15.

## DESCRIPTION OF AN APPARATUS FOR RECORDING THE MEAN OF THE TIMES OF A SET OF OBSERVATIONS, BY C. S. PEIRCE, ASSISTANT IN THE UNITED STATES COAST SURVEY.

The object of the contrivance is to enable an observer, after he has touched a key at each one of several observations, which succeed at a constant interval of a few seconds, immediately to read off on a dial the mean of the times of the observations to hundredths or thousandths of a second, thus avoiding the delay, labor, and error involved in reading chronograph-sheets.

Suppose the number of observations in a set is $n$. Then, if $t_i$ is the time of the $i$th observation the mean time is—

$$\frac{1}{n} \Sigma_i t_i \ .$$

Let $S_i$ be ~~double~~ the number of whole seconds in $t_i$, and let $s_i$ be the fraction of a second. Then—

$$\frac{1}{n} \Sigma_i t_i = \frac{1}{n} \Sigma_i S_i + \Sigma_i \frac{s_i}{n} \ .$$

The problem thus divides itself into two, to determine each term of the second member of this equation. This division of the problem constitutes the first essential character of the method here to be described.

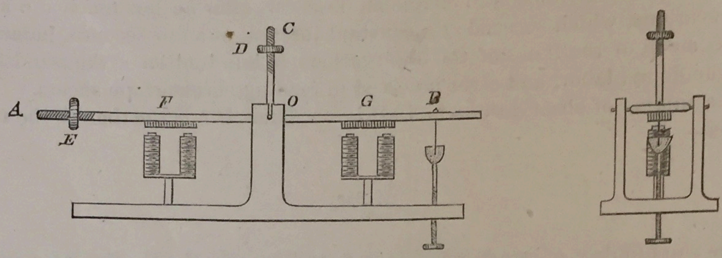
If the observations occur at irregular and unknown intervals, the observer may separately note $S_i$ for each observation, without any particular apparatus, and so calculate the first term. But if the observations occur at intervals approximately known, the first term can be determined with less trouble. Suppose, for instance, that the observations, like transits of stars, are known to occur at intervals nearly symmetrical about the middle one. Then, if there exist~~s~~ any easy means of determining the time of this one accurately to the one $n$th part of a second, this will be equal to the first term, provided the observations follow one another with sufficient regularity. But if $n$ be too great for this, or if it be an even number, the observer may note, by any simple means, the times of the first and last observations. These times need then only be noted to two $n$ths of a second, and so for any larger numbers. A transit-observer may conveniently use seven wires, and note the times over the second and sixth wires to a quarter of a second. When $n$ is greater, a marking-watch may be conveniently used. In using this instrument, the observer need not seek to distinguish the different observations of a set, as their order does not affect the mean value.

I have now to describe the means by which I would determine the value of the second term—

$$\Sigma_i \frac{s_i}{n} \ .$$

Suppos~~ing~~e that we have the means of registering the sum of the fractions $s_i$. Then to register, instead, their mean, we may use one of the following methods: First, we may regulate the registering apparatus by a "regulateur Villarceau." This may be made to run at any desired rate within certain limits with great accuracy; and it should be made to run at one $n$th of the rate required for the registry of the sum of $s_i$. Second, we may have a frictional connection between two solids of revolution. Third, we may perform the required division by the graduation of the dial by changing from one dial-face to another. But the simple division of $\Sigma s$ by $n$ is so very easily performed that it would hardly be worth while to make the necessary adjustment of the apparatus to put any of these methods into practice. I will, therefore, proceed at once to describe a contrivance for registering $\Sigma s$.

We require for this purpose three special pieces of apparatus besides the usual break-circuit chronometer. The first is a Hipp's chronoscope. Only, it would generally be better to have an instrument on a similar plan but registering hundredths instead of thousandths of a second, and running for five minutes at least. The essence of the instrument is a train of clock-work, running rapidly and regularly, and a dial with a hand connected with wheels, which are thrown into gear with the train when a certain galvanic circuit is made (or broken), and which are thrown out and stopped when the circuit is broken (or made). The second instrument needed is an observing-key, made something like a piano-forte key, with a metallic hammer, for making a very short galvanic connection. The third instrument is a peculiar relay, constructed as ~~follows:~~ shown in the figure,




O is a fixed axis, about which turns a lever, A B, which is provided with a vertical arm, O C. Upon this arm, there is a movable counterpoise, D, for raising or lowering the center of mass. On the arm A there is another weight, E, to adjust the balance of the lever. At the end B there is a platinum point, which dips into a mercury-cup, the height of which is adjustable by a screw. At F and G are armatures, and below each a small electro-magnet. The lever will turn to a limited extent, and is so balanced that it will remain with either end down when it is thrown from one position to the other.

Four batteries are now to be arranged in three connections, as follows:*

1st, Copper; mercury-cup; chronoscope; zinc.

2d, Copper; electro-magnet G; key; zinc.

3d, $\left\{\begin{array}{l}\text{Copper, first battery;}\\ \text{Zinc, second battery;}\end{array}\right\}$ electro-magnet F; $\left\{\begin{array}{l}\text{chronometer; zinc, first battery.}\\ \text{copper, second battery.}\end{array}\right.$

Before the observations begin, the A end of the relay will be down, and there will be no current through the chronoscope, and the hands will be still. When the key is touched, the electro-magnet G is made effective, the B end of the lever goes down, the first current is made, and the chronoscope-hand moves. This continues until the next chronometer-second, when the circuit through the first battery of the third connection is broken so that the second battery is no longer neutralized as it should have been at first; the electro-magnet F is made, the B end of the relay goes down, the circuit through the mercury-cup is broken, and the chronoscope stops. By reading the dial of the chronoscope before and after the set of observations, we have the quantity—

$$n - \Sigma_i s_i \; .$$

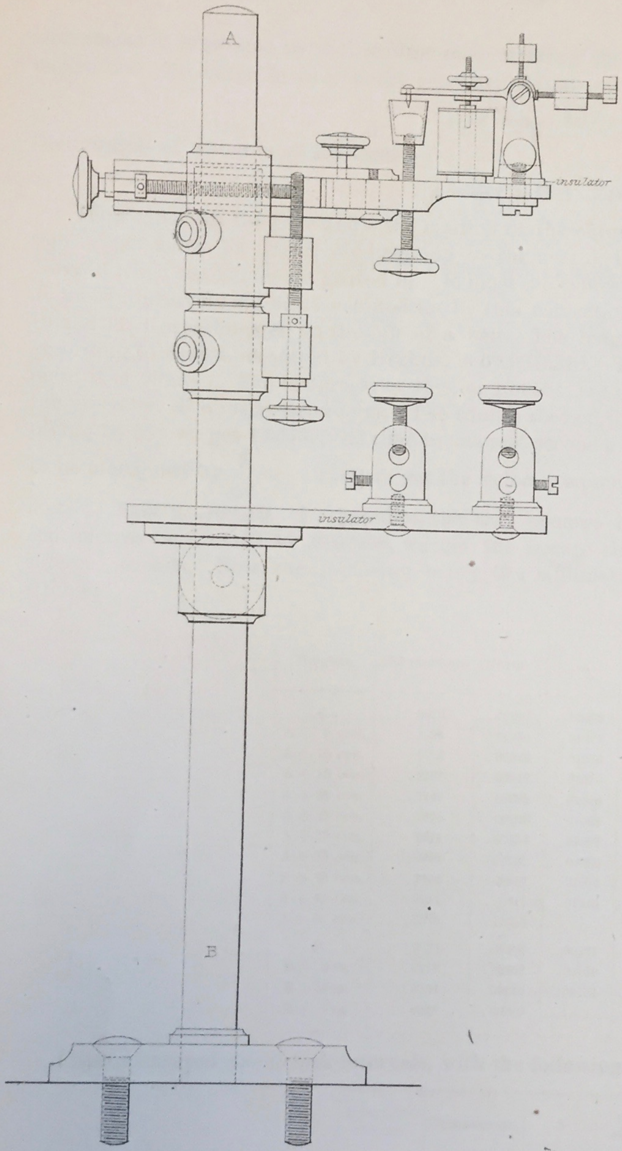
The interference of the observer's signal with the chronometer-second may produce either of two effects: it will either add one second to the recorded sum or will cause the omission of one observation. Either effect is easily detected in the result.

I have had constructed a relay of the sort described above, only replacing the magnet F by an agate which can be struck by a pendulum for determining gravity.†

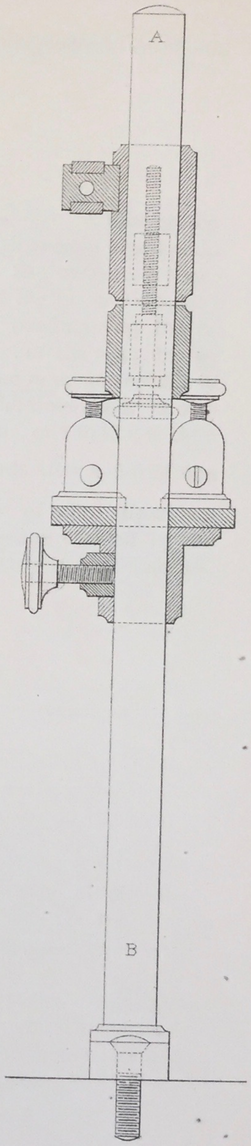
In the Coast Survey Report for 1870, page 212, I have shown the existence of a correction to the times as given by Hipp's chronoscope, owing to the inertia and friction of the wheels connected with the hands, in consequence of which, as soon as they are geared in, the movement begins to be retarded and they move slower and slower during three-fourths of a second. As the

* A still better arrangement would be to have a make-circuit chronometer.

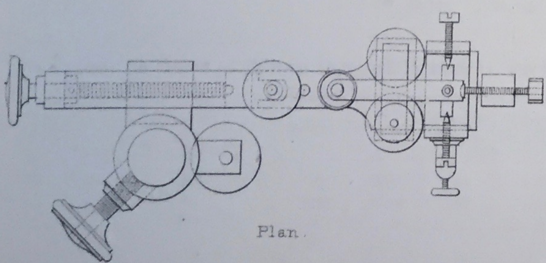
† The details of the instrument are by Mr. Breguet, who has been kind enough to interest himself in the matter, and to carry out the construction in the most perfect manner.




Elevation




Section A.B



Plan.

APPARATUS FOR RECORDING A

MEAN OF OBSERVED TIMES

devised by

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U.S. COAST SURVEY

1875

instrument is regulated by a vibrating reed striking the teeth of a revolving wheel, we should expect the retardation to vary as the velocity, so that—

$$D_t^2 s = a - b D_t s ,$$

the integral of which is of the form—

$$s = \frac{a}{b} t + B\,(1 - e^{-bt})$$

Here $t$ denotes the time and $s$ the reading of the chronoscope. The existence of this correction is also shown by attentively listening to the note of the reed, which is distinctly heard to be lowered when the hands are geared in. It must be confessed, however, that the measured times of known intervals are not accounted for by this correction; for example, at Berlin, in 1876, May 10 and 11, I experimented on the fall of a ball. The length of the seconds' pendulum at Berlin, according to Bessel, as stated by Bruhns, is $0^{m}.9942318$. This is for the level of the sea; for my point it is .994223. The reciprocal of this, or 1.00581, is the square of the time in mean seconds of the vibration of a metre-pendulum. The square root of this, or 1.00290, is the time itself. Multiplying by $\frac{366}{365}$ we get 1.00565, the time in sidereal seconds. The square of this, or 1.01133, is then to be multiplied by $\frac{1}{\eth^2}$ or, .101321 to get the velocity square in sidereal seconds after a fall of one metre. This is .102469. If, therefore, the true squares by the chronoscope of a fall through 5 centimetres be divided by .0102469, we get the square time by the chronoscope divided by the sidereal seconds. Now, then, A being nearly 40.1 millimetres and B being nearly 19.6 millimetres, I found—

| Heights. | Chronoscope. | (Chron.)² | Δ | Δ / .01025 | Chron. / Sid. sec. |
|---|---|---|---|---|---|
| A. | .0919 | .00845 | .01090 | 1.064 | 1.032 |
| A + 5 cen. | .1391 | .01935 | .01037 | 1.012 | 1.006 |
| A + 10 cen. | .1724 | .02972 | .01076 | 1.050 | 1.025 |
| A + 15 cen. | .2012 | .04048 | .01010 | .986 | .993 |
| A + 20 cen. | .2249 | .05058 | .01048 | 1.023 | 1.011 |
| A + 25 cen. | .2471 | .06106 | .01028 | 1.003 | 1.001 |
| A + 30 cen. | .2671 | .07134 | .01097 | 1.071 | 1.035 |
| A + 35 cen. | .2869 | .08231 | .00986 | .962 | .981 |
| A + 40 cen. | .3036 | .09217 | .01094 | 1.068 | 1.034 |
| A + 45 cen. | .3211 | .10311 | .01046 | 1.021 | 1.010 |
| A + 50 cen. | .3370 | .11357 | ........ | ........ | ........ |
| B. | .0637 | .00406 | .00101 | .986 | .993 |
| B + 5 m. | .0712 | .00507 | .00108 | 1.054 | 1.027 |
| B + 10 m. | .0784 | .00615 | .00104 | 1.015 | 1.007 |
| B + 15 m. | .0848 | .00719 | ........ | ........ | ........ |

I also measured some clock-intervals, with the following results:

| Interval in sid. sec. | Chronoscope. | Δ | Chron. / sid. sec. |
|---|---|---|---|
| 1 | .996 | .996 | 0.996 |
| 2 | 1.993 | .997 | 0.997 |
| 10 | 9.956 | 7.963 | 0.9954 |
| 50 | 49.745 | 39.789 | 0.9947 |

If we take the times of falling through A and B, we have—

| Height. | (Chron.)² | (Chron.²) / Height. | (Chron.)² / 205 height. | Chron. / sid. sec. |
|---|---|---|---|---|
| .0401 | .00845 | .211 | 1.03 | 1.02 |
| .0196 | .00406 | .207 | 1.01 | 1.01 |

~~If we compare~~ The true time of falling A + 50 cen. is 0.3291 sid. seconds, for ~~with the record of 1 second~~, we have $54.01 \times .00205 = .1083 = (.3291)^2$. Hence the difference between this and 1 second ~~of time~~ is 0.6709 sidereal second. The difference of the chronoscope results is 0.659, and the ratio is only .982.

That there really is a retardation is certain both *à priori* and from the sound; and it is shown by the rates from the clock-seconds. But this can hardly account for the discrepancy between the measures with the clock and fall-apparatus, for the last given rate connecting the two is too small. I am inclined to think that there may be a correction of the fall-experiments, proportional to the momentum at impact, and, therefore, to the time. Experiments should be made with pendulums of different lengths. The relay above described, with the addition of a circuit-reverser, will render such experiments easy.

In regard to the accuracy of Hipp's chronoscope, I may mention that the chronoscope-times given above for the falls are the means of ten observations each. It may, then, be calculated from the agreement of the resulting ratios in the last column that the probable error of a single observation but slightly exceeds one thousandth of a second. In using the instrument for the automatic record of pendulum-transits, then, it will be quite sufficient to have ten observations in a set. This will give the intervals accurately to a thousandth of a second, or as accurately as the method of coincidences.

Let us now briefly consider the effect of the resistance of the lever shown in the plate upon the motion of the pendulum. Owing to the elasticity of the material, we may consider the impact to be instantaneous and to produce a reduction of the velocity of the pendulum in a fixed ratio. Let $t$ be the variable time which is occupied by the pendulum in swinging from the vertical position to one having an angle, $\varphi$, from the vertical; let $v$ be the angular velocity at that instant; T, the period of oscillation; $\Phi$, the amplitude of oscillation; $\pi$, the ratio of the circumference to the diameter, and $\frac{1}{1+i}$ the ratio of $v$ just before the impact to $v$ just after. Let $\delta t$ and $\delta \Phi$ denote variations of $t$ and $\Phi$ produced by the impact. Then we have, from the common theory of the pendulum—

$$t = \frac{\mathrm{T}}{\pi} \arctan \frac{\varphi \pi}{v \mathrm{T}},$$

$$\Phi = \sqrt{\frac{v^2 \mathrm{T}^2}{\pi^2} + \varphi^2}.$$

In these equations we are to multiply $v$ by $(1+i)$ and subtract from the products the above unchanged values to obtain $\delta t$ and $\delta \Phi$. Developed by Taylor's theorem, and neglecting all but the first two terms, we have—

$$\delta \Phi = \frac{v^2 \mathrm{T}^2}{\varphi \pi^2} i = \Phi \left( \cos \frac{\pi t}{\mathrm{T}} \right)^2 i = \frac{\Phi^2 - \varphi^2}{\Phi} i,$$

$$\delta t = \frac{\frac{\varphi \pi}{v \mathrm{T}}}{\sqrt{1 + \left( \frac{\varphi \pi}{v \mathrm{T}} \right)^2}} i = \frac{\mathrm{T}}{\pi} \sin \frac{t}{\mathrm{T}\pi} \cdot i = \frac{\mathrm{T}}{\pi} \sqrt{1 - \frac{\delta \Phi}{\varphi i}} \cdot i = \frac{\mathrm{T}}{\pi} \frac{\varphi}{\Phi^2 - \varphi^2} \delta \Phi = \frac{\mathrm{T}}{\pi} \frac{\varphi}{\Phi} i$$

The quantity $i$ is twice the ratio of the virtual mass of the lever to the sum of those of pendulum and lever. By the virtual mass, I mean the square of the moment of the momentum divided by the moment of inertia. It is safe to say that $i$ does not exceed $\frac{1}{1000}$. And as $\frac{\varphi}{\Phi}$ can easily be reduced to $\frac{1}{20}$, the effect of the resistance can hardly in ten vibrations be perceptible. However, its amount may be calculated by observing $\varphi$ and $\delta \Phi$.

It will be observed that the resistance shortens the time of oscillation if the impact occurs while the pendulum is moving upward, and lengthens it in the reverse case. Hence, there would be no accumulation of the effect in ten transits over what there would be in two, were it not for the thickness of the agate.

The following results of successive series of 298 swings each, measured with the above-described instrument, by ten transits every five minutes, are a fair specimen of the results obtained.

APRIL 8.

| Heavy end up. | Heavy end down. |
|---|---|
| $299^s.9463$ | $299^s.9162$ |
| 299 . 9436 | 299 . 9153 |
| 299 . 9457 | 299 . 9147 |
| 299 . 9427 | 299 . 9129 |
| 299 . 9439 | 299 . 9132 |

APRIL 7.

| Heavy end down. | Heavy end up. |
|---|---|
| $299^s.9215$ | $299^s.9549$ |
| 299 . 9176 | 299 . 9552 |
| 299 . 9167 | 299 . 9564 |
| 299 . 9179 | 299 . 9513 |
| 299 . 9176 | 299 . 9525 |

The weights to be assigned to successive intervals of a set of 5 are 5, 8, 9, 8, 5, and this gives for April 8—

$$\text{Heavy end up} \quad T_2 = 1^s.006525, T_2^2 = 1.013093$$
$$\text{Heavy end down} \quad T = 1.006424, T^2 = 1.012889$$
$$T^2 \text{ (corr'd for resistance of air, etc.)} = \frac{39\,T_1^2 - 17\,T_2^2}{22} = 1.012731$$

For April 7—

$$T_1 = 1.006436, T^2 = 1.012913$$
$$T_2 = 1.006558, T_2^2 = 1.013159$$
$$T^2 \text{ (corr'd)} = 1.012731$$

This is expressed in chronograph-seconds. The results of the two days are identical to the last figure.