Microeconomics I
Solutions to Problem Set 5

1. $f(S,L) = S^{0.5}L^{0.5}$. We need to derive the total cost function in countries A and B and compare them.

In country A,the cost function comes from

$$\min_{S,L\geq 0} 7S + 7L$$
$$\text{s.t. } \sqrt{SL} \geq Q$$

where Q is total output. Using the first-order conditions of the problem and the constraint satisfied with equality, we obtain

$$\frac{L}{S} = 1 \text{ and } \sqrt{SL} = Q$$

Thus, as a function of Q, inputs S and L are

$$S = L = Q$$

and the total cost function in country A is:

$$C^A(Q) = 7Q + 7Q = 14Q$$

Similarly, in country B,the cost function comes from

$$\min_{S,L\geq 0} 8S + 6L$$
$$\text{s.t. } \sqrt{SL} \geq Q$$

where Q is total output. Using the first-order conditions of the problem and the constraint satisfied with equality, we obtain

$$\frac{L}{S} = \frac{4}{3} \text{ and } \sqrt{SL} = Q$$

Thus, as a function of Q, inputs S and L are

$$S = \sqrt{\frac{3}{4}}Q;\ L = \sqrt{\frac{4}{3}}Q$$

and the total cost function in country B is:

$$C^B(Q) = 8\sqrt{\frac{3}{4}}Q + 6\sqrt{\frac{4}{3}}Q = 8\sqrt{3}Q$$

Since $14 > 8\sqrt{3}$, the firm should locate in country B.

2. $Q(K,L,M) = K^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}}$ and $r = 1$, $w = 1$, and $m = 1$.

a) Marginal products are

$$\begin{aligned} MP_K &= \frac{1}{3}K^{\frac{-2}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}} \\ MP_L &= \frac{1}{3}K^{\frac{1}{3}}L^{\frac{-2}{3}}M^{\frac{1}{3}} \\ MP_M &= \frac{1}{3}K^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{-2}{3}} \end{aligned}$$

In order to verify returns to scale,

$$Q(\lambda K, \lambda L, \lambda M) = (\lambda K)^{\frac{1}{3}}(\lambda L)^{\frac{1}{3}}(\lambda M)^{\frac{1}{3}} = \lambda K^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}} = \lambda Q(K,L,M)$$

hence, constant returns to scale.

b) The problem reads:

$$\begin{aligned} &\min_{K,L,M\geq 0} K + L + M \\ \text{s.t. } K^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}} &\geq Q \end{aligned}$$

and combining the first-order conditions and using the constraint it is easy to see that:

$$K = L = M = Q$$

Hence, $C(Q) = 3Q$.

c) In the short run, $Q(\overline{K}, L, M) = \overline{K}^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}}$. Setting up and solving the firm's short-run cost minimization problem, it is easy to see that $L = M$. Hence,

$$Q = \overline{K}^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}} = \overline{K}^{\frac{1}{3}}L^{\frac{2}{3}} \Rightarrow L = M = \frac{Q^{\frac{3}{2}}}{2}$$

and the cost function is $C(Q) = 4 + \frac{Q^{\frac{3}{2}}}{2} + \frac{Q^{\frac{3}{2}}}{2} = 4 + Q^{\frac{3}{2}}$.

d) In the long run, when $Q = 4$, $K = 4 = L = M$. In the short run, when $Q = 4$ and $\overline{K} = 4$, $L = M = \frac{4^{\frac{3}{2}}}{2} = 4$.

3. The cost-minimization problem is:

$$\begin{aligned} &\min_{K,L\geq 0} 50K + 8L \\ \text{s.t. } K^{0.4}L^{0.4} &\geq Q \end{aligned}$$

then, from the first-order conditions of the problem and the constraint satisfied with equality,

$$\frac{L}{K} = \frac{25}{4} \Rightarrow L = \frac{25}{4}K \text{ and } K^{0.4}L^{0.4} = Q$$

Thus, $K = \frac{2}{5}Q^{\frac{5}{4}}$ and $L = \frac{5}{2}Q^{\frac{5}{4}}$. Then, the cost function is

$$C(Q) = 50 \cdot \frac{2}{5}Q^{\frac{5}{4}} + 8 \cdot \frac{5}{2}Q^{\frac{5}{4}} = 40Q^{\frac{5}{4}}$$

and the average cost function is

$$AC(Q) = \frac{C(Q)}{Q} = 40Q^{\frac{1}{4}}$$

This average cost function is increasing in Q. This is a direct consequence of the fact that the production function exhibits decreasing returns to scale. In other words, the production function exhibiting DRS implies a convex total cost function, and hence, an increasing average cost function.

4. $y = f(K, L) = KL$. The conditional labor demand function $L(y, w, r)$ is obtained from the cost minimization problem:

$$\begin{aligned} &\min_{S,L\geq 0} rK + wL \\ \text{s.t. } KL &\geq y \end{aligned}$$

Therefore,

$$\frac{L}{K} = \frac{r}{w} \Rightarrow K = \frac{w}{r}L \text{ and } KL = y$$

yielding $L = \sqrt{\frac{r}{w}y}$.

5. The firms will supply strictly positive quantities since $MC_1(y) > AVC_1(y)$ ($y > \frac{y}{2}$) and $MC_2(y) > AVC_2(y)$ ($\frac{y}{5} > \frac{y}{10}$) for any $y > 0$.

The supply of a firm is determined by the following condition: $p = MC_1(y)$ that is $p = y$ for the first 100 firms and $p = MC_2(y)$ so that $y = 5p$ for the other 60 firms. The supply of the first 100 firms is then $S_1(p) = 100p$ and the supply of the other 60 firms is then $S_2(p) = 60 \times 5p = 300p$. The supply of the industry is then: $S(p) = S_1(p) + S_2(p) = 400p$.

6. a) We can express average non-sunk costs as: $ANSC(q) = \frac{NSFC}{q} + q$, and marginal costs as: $MC(q) = 2q$. The marginal cost curve intersects the average non-sunk cost curve at its minimum. This occurs at a level of output such that $\frac{NSFC}{q} + q = 2q \Rightarrow q = \sqrt{NSFC}$.

We are also told that the minimum of the average non-sunk costs is 20, since the firm decides to shut down if the price falls below this level. Hence, at $q = \sqrt{NSFC}$, the value of the marginal cost and of the average non-sunk cost is 20. Thus, $20 = 2\sqrt{NSFC} \Rightarrow NSFC = 100$.

b) Since $p = MC$, if $q = 20$ then $p = 40$. $Q(40) = 240 - \frac{40}{2} = 220 \Rightarrow N = 11$.

7. a) Individual supply is $p = \frac{q}{1000} \Rightarrow q = 1000p$. Therefore, industry supply is $Q = 60q = 60000p$.

b) In the short run, $60000p = 150000 - 15000p \Rightarrow p = 2$, $Q = 120000$, $q = 2000$.

c) Since the marginal cost curve intersects the average cost curve at its minimum, and this minimum is the long-run price, $MC(1000) = 1 = p^{LR}$. Then, $Q = 135000$ and $N = \frac{135000}{1000} = 135$.

8. a) The long-run equilibrium price is the minimum value of the average cost curve. Since $AC(q) = 40 - 6q + \frac{1}{3}q^2$, the minimum occurs when $-6 + \frac{2}{3}q = 0 \Rightarrow q = 9$ and the value of $AC(9) = 13 = p^{LR}$.

b) As it was shown in the previous part, each firm produces $q = 9$.

c) $Q = 2200 - 1300 = 900 \Rightarrow N = 100$.

d) We need A such that $N = 200$. Since costs are unaffected, each active firm will produce $q = 9$, implying a total output of 1800. The equilibrium price will remain at 13. Then, $1800 = A - 1300 \Rightarrow A = 3100$.

9. a) and b) First, notice that Policy I takes us back to the autarky equilibrium. This equilibrium is the intersection of demand and the domestic supply, i.e.

$$10 - q = 10 + q \Rightarrow q = 50, \;\; p = 60$$

By contrast, in the free-trade equilibrium the domestic price equals the world price, hence, $p = 30$ and $Q^d = 80$, $Q^s = 20$ and imports are 60. With free trade, consumer surplus is $\frac{1}{2} \cdot 80 \cdot 80 = 3200$ and producer surplus is $\frac{1}{2} \cdot 20 \cdot 20 = 200$.

Under Policy I consumer surplus is $\frac{1}{2} \cdot 50 \cdot 50 = 1250$, and domestic producer surplus is $\frac{1}{2} \cdot 50 \cdot 50 = 1250$. Thus, $DWL = 700$.

Policy II makes the outcome be the same as in the free-trade equilibrium, and thus, $DWL = 0$.

Finally, Policy III introduces a \$10 tariff. This means that the domestic price will be $p^w + t = 30 + 10 = 40$. Consumption is 70, and imports are reduced to 40. Consumer surplus is now $\frac{1}{2} \cdot 70 \cdot 70 = 2450$, whereas domestic producers surplus is $\frac{1}{2} \cdot 30 \cdot 30 = 450$. Government revenues are $10 \cdot 40 = 400$. This means that total surplus is 3300, and hence $DWL = 100$. This deadweight loss can be decomposed into output that is not produced (50) and output that used to be produced by foreign producers that is now produced by domestic (less efficient) firms (50). An import quota of 40 would have the same effect on the domestic price, although the deadweight loss would increase by 400, since government revenues would be zero.

If the government wanted to eliminate imports by subsidizing domestic producers, it would do so introducing a subsidy of 60 or above per unit produced. This way, the transaction price in the domestic market would be 30 (or below if it chooses a greater subsidy), eliminating the possibility of foreign producers selling in the domestic market.

10. First, it is worth noticing that the domestic supply is flat at zero up to 30 units, where it becomes vertical. This means that the autarky price would be 70.

a) The world price is 40. Since the autarky price is 70, the tariff must be at least 30 dollars if it is to be prohibitive.

b) We need to determine how much consumers are willing to pay so that the tariff is not passed and how much producers are willing to pay to have it passed. In order to do so, we need to analyze how CS and PS change if the tariff is introduced.

Without the tariff, $CS = \frac{1}{2}60 \cdot 60 = 1800$, whereas $PS = 40 \cdot 30 = 1200$.

With the tariff, $CS = \frac{1}{2}30 \cdot 30 = 450$, whereas $PS = 70 \cdot 30 = 2100$.

Thus, producers would be willing to pay up to $\Delta PS = 900$, whereas consumers would be willing to pay up to $\nabla CS = 1350$. Thus, consumers are willing to pay more, and therefore, the tariff is not introduced.

c) The lump-sum tax makes CS be, without the tariff, $CS = 1800 - S$. Hence, $\nabla CS = 1350 - S$. Therefore, the value of S that induces consumers to bid the same as producers is such that $1350 - S = 900 \Rightarrow S = 450$. So, if $S \geq 450$, the tariff is introduced.