Microeconomics I
Problem Set 5

1. As head of the planning commission of Eastern Motors, your job is to determine where to locate a new plant. The only inputs used in your cars are steel and labor and the production function is a Cobb-Douglas where $f(S, L) = S^{0.5}L^{0.5}$, where S is tons of steel and L is units of labor. You can locate your plant either in country A or country B. In country A, steel costs 7 euros a ton and labor costs 7 euros per unit. In country B, steel costs 8 euros per ton and labor costs 6 euros per unit. In which country should the company locate its new plant so as to minimize costs per unit of output?

2. Suppose that a firm uses three inputs to produce its output: capital (K), labor (L), and materials (M). The firm's production function is given by $Q(K, L, M) = K^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}}$. The prices of capital, labor, and materials are $r = 1$, $w = 1$, and $m = 1$, respectively.

a) Provide expressions for the marginal products of capital, labor, and materials. Does this production function exhibit increasing, decreasing, or constant returns to scale?

b) What is the solution to the firm's long-run cost minimization problem given that the firm wants to produce Q units of output?

c) What is the solution to the firm's short-run cost minimization problem given that the firm wants to produce Q units of output and capital is fixed at $\overline{K}$?

d) Verify that when $Q = 4$ and $\overline{K} = 4$, which is the long-run cost minimizing quantity of capital when $Q = 4$, the short-run and long-run quantities of labor and materials are the same.

3. Derive the long-run total cost function if $w = 8$, $r = 50$, and $Q(K, L) = K^{0.4}L^{0.4}$. What is the expression for the average cost function? How do your results depend on returns to scale of the production function?

4. A firm has the production function $y = f(K, L) = KL$, where K is the amount of capital and L is the amount of labor it uses as inputs. The cost per unit of capital is r and the cost per unit of labor is a wage w. Determine the conditional labor demand function $L(y, w, r)$.

5. In the short run, the bicycle industry includes 100 firms with the cost curve $c_1(y) = 2 + \frac{y^2}{2}$ and 60 firms with the cost curve $c_2(y) = \frac{y^2}{10}$. No new firms can enter the industry. What is the industry supply curve?

6. A market contains a group of identical firms. Each firm acts as a price taker and has a short-run total cost function $SRTC(q) = 250 + q^2$, where q is the annual output of each firm. A study reveals that part of the \$250 fixed cost

is sunk, and each firm will produce if the price exceeds $20 per unit and will shut down if price is less than $20 per unit. The market demand curve for the industry is $Q(p) = 240 - \frac{p}{2}$, where p is the market price.

a) How much of the fixed cost is sunk for each firm?

b) At the current market price, each firm produces 20 units. What is the market price and how many firms are there in this industry?

7. Demand for roasted coffee is given by $Q_d = 150000 - 15000p$, where p is the market price. The market for roasted coffee is perfectly competitive. Each firm has access to the same technology which yields a marginal cost curve given by $MC(q) = \frac{q}{1000}$, where q is the quantity supplied by an individual firm. There are initially 60 firms supplying roasted coffee.

a) What is the market supply curve?

b) What are the short-run equilibrium price and quantity in the market? How much coffee does each individual firm supply?

c) Each firm's average cost curve achieves its minimum point when it produces $q = 1000$. In the long-run equilibrium, what are the market price and quantity? How many firms are there in the long run?

8. Propylene is used to make plastic. The propylene industry is perfectly competitive, and each producer has a long-run total cost function given by $TC(q) = 40q - 6q^2 + \frac{1}{3}q^3$

The market demand curve for propylene is $Q(p) = 2200 - 100p$.

a) What is the long-run equilibrium price in this industry?

b) At this price, how much would an individual firm produce?

c) How many firms are in the propylene market in a long-run competitive equilibrium?

d) Suppose the demand curve shifted so that it is now $Q(p) = A - 100p$. How large would A have to be so that in the new long-run competitive equilibrium, the number of propylene firms was twice what it was in the initial long-run equilibrium?

9. Suppose that the domestic demand for computer chips is $p^d = 110 - q^d$, where q^d is the number of units of chips demanded domestically when the price is p^d. The domestic supply is $p^s = 10 + q^s$, where q^s is the number of units of chips supplied domestically when domestic suppliers receive a price p^s. Foreign suppliers would be willing to supply any number of chips at a price of $30. The government is contemplating three possible policies:

Policy I: The government decides to ban imports of chips.

Policy II: Imports of chips are allowed with no tariff.

Policy III: The government allows imports, but imposes a tariff of $10 per unit.

a) For each of the three policies, what is the size of the domestic producer surplus, the consumer surplus and total welfare?

b) What is the welfare-maximizing policy? What is the size of the deadweight loss in each of the three cases?

10. Demand for steel in Aruba, which is a small economy relative to the world total, is given by $Q = 100 - p$, where output is expressed in tons per day and price in dollars. Domestic producers combined can produce up to 30 tons per day, at zero marginal cost. The world price of steel is 40 dollars and there are initially no barriers to trade in steel of any kind.

The Aruban government is thinking of introducing a prohibitive tariff, i.e. one that completely eliminates steel imports. However, the decision will not be made based on economic efficiency, but on lobbying efforts by consumers and domestic producers. Specifically, consumers can lobby the government by making a lump-sum payment to avoid the tariff being introduced, and producers can lobby by making a lump-sum payment so as to induce the government to introduce the tariff. The government will choose depending on who is willing to pay the most.

a) What is the minimum amount of the prohibitive tariff?

b) Will the tariff be introduced? Justify your answer numerically and graphically.

Suppose now that the Aruban government announces that a lump-sum tax on consumers of S will be introduced only if no tariff is in place. Thus, if the prohibitive tariff is passed, there will be no tax.

c) What is the minimum value of S that would make domestic producers successful in their lobbying efforts (i.e. that would induce them to bid more than consumers)?