

# Microeconomics I

## Problem Set 4

1. For each of the following production functions, calculate  $MP_1$ ,  $MP_2$ , and  $TRS$ . Also determine whether the functions exhibit decreasing, constant, or increasing returns to scale:

- a)  $f(x_1, x_2) = x_1^4 x_2^4$
- b)  $f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$
- c)  $f(x_1, x_2) = \ln x_1 + \ln x_2$
- d)  $f(x_1, x_2) = 5x_1 + 3x_2$
- e)  $f(x_1, x_2) = \min\{x_1, x_2\}$

2. A firm has the production function  $f(x_1, x_2) = (x_1^b + x_2^b)^c$ , where  $b > 0$  and  $c > 0$ ,  $x_1 > 0$  and  $x_2 > 0$ . Show that the firm exhibits increasing returns to scale if and only if  $bc > 1$ .

3. Show that the production functions  $f(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}$  and  $g(x_1, x_2) = x_1 + \min\{x_1, x_2\}$  exhibit constant returns to scale, whenever  $x_1 > 0$  and  $x_2 > 0$ . Show that  $f(x_1, x_2) = 3x_1^{0.4} + 6x_2^{0.7}$  exhibits decreasing returns to scale, whenever  $x_1 > 0$  and  $x_2 > 0$ .

4. When Farmer Hoglund applies  $N$  kilos of fertilizer, the marginal product of fertilizer is  $1 - N/200$  bushels of corn. If the price of corn is 4 euros per bushel and the price of fertilizer is 1.2 per kilo, then how many kilos of fertilizer should Farmer Hoglund use in order to maximize his profits?

5. A firm has two variable factors and a production function  $f(x_1, x_2) = \sqrt{2x_1 + 4x_2}$ .

(a) Draw the isoquants corresponding to an output of 3 and to an output of 4.

(b) If the price of the output is 4, the price of factor 1 is 2, and the price of factor 2 is 3, find the profit-maximizing amount of factor 1, the profit-maximizing amount of factor 2, and the profit-maximizing level of output.

6. Let a production function be  $f(K, L) = \frac{L}{2} + \sqrt{K}$ , where  $L$  is the amount of labor used and  $K$  is the amount of capital used.

(a) Show that returns to scale are decreasing. Show that the marginal product of labor is constant.

(b) In the short run, capital is fixed at 4 units and labor is variable. Draw output as a function of labor in the short run and draw the marginal product of labor as a function of labor in the short run. The average product of labor is defined as total output divided by the amount of labor input. Draw the average product of labor as a function of labor in the short run.

(c) What is the short-run maximizing profit amount of labor if the cost of labor is  $w_L = 1$  and the output price is 1 euro per unit?

7. A firm has a production function described as follows. “Weekly output is the square root of the minimum of the number of units of capital and the number of units of labor employed per week.” Suppose that in the short run this firm must use 16 units of capital but can vary its amount of labor freely.

(a) Write down a formula that describes the marginal product of labor in the short run as a function of the amount of labor used. (Be careful at the boundaries.)

(b) If the wage  $w$  is 1 euro and the price of output  $p$  is 4 euros, how much labor will the firm demand in the short run?

(c) Answer question (b) for  $w = 1$  euro and  $p = 10$  euros?

(d) Write down an equation for the firm’s short-run demand for labor as a function of  $w$  and  $p$ .