Microeconomics I
Solutions to Problem Set 3

1. Her utility maximization problem is

$$\max_{x,y} \min\{4x, 2y\}$$
$$\text{s.t. } p_x x + p_y y \leq m$$

Since the utility function is not differentiable at the optimal bundle, the condition that substitutes for the equality of the MRS and the ratio of prices is $4x = 2y$. Plugging this into the budget constraint, we obtain:

$$p_x x + 2p_y x = m \Rightarrow x = \frac{m}{p_x + 2p_y}$$

2. $U(x,y) = \sqrt{x} + \sqrt{y}$ and prices are $p_y = 1$, $p_x = 5$.

a) First, corner solutions can be ruled out. Notice that $MRS = \sqrt{\frac{y}{x}}$ and

$$\lim_{x\to 0} \sqrt{\frac{y}{x}} = +\infty \text{ and } \lim_{y\to 0} \sqrt{\frac{y}{x}} = 0$$

Now, if $I = 60$, the optimal bundle comes from

$$\sqrt{\frac{y}{x}} = \frac{5}{1} \Rightarrow y = 25x \text{ and } 5x + y = 60$$

which implies that $x = 2$, $y = 50$.

b) Mark's demand for x, given that $I = 60$ and $p_y = 1$ also comes from the equality $MRS = \frac{p_x}{p_y}$ and the budget constraint satisfied with equality. These two equalities read

$$\frac{y}{x} = p_x^2 \text{ and } p_x x + y = 60$$

and thus, the demand for good x is

$$x = \frac{60}{p_x(1 + p_x)}$$

c) If $p_x = p_y = 1$ and income is now a variable, m, notice that the previous equalities become

$$\frac{y}{x} = 1 \text{ and } x + y = m$$

and thus, we can write Mark's Engel curve for good x as $x = \frac{m}{2}$. Since $\frac{\partial x}{\partial m} > 0$, it is a normal good.

3.

a) Miss Muffet's utility function is $U(C,W) = \min\{2C, W\}$. This implies that at the optimum, $W = 2C$. Combining this with the budget constraint $C + 0.75W = 22$, we have

$$C = \frac{44}{5};\ W = \frac{88}{5}$$

b) Since $W = 2C$, $C = \frac{W}{2}$. Thus, if $p_C C + p_W W = m$, then the expression for the demand for whey is

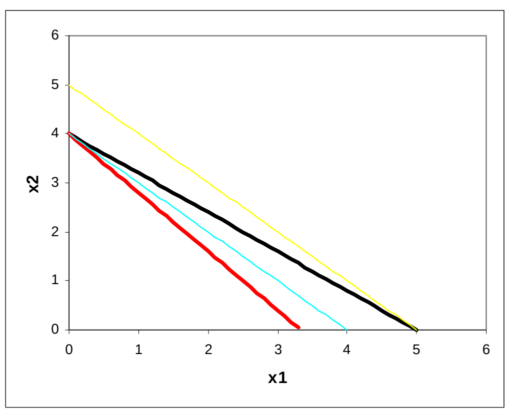
$$W = \frac{2m}{2p_W + p_C}$$

c) Assume you may still consume fractional units of curds. If you can not consume fractional units of whey, then you have to compare the utility of bundles $(9.25, 17)$ and $(8.5, 18)$, which are the closest to the optimum in part (a) that are on the budget line. It turns out that the level of utility is the same in both, hence both are the solution to her utility maximization problem.

4.

a) Let x_1 be consumption of peach juice, and x_2 orange juice. A utility function that describes his preferences is $u(x_1, x_2) = x_1 + x_2$. When $p_1 = 1$, $p_2 = 1.25$, the optimum was $(5, 0)$. When $p_1 = 1.5$, the optimum is $(0, 4)$.

b) His utility decreases from 5 to 4. Graphically, thicker lines are budget constraints; thinner lines are indifference curves.



c) Under the new prices, given that he only consumes orange juice, he must be given an extra 1.25 dollars to enjoy the same level of utility as before the price change.

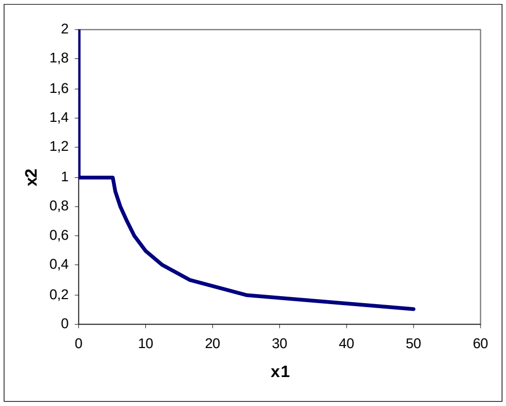
d) If $p_1 = 1$, demand for orange juice is:

$$x_2(p_2) = \begin{cases} 0 \text{ if } p_2 > 1 \\ [0,5] \text{ if } p_2 = 1 \\ \frac{5}{p_1} \text{ if } p_2 < 1 \end{cases}$$

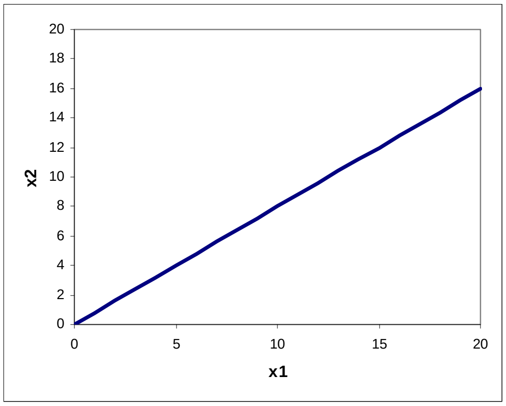
If $p_1 = 1.5$, demand for orange juice is:

$$x_2(p_2) = \begin{cases} 0 \text{ if } p_2 > 1.5 \\ [0, \frac{10}{3}] \text{ if } p_2 = 1.5 \\ \frac{5}{p_1} \text{ if } p_2 < 1.5 \end{cases}$$

Graphically, when $p_1 = 1$,



e) If $p_1 = 1.5$, $p_2 = 1.25$, then he consumes orange juice only. Hence, $x_2 = \frac{100m}{125} = \frac{4m}{5}$. Graphically,



5. We do not know. The old optimal bundle is no longer feasible, although it might be the case that she is now able to afford some other bundle that she prefers to the old optimum (recall that although the price of the first good increases, the price of the second increases; this makes some bundles affordable which were not at the original prices). It all depends on the shape of her indifference curves.

6. First, income m' that makes the initial optimum just affordable when the price of good x is 4 is:

$$m' = m + x(3, m)\Delta p$$

Therefore, $m' = 419 + 5.5 = 424.5$. Knowing this, we can find:

$$x(4, m') = x(4, 424.5) = 0.625$$

implying that $DS = 0.625 - 5.5 = -4.875$ and $DI = 0.35 - 0.625 = -0.275$. Thus, the correct answer is (b) (taking into account that the answer uses two decimals instead of three).

7. $U(x_1, x_2) = 2(\ln x_1) + x_2$. Her utility function is quasilinear with respect to good 2, which implies that her consumption of good 1 will not change if income changes. Hence, she will consume 10 units of good 1 after her income doubles. To verify this, notice that the conditions that characterize her optimal bundle are:

$$\frac{2}{x_1} = \frac{p_1}{p_2} \text{ and } p_1 x_1 + p_2 x_2 = m$$

which implies that $x_1 = \frac{2p_2}{p_1}$, while $x_2 = \frac{m}{p_2} - 2$ (provided that income is high enough so that the solution is interior).

8. $U(r, z) = z + 120r - r^2$. Again, the utility function is quasilinear with respect to the consumption of zinnias. This implies that the consumption of roses will not respond to changes in income, and she will devote the full addition in land to the consumption of zinnias. Hence, the correct answer is (d).

9. The only choice consistent with the statement is that his preferences are not homothetic (e). With homothetic preferences, when income increases, consumption of every good increases in the same proportion. Hence, we should expect his consumption of hamburgers to increase after the increase in his income.

10. With perfect complements, the substitution effect is zero, and thus, the income effect fully accounts for the change in quantity demanded (c). This is because the optimal bundle once one adjusts the consumer's income so that it just affords the initial optimum remains the same, hence making the substitution effect be zero.