Microeconomics I
Solutions to Problem Set 2

1. Let x_1 denote consumption of Cheerios, and x_2, consumption of Golden Grahams. Since he is always willing to give up 1 ounce of Cheerios for exactly 2 ounces of Golden Grahams, regardless of how much of each type of cereal he consumes, the absolute value of the slope if his indifference curves, and hence, his MRS is 2. Notice that since

$$MRS = \frac{MU_1}{MU_2} = 2$$

one utility function that represents his preferences is $u(x_1, x_2) = 2x_1 + x_2$.

Given his income and prices, his budget constraint is: $x_1 + 0.8x_2 = 10$. In this case, since the MRS is constant and is differenent from the price ratio, the solution will imply consumption of one of the goods being set to zero. In particular, since

$$2 = MRS = \frac{MU_1}{MU_2} > \frac{p_1}{p_2} = \frac{10}{8}$$

he will allocate all his income to consumption of x_1. Thus, the optimal bundle is $(10, 0)$. The MRS at the optimal bundle is 2, same as at any other bundle.

2.

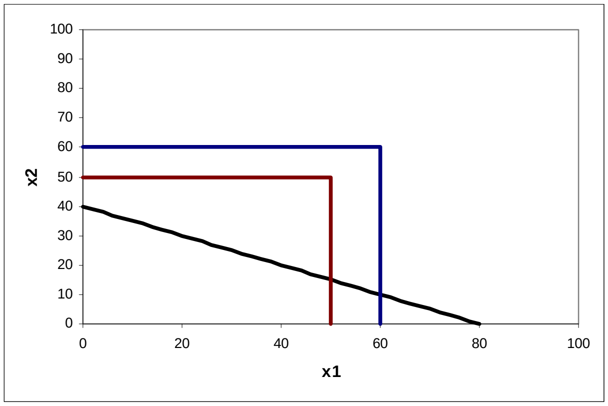
a) Since she spends a total of 400 minutes studying, and given that $m_1 = 5x_1$ and that $m_2 = 10x_2$, the set of feasible grades is:

$$5x_1 + 10x_2 \leq 400$$

Since Nancy's overall grade is the maximum score, her utility function will be

$$u(x_1, x_2) = \max\{x_1, x_2\}$$

Graphically, the budget constraint and two indifference curves look like:



b) Nancy will maximize utility choosing either $x_1 = 0$ or $x_2 = 0$. If $x_1 = 0$, then $x_2 = \frac{400}{10} = 40$, whereas if $x_2 = 0$, then $x_1 = \frac{400}{5} = 80$. Thus, the optimal combination of x_1, x_2 is $(80, 0)$. What she does is to allocate all her time to the easier exam for her, considering that the final grade is the maximum of the two.

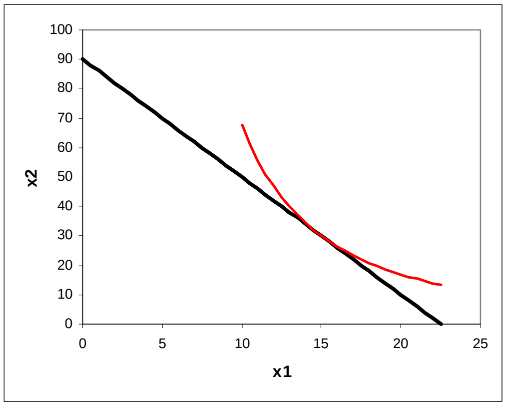
3.

a) We obtain the optimal bundle applying $MRS = \frac{p_P}{p_C}$ together with the budget constraint satisfied with equality. In this part, the budget line is $8P + 2C = 180$. Hence,

$$\begin{aligned} \frac{2C}{P} &= \frac{8}{2} = 4 \text{ and} \\ 8P + 2C &= 180 \end{aligned}$$

which implies that the optimal bundle is $(15, 30)$.

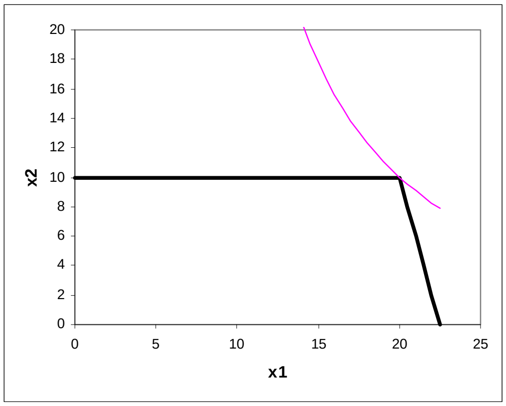
Graphically,



b) Now the budget constraint is characterized by:

$$C = \begin{cases} 10 \text{ if } P \leq 20 \\ 90 - 4P \text{ if } P > 20 \end{cases}$$

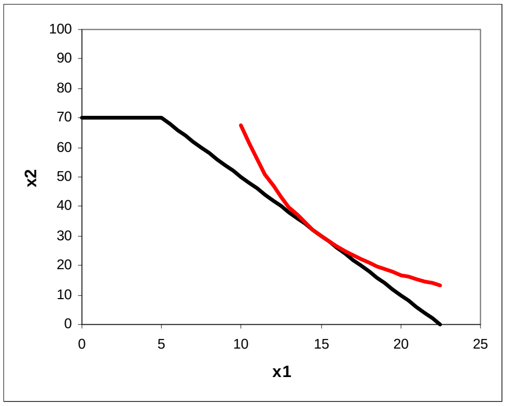
i.e. it is flat up to 20 pizzas and then it follows the same expression as in part a. This reflects the fact that you can not use coupons to increase your consumption of phone calls. The optimal bundle is $(20, 10)$. You should be able to verify that for any just affordable bundle such that $P > 20$, the MRS is less than the ratio of prices, which induces you to increase your consumption of phone calls. This is why your optimal bundle is $(20, 10)$. Graphically,



c) Now the budget constraint is characterized by:

$$C = \begin{cases} 70 \text{ if } P \leq 5 \\ 90 - 4P \text{ if } P > 5 \end{cases}$$

i.e. it is flat up to 5 pizzas and then it follows the same expression as in part a. Notice that now bundle $(15, 30)$ is affordable. Thus, this is the optimal bundle now. Graphically,

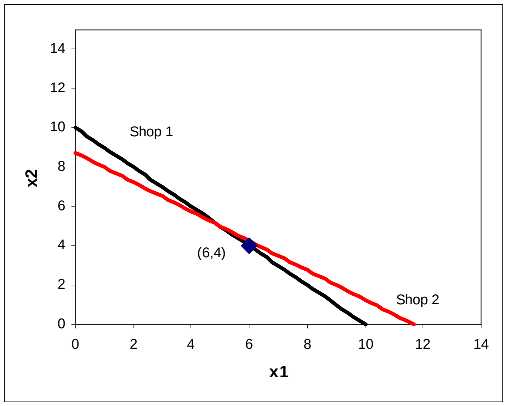


4.

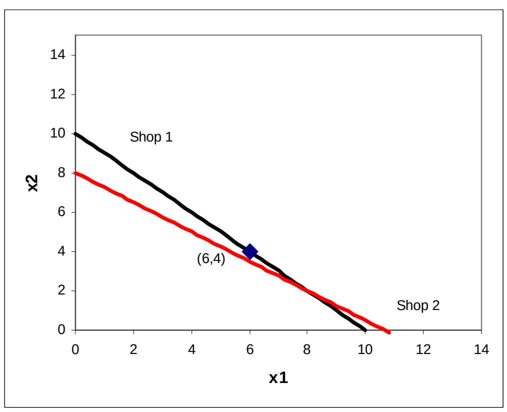
c) is correct. Notice that George's preferences are non-convex, which implies that he will choose a corner solution. The point of tangency between the budget line and an indifference curve is indeed a minimum. If he consumes good x only, he consumes bundle $\left(\frac{49}{5}, 0\right)$, whereas if he consumes good y only, he consumes bundle $\left(0, \frac{49}{11}\right)$, and $u\left(\frac{49}{5}, 0\right) > u\left(0, \frac{49}{11}\right)$.

5. The optimal bundle in shop A is $(6,4)$, where the first entry is wine and the second entry, cheese. It all depends on whether $(6,4)$ is affordable in shop B.

a) If $F = 1.5$, since bundle $(6,4)$ is more than affordable, he will choose to enter shop B.



a) If $F = 1.8$, bundle $(6,4)$ is not affordable. However, we can not conclude that he will choose shop A, since some bundles that are affordable in B are not affordable in A. Then, the conclusion depends on the exact shape of his indifference curves, and we do not have information on that. Graphically,



6. Let (x_1^S, x_2^S) be your purchases of chocolate and raspberry frozen yogurt at the shop, where prices are p_1^S, p_2^S. Now Jerry runs into Newman, who will buy from or sell to Jerry as many pints of the two flavors Jerry wishes to transact, at prices $p_1^N = p_1^S$ and $p_2^N > p_2^S$. This implies that the following bundles are affordable to Jerry:

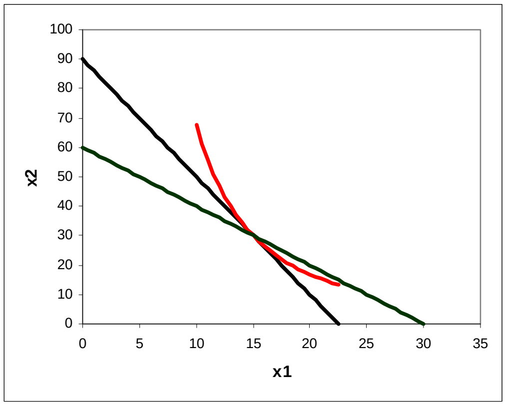
$$p_1^N x_1 + p_2^N x_2 \leq p_1^N x_1^S + p_1^N x_1^S$$

Notice that this budget line goes through Jerry's optimal bundle at the prices posted at the shop. Then, given the prices posted by Newman, bundle (x_1^S, x_2^S) is not optimal. In particular, notice that at (x_1^S, x_2^S)

$$\frac{MU_1}{p_1^N} > \frac{MU_2}{p_2^N}$$

which means that if he increases his consumption of chocolate frozen yogurt (and reduces that of raspberry), he is better off, except in the case of the two favors being perfect complements.

Graphically, the initial budget line is the black one, which is tangent to the red indifference curve. Newman's prices imply that the new budget line for Jerry is the green one, which goes through the initial optimum. The new optimum will be to the left of the initial optimum.



7. $U(C_1, C_2) = 2\ln(C_1) + \ln(C_2)$

a) If he can neither borrow nor lend, his period 2 consumption will be at most equal to his period 1 savings. Thus, his budget constraint is $C_1 + C_2 \leq 75000$. Therefore, the relevant ratio of prices is 1 and the equalities that will allow us to determine the optimal bundle are:

$$\frac{2C_2}{C_1} = 1 \text{ and } C_1 + C_2 = 75000$$

which imply that $C_1 = 50000$, $C_2 = 25000$. He will save 25000 euros for period 2.

b) If we now assume he can borrow and lend at an interest rate $r > 0$, his budget constraint becomes $C_1 + \frac{C_2}{1+r} \leq 75000$. To understand this, notice that if he wants to increase his period 2 consumption by one euro, he will have to give up $\frac{1}{1+r}$ euros of period 1 consumption (he lends $\frac{1}{1+r}$ in period 1 and what he gets back in period 2 is the principal plus interests, i.e. $(1+r)\frac{1}{1+r} = 1$). Notice that the constraint can be interpreted as the present value of consumption equals the present value of income. In particular, if he were to receive some income in period 2, it would add to the right-hand side of the constraint, but multiplied by $\frac{1}{1+r}$. Also notice that the relevant ratio of prices is $(1+r)$. Therefore, the new optimum is given by

$$\frac{2C_2}{C_1} = 1 + r \text{ and } C_1 + \frac{C_2}{1+r} = 75000$$

which implies that

$$C_2 = \frac{75000(1+r)}{3}$$

c) An increase in the interest rate would make him consume more in the second period. To see this, notice that $\frac{\partial C_2}{\partial r} > 0$. This is because an increase in the interest rate reduces the price of period 2 consumption, thus inducing him to consume more of this good.