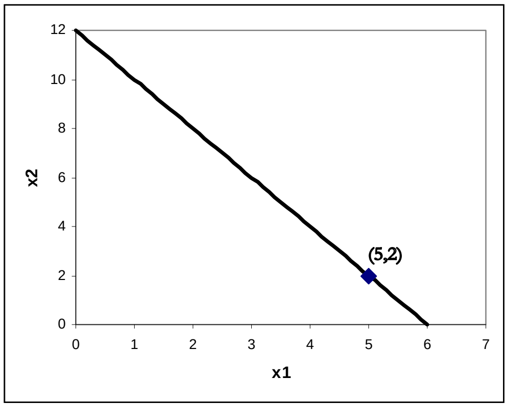
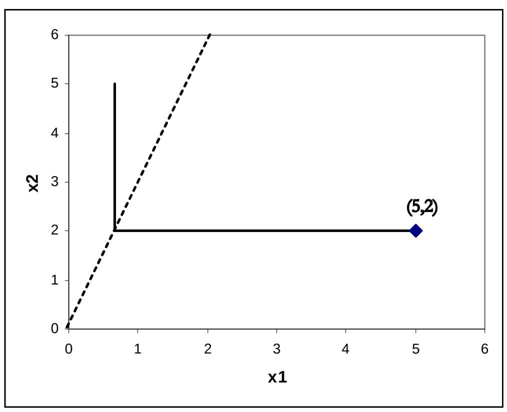
Microeconomics I
Solutions to Problem Set 1

1. Let x_1 be wine and x_2 be cheese.

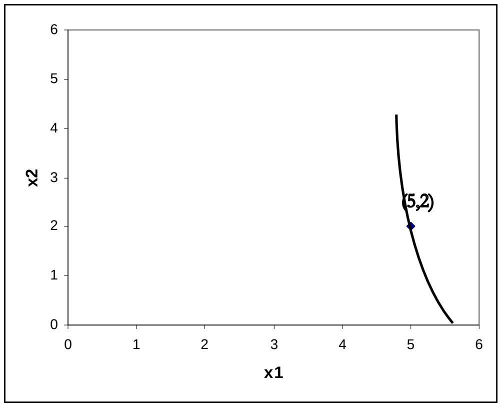
a) Since Bob is willing to give up 1 unit of wine for exactly 2 units of cheese, his marginal rate of substitution is constant. A utility function that satisfies this property is $u(x_1, x_2) = 2x_1 + x_2$. $MRS = 2$, constant. Graphically,



b) Jimmy sees wine and cheese as perfect complements. A utility function that describes these preferences is $u(x_1, x_2) = \min\{3x_1, x_2\}$. At bundle $(5, 2)$, the MRS is zero. Graphically,

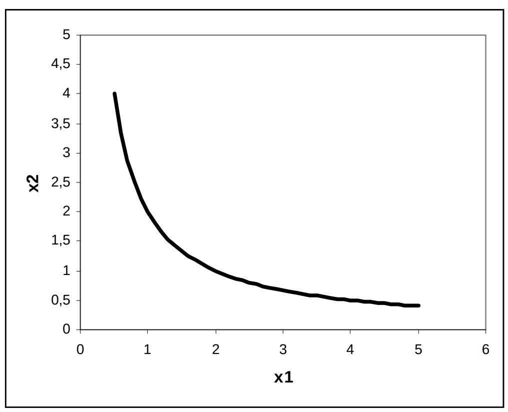


c) We can not provide an exact utility function with the information given. The fact that $MRS > 2$ implies that the slope of the indifference curve never falls below 2. Graphically,



2. We have to check that the utility function increases in x_1 and/or x_2. A sufficient condition is both marginal utilities ($MU_1 = \frac{\partial u}{\partial x_1}$ and $MU_2 = \frac{\partial u}{\partial x_2}$) being non-negative, with at least one of them being positive. All of them are monotone. Whether or not they satisfy convexity can be seen from the shape of an arbitrary indifference curve.

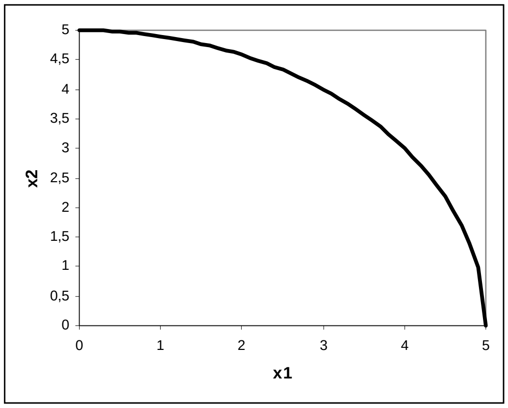
a) $MU_1 = x_2 > 0$; $MU_2 = x_1 > 0$. The shape of a typical indifference curve is:



It may be seen that it satisfies convexity.

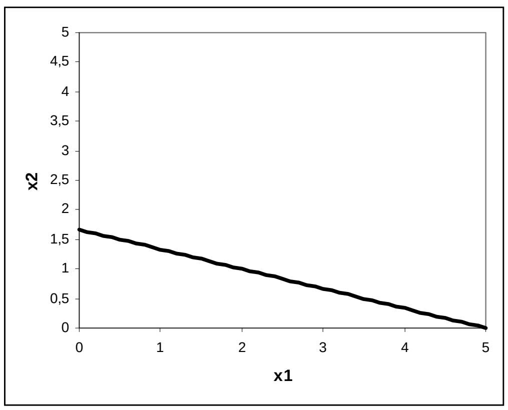
b) $MU_1 = \frac{1}{x_1} > 0$; $MU_2 = \frac{1}{x_1} > 0$. The graph is the same as in part a.

c) $MU_1 = 2x_1 > 0$; $MU_2 = 2x_2 > 0$. The shape of a typical indifference curve is:



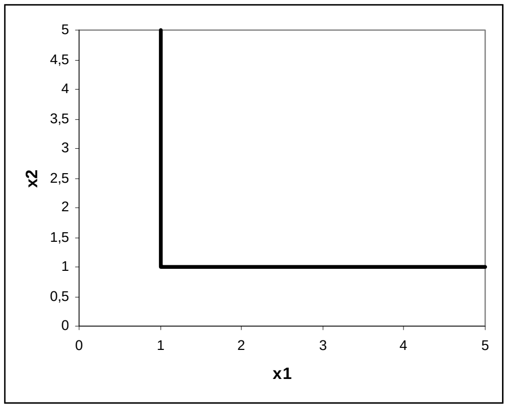
In this case, convexity is not satisfied.

d) $MU_1 = 1 > 0$; $MU_2 = 3 > 0$. The shape of a typical indifference curve is:



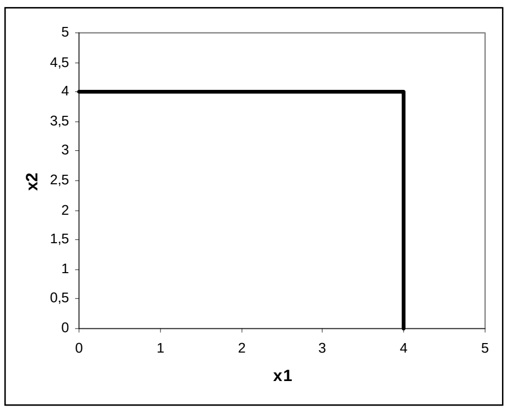
Convexity is satisfied.

e) $MU_1 = \begin{cases} 1 \text{ if } x_1 < x_2 \\ 0 \text{ if } x_1 \geq x_2 \end{cases}$; $MU_2 = \begin{cases} 1 \text{ if } x_2 < x_1 \\ 0 \text{ if } x_2 \geq x_1 \end{cases}$. The shape of a typical indifference curve is:



Convexity is satisfied.

f) $MU_1 = \begin{cases} 0 \text{ if } x_1 < x_2 \\ 1 \text{ if } x_1 \geq x_2 \end{cases}$; $MU_2 = \begin{cases} 0 \text{ if } x_2 < x_1 \\ 1 \text{ if } x_2 \geq x_1 \end{cases}$. The shape of a typical indifference curve is:



Convexity is not satisfied.

3.

a) $MU_1 = 4x_1^3 x_2^4$; $MU_2 = 4x_1^4 x_2^3$; $MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$.

b) $MU_1 = \frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{1}{4}}$; $MU_2 = \frac{1}{4} x_1^{\frac{1}{4}} x_2^{-\frac{3}{4}}$; $MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$. Hence, this utility function represents the same preferences as that in part a.

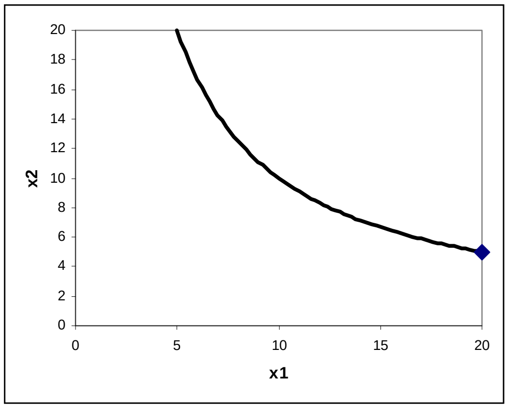
c) $MU_1 = \frac{1}{x_1}$; $MU_2 = \frac{1}{x_2}$; $MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$.

d) $MU_1 = 5$; $MU_2 = 3$; $MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{5}{3}$.

e) $MU_1 = \begin{cases} 1 \text{ if } x_1 < x_2 \\ 0 \text{ if } x_1 \geq x_2 \end{cases}$; $MU_2 = \begin{cases} 1 \text{ if } x_2 < x_1 \\ 0 \text{ if } x_2 \geq x_1 \end{cases}$; $MRS_{1,2} = \frac{MU_1}{MU_2} = \begin{cases} \infty \text{ if } x_1 < x_2 \\ \text{does not exist if } x_1 = x_2 \\ 0 \text{ if } x_1 > x_2 \end{cases}$.

4.

a) The indifference curve is:



b) The set of bundles that Charlie prefers to $(10, 15)$ is the set of bundles on or above the indifference curve that contains $(10, 15)$. The set of bundles such that Charlie weakly prefers $(20, 5)$ to these bundles is the set of bundles on or below the indifference curve that contains $(20, 5)$.

c) True.

d) True.

e) True.

f) False.

g) True.

h) The expression for the MRS (always in absolute value) is $\frac{x_B}{x_A}$. Thus, the MRS at these points is 1, 4, and $\frac{1}{4}$, respectively.

i) 4.

j) $\frac{1}{4}$.

k) They exhibit decreasing (in absolute value) MRS. This implies convexity of preferences.