

Microeconomics I

Problem Set 1

1. Assume there are only two goods: wine and cheese, and three different individuals, whose preferences can be described as follows:

Bob likes both wine and cheese. He has a constant marginal rate of substitution (wine for cheese): he is always willing to give up 1 unit of wine for exactly 2 units of cheese.

Jimmy demands specific proportions of wine and cheese: 1 part wine for every 3 parts cheese, and any wine or cheese in excess of that proportion is worth nothing to him.

Peter likes both wine and cheese and has a declining marginal rate of substitution. However, at every bundle, his marginal rate of substitution is greater than 2.

a) Find a utility function that describes Bob's preferences. Draw the indifference curve that contains bundle $(5, 2)$ and write the equation of that indifference curve, given the utility function that you provided. What is the MRS at that point?

b) Find a utility function that describes Jimmy's preferences. Draw the indifference curve that contains bundle $(5, 2)$ and write the equation of that indifference curve, given the utility function that you provided. What is the MRS at that point?

c) Sketch Peter's indifference curve containing bundle $(5, 2)$. How does the fact that $MRS > 2$ translate into your graph?

2. Determine whether the preferences described by the following utility functions satisfy monotonicity. Also show graphically whether they satisfy convexity:

- a) $U(x_1, x_2) = x_1 x_2$
- b) $U(x_1, x_2) = 2 + \ln x_1 + \ln x_2$
- c) $U(x_1, x_2) = x_1^2 + x_2^2$
- d) $U(x_1, x_2) = x_1 + 3x_2$
- e) $U(x_1, x_2) = \min \{x_1, x_2\}$
- f) $U(x_1, x_2) = \max \{x_1, x_2\}$

3. For each of the following utility functions, calculate MU_1 , MU_2 , $MRS_{1,2}$ (MRS in absolute value):

- a) $U(x_1, x_2) = x_1^4 x_2^4$
- b) $U(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$
- c) $U(x_1, x_2) = \ln x_1 + \ln x_2$
- d) $U(x_1, x_2) = 5x_1 + 3x_2$
- e) $U(x_1, x_2) = \min \{x_1, x_2\}$

4. Charlie likes both apples and bananas. He consumes nothing else. The consumption bundle where Charlie consumes x_A bushels of apples per year and x_B bushels of bananas per year is written as (x_A, x_B) . Last year, Charlie consumed 20 bushels of apples and 5 bushels of bananas. It happens that the set of consumption bundles (x_A, x_B) such that Charlie is indifferent between (x_A, x_B) and $(20, 5)$ is the set of all bundles such that $x_B = \frac{100}{x_A}$. The set of bundles (x_A, x_B) such that Charlie is just indifferent between (x_A, x_B) and bundle $(10, 15)$ is the set of bundles such that $x_B = \frac{150}{x_A}$.

(a) Plot several points that lie on the indifference curve that passes through point $(20, 5)$, and sketch this curve. Do the same for the indifference curve passing through point $(10, 15)$.

(b) Draw the set of commodity bundles that Charlie weakly prefers to bundle $(10, 15)$. Shade in the set of commodity bundles such that Charlie weakly prefers $(20, 5)$ to these bundles.

For each of the following statements about Charlie's preferences, write "true" or "false".

(c) $(30, 5) \sim (10, 15)$.

(d) $(10, 15) \succ (20, 5)$.

(e) $(20, 5) \succ (10, 10)$.

(f) $(24, 4) \succ (11, 9.1)$.

(g) $(11, 14) \succ (2, 49)$.

(h) Remember that Charlie's indifference curve through point $(10, 10)$ has the equation $x_B = \frac{100}{x_A}$. Find Charlie's marginal rate of substitution at points $(10, 10)$, $(5, 20)$ and $(20, 5)$.

(i) What is his marginal rate of substitution at point $(5, 20)$?

(j) What is his marginal rate of substitution at point $(20, 5)$?

(k) Do the indifference curves you have drawn for Charlie exhibit diminishing marginal rate of substitution?